CAREERS360

PRACTICE Series

RBSE Class 12

Mathematics

Previous Year Questions with Detailed Solution

RBSE Class 12 Maths Question with Solution - 2024

SECTION-A

1. Multiple Choice Questions :

i) Let R be the relation in the set $\{1,2,3,4\}$ given by $R=\{(1,2),(2,2),(1,1),$ $(4,4),(1,3),(3,3),(3,2)\}$ choose the correct answer in the given options.
A) R is reflexive and symmetric but not transitive.
B) R is reflexive and transitive but not symmetric.
C) R is symmetric and transitive but not reflexive.
D) R is an equivalence relation.

Solution:

Let's analyze the given relation $R=\{(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$ on the set $\{1,2,3,4\}$.
- Reflexive: A relation is reflexive if (a,a) is in R for every element a in the set. For the set $\{1,2,3,4\}$, the reflexive pairs should be $(1,1),(2,2),(3,3),(4,4)$.
- In R, we have $(1,1),(2,2),(3,3),(4,4)$, so the relation is reflexive.
- Symmetric: A relation is symmetric if $(a,b)\in R$ implies $(b,a)\in R$. Let's check:
- $(1,2)\in R$, but $(2,1)\notin R$.
- Therefore, the relation is not symmetric.
- Transitive: A relation is transitive if $(a,b)\in R$ and $(b,c)\in R$ implies $(a,c)\in R$. Let's check:
- $(1,2)\in R$ and $(2,2)\in R$, but there is no need for $(1,2)$ to satisfy transitivity as (a,a) is already in the relation.
- Similarly, no other violation occurs.
- Therefore, the relation is transitive.

Conclusion:
The relation is reflexive and transitive, but not symmetric. Therefore, the correct option is:
B) R is reflexive and transitive but not symmetric.

ii) The principal value of $\operatorname{cosec}^{-1}(2)$ is :-
A) $\frac{\pi}{2}$
B) $\frac{\pi}{3}$
C) $\frac{\pi}{6}$
D) π

Solution:

To find the principal value of $\operatorname{cosec}^{-1}(2)$, we need to find an angle θ such that:

$$\text{cosec}(\theta) = 2$$

This means:

$$\frac{1}{\sin(\theta)} = 2 \implies \sin(\theta) = \frac{1}{2}$$

The angle θ for which $\sin(\theta) = \frac{1}{2}$ in the principal range of the cosecant inverse function (which is $[0, \pi]$, excluding $\frac{\pi}{2}$) is:

$$\theta = \frac{\pi}{6}$$

Thus, the principal value of $\text{cosec}^{-1}(2)$ is:

$$\frac{\pi}{6}$$

The correct answer is:

$$\text{C) } \frac{\pi}{6}$$

iii) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ then, $(2A - B)$ will be :

A) $\begin{bmatrix} 1 & -5 & 2 \\ 5 & 6 & 0 \end{bmatrix}$

) $\begin{bmatrix} 5 & 6 & 0 \\ 1 & -5 & 3 \end{bmatrix}$

C) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

D) $\begin{bmatrix} -1 & 3 & 5 \\ 5 & 6 & 0 \end{bmatrix}$

Solution:

We are given matrices A and B :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

Step 1: Compute $2A$

Multiply each element of matrix A by 2 :

$$2A = 2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix}$$

Step 2: Compute $2A - B$

Subtract matrix B from $2A$:

$$2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

Perform the subtraction element-wise:

$$2A - B = \begin{bmatrix} 2-3 & 4-(-1) & 6-3 \\ 4-(-1) & 6-0 & 2-2 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$$

iv) If $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$; then the value of x is :

A) 2
B) 0
C) 1
D) -1

Solution:

Given the equation:

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Step 1: Compute the determinant on the left-hand side (LHS)

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = (2 \times 5) - (3 \times 4) = 10 - 12 = -2$$

Step 2: Compute the determinant on the right-hand side (RHS)

$$\begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix} = (x \times 5) - (3 \times 2x) = 5x - 6x = -x$$

Step 3: Set up the equation

$$-2 = -x$$

Solving for x :

$$x = 2$$

Final Answer:
The correct value of x is A) 2 .

v) If $2x + 8y = \sin x$, then $\frac{dy}{dx}$ is :

A) $\frac{\sin x - 2}{8}$

B) $\frac{\cos x - 2}{8}$

C) $\frac{\cos x + 2}{2}$

D) $\frac{\cos x + 2}{3}$

Solution:

Step 1: Differentiate both sides of the equation with respect to x
- The derivative of $2x$ with respect to x is 2 .
- The derivative of $8y$ with respect to x is $8\frac{dy}{dx}$ (using the chain rule).
- The derivative of $\sin x$ with respect to x is $\cos x$.

Thus, differentiating both sides gives:

$$2+8\frac{dy}{dx}=\cos x$$

Step 2: Solve for $\frac{dy}{dx}$
To isolate $\frac{dy}{dx}$, subtract 2 from both sides:

$$8\frac{dy}{dx}=\cos x-2$$

Now divide both sides by 8 :

$$\frac{dy}{dx}=\frac{\cos x-2}{8}$$

Final Answer:
The correct answer is:

$$\left\lceil\frac{\overline{\downarrow jx-2}}{8}\right\rceil$$

vi) In which of the following intervals is $y=x^2e^{-x}$ increasing?

A) $(1,0)$

B) - $(2,0)$

C) $(2,-\infty)$

D) $(0,2)$

Solution:

Step 1: Compute the derivative of $y=x^2e^{-x}$
Using the product rule:

$$\frac{d}{dx}\left(x^2e^{-x}\right)=\frac{d}{dx}\left(x^2\right)\cdot e^{-x}+x^2\cdot\frac{d}{dx}\left(e^{-x}\right)$$

The derivative of x^2 is $2x$, and the derivative of e^{-x} is $-e^{-x}$. Therefore:

$$\frac{d}{dx}\left(x^2e^{-x}\right)=2xe^{-x}+x^2\left(-e^{-x}\right)$$
$$\frac{d}{dx}\left(x^2e^{-x}\right)=e^{-x}\left(2x-x^2\right)$$

Step 2: Analyze the sign of the derivative
For $y=x^2e^{-x}$ to be increasing, the derivative $e^{-x}\left(2x-x^2\right)$ must be positive:

$$e^{-x}\left(2x-x^2\right)>0$$

Since e^{-x} is always positive, we only need to solve:

$2x - x^2 > 0$
Factoring:

$x' \downarrow -x) > 0$

Step 3: Solve the inequality $x(2-x) > 0$
This inequality holds when $0 < x < 2$. Therefore, the function is increasing in the interval $(0, 2)$.
Final Answer:
The correct answer is:

$D)(0, 2)$

vii) The value of $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

A) $\sec x - x + c$

B) $\sec x \tan x + c$

C) $\tan x + x^2 + c$

D) $\tan x - x + c$

Solution:

Step 1: Simplify the integrand
Using the trigonometric identities:

$$\sec^2 x = \frac{1}{\cos^2 x}, \quad \csc^2 x = \frac{1}{\sin^2 x}$$

The integrand simplifies as follows:

$$\frac{\sec^2 x}{\csc^2 x} = \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

Step 2: Evaluate the integral
We now need to compute the integral of $\tan^2 x$:

$$\int \tan^2 x dx$$

Using the identity:

$$\tan^2 x = \sec^2 x - 1$$

The integral becomes:

$$\int (\downarrow^2 x - 1) dx$$

Step 3: Solve the integral
We can split this into two integrals:

$$\int \sec^2 x dx - \int 1 dx$$

The integral of $\sec^2 x$ is $\tan x$, and the integral of 1 is x. Therefore:

$\int \sec^2 x dx - \int 1 dx = \tan x - x + C$
Final Answer:
The value of the integral is:

$\tan x - x + C$

viii) The area of the region bounded by the circle $x^2 + y^2 = 9$ in the first quadrant is:

A) 9π

B) $\frac{3\pi}{4}$

C) $\frac{9\pi}{4}$

D) 3π

Solution:

We are given the equation of the circle:

$x^2 + y^2 = 9$
This represents a circle with center at $(0,0)$ and radius $r = 3$. We are asked to find the area of the region bounded by the circle in the first quadrant.

Step 1: Area of the full circle
The area of a circle is given by the formula:

$$\text{Area of the full circle } = \pi r^2 = \pi(3)^2 = 9\pi$$
Step 2: Area in the first quadrant
Since the first quadrant represents one-fourth of the total area of the circle, we divide the total area by 4 :

$$\text{Area in the first quadrant } = \frac{1}{4} \times 9\pi = \frac{9\pi}{4}$$
Final Answer:
The area of the region bounded by the circle in the first quadrant is:

$\frac{9\pi}{4}$

ix) Area of the region bounded by the curve $y^2 = 4x, y$ - axis and the line $y = 3$ is:

A) 2

B) $\frac{9}{4}$

C) $\frac{9}{8}$

D) $\frac{9}{2}$

Solution:

Step 1: Express x in terms of y
From the given curve $y^2=4x$, we can solve for x as:

$$x=\frac{y^2}{4}$$

Step 2: Set up the integral for the area
The area we want to find is bounded by the curve $x=\frac{y^2}{4}$, the y-axis (which is $x=0$), and the line $y=3$. The limits of integration are from $y=0$ to $y=3$.

The area between the curve and the y-axis can be written as:

$$\text{Area }=\int_0^3\left(\frac{y^2}{4}-0\right)dy$$

Step 3: Compute the integral

$$\int_0^3 \frac{y^2}{4} dy=\frac{1}{4} \int_0^3 y^2 dy$$

Now, compute the integral of y^2 :

$$\int {}^{2}_{\downarrow} dy=\frac{y^3}{3}$$

Substitute this back:

$$\frac{1}{4}\left[\frac{y^3}{3}\right]_0^3=\frac{1}{4} \times \frac{27}{3}=\frac{1}{4} \times 9=\frac{9}{4}$$

Final Answer:
The area of the region bounded by the curve, the y-axis, and the line $y=3$ is:

$$\boxed{\frac{9}{4}}$$

x) The degree of the differential equation $\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$ is

A) 1

B) 2

C) 3

D) 4

Solution:

We are given the differential equation:

$$\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$$

Step 1: Identify the order of the differential equation
The order of a differential equation is the highest order derivative present in the equation. In this case, the highest order derivative is $\frac{d^2s}{dt^2}$, which is a second-order derivative. Thus, the order of the equation is 2 .

Step 2: Define the degree of the differential equation
The degree of a differential equation is the power of the highest order derivative, provided the equation is a polynomial in derivatives. Here, the highest order derivative is $\frac{d^2 s}{dt^2}$, and it appears to the first power (since it is multiplied by $3s$, but not raised to any power).

Thus, the degree of the differential equation is 1.
Final Answer:
The degree of the differential equation is:
1

xi) If $\vec{a}$ is a nonzero vector of magnitude ' a ' and λ a nonzero scalar, then $\lambda \vec{a}$ is unit vector if

A) $\lambda=1$

B) $\lambda=-1$

C) $a=|\lambda|$

D) $a=\frac{1}{|\lambda|}$

Solution:

To find when $\lambda \vec{a}$ is a unit vector, we need the magnitude of $\lambda \vec{a}$ to be 1 . Here's the reasoning:
Step 1: Magnitude of $\lambda \vec{a}$
The magnitude of the vector $\lambda \vec{a}$ is given by:

$$|\lambda \vec{a}|=|\lambda||\vec{a}|$$

Let the magnitude of $\vec{a}$ be a, so:

$$|\lambda \vec{a}|=|\lambda| a$$

Step 2: Condition for a unit vector
For $\lambda \vec{a}$ to be a unit vector, its magnitude must be 1 :

$$|\lambda| a=1$$

Solving for a, we get:

$$\downarrow=\frac{1}{|\lambda|}$$

xii)The direction cosine of y-axis is :

A) $0,0,0$

B) $1,0,0$

C) $0,1,0$

D) $0,0,1$
Solution:

The direction cosines of a vector are the cosines of the angles that the vector makes with the coordinate axes.

For the vector aligned along the y-axis:
- It makes an angle of 90° with the x-axis and the z-axis, so $\cos\left(90^{\circ}\right)=0$ for these axes.
- It makes an angle of 0° with the y-axis, so $\cos\left(0^{\circ}\right)=1$ for the y-axis.

Thus, the direction cosines for the y-axis are:

$(0,1,0)$

xiii) The direction cosines of the line passing through the two points $(-2,4,-5)$ and $(1,2,3)$ is :

A) $\frac{3}{\sqrt{70}}, \frac{2}{\sqrt{70}}, \frac{8}{\sqrt{70}}$

B). $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

C) $\frac{2}{\sqrt{77}}, \frac{-3}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

D) $\frac{8}{\sqrt{13}}, \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}}$

Solution:

Step 1: Find the direction ratios of the line
The direction ratios are the differences between the corresponding coordinates of the two points:

$$\text{Direction ratios } =(1-(-2), 2-4, 3-(-5))=(3,-2,8)$$

So, the direction ratios of the line are $3,-2,8$.
Step 2: Find the magnitude of the direction ratios
The magnitude is given by:

$$\sqrt{3^2+(-2)^2+8^2}=\sqrt{9+4+64}=\sqrt{77}$$

Step 3: Find the direction cosines
The direction cosines are the direction ratios divided by the magnitude:

$$\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

xiv) If $\mathrm{P}(\mathrm{A})=0.8, \mathrm{P}(\mathrm{B})=0.5$ and $\mathrm{P}(\mathrm{B}/\mathrm{A})=0.4$, then the value of $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$ is:

A) 0.32

B) 0.20

C) 0.40

D) 0.64

Solution:

We are given the following information:
- $\mathrm{P(A)=0.8}$
- $\mathrm{P(B/A)=0.4}$

We need to find $P(A \cap B)$.
Step 1: Use the conditional probability formula
The formula for conditional probability is:

$$\mathrm{P(\,B/A)=\frac{P(\,A\cap B)}{P(\,A)}}$$

We can rearrange this formula to solve for $\mathrm{P(A \cap B)}$:

$$\mathrm{P(\,A\cap\,B)=P(\,B/A)\times P(\,A)}$$

Step 2: Substitute the values
Substitute $\mathrm{P(B/A)=0.4}$ and $\mathrm{P(A)=0.8}$ into the formula:

$$\mathrm{P(\,A\cap\,B)_{\downarrow}0.4\times 0.8=0.32}$$

xv) Two cards are drawn at random and without replacement from a pack of 52 playing cards, then the probability that both the cards are black is :

A) $\frac{26}{52}$

B) $\frac{52}{102}$

C) $\frac{25}{51}$

D) $\frac{1}{2}$

Solution:

Step 1: Total number of black cards
A standard deck of 52 playing cards consists of 26 black cards (13 spades and 13 clubs).
Step 2: Probability of drawing two black cards
The probability of drawing the first black card is:

$$\frac{26}{52}$$

After drawing one black card, there are now 25 black cards left, and the total number of remaining cards is 51 . So, the probability of drawing a second black card is:

$$\frac{25}{51}$$

Step 3: Multiply the probabilities
The probability that both cards are black is the product of the individual probabilities:

$$\frac{26}{52}\times\frac{25}{51}=\frac{1}{2}\times\frac{25}{51}=\frac{25}{102}$$

2. Fill in the blanks :

i) $\sin^{-1} x$ **is a function whose domain is**

Solution:

The function $\sin^{-1} x$, also known as the arcsine function, is defined as the inverse of the sine function. For $\sin^{-1} x$ to be valid, the input value x must lie within the range of the sine function, which is:

$[-1, 1]$
Therefore, the domain of $\sin^{-1} x$ is:

$x \in [-1, 1]$

ii) The value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is

Solution:

Step 1: Understanding the range of $\sin^{-1}$
The range of $\sin^{-1} x$ (also known as arcsine) is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This means that the result of $\sin^{-1} y$ will always be an angle within this interval.

Step 2: Calculate $\sin \frac{2\pi}{3}$
We know that:

$\sin \frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin \frac{\pi}{3}$
Since $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$, we have:

$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
Step 3: Apply the $\sin^{-1}$ function
Now, we need to find $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$, but we must ensure the answer is in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
The angle whose sine is $\frac{\sqrt{3}}{2}$ and lies in this range is:

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

Step 4: Consider the behavior of inverse sine
Since the original angle $\frac{2\pi}{3}$ is not within $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we map it to an equivalent angle in this range. The equivalent angle for $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is:

$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

iii) The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

Solution:

Step 1: Understanding the range of $\cos^{-1}$
The principal value of $\cos^{-1} x$ lies in the range $[0, \pi]$. So the result of $\cos^{-1} y$ must be an angle in this interval.

Step 2: Identify the angle
We know that:

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
Thus:

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

iv) If $y = \cos \sqrt{x}$, then the value of $\frac{dy}{dx}$ will be

Solution:

We know that the derivative of $\cos u$ with respect to u is $-\sin u$. In this case, $u = \sqrt{x}$, so we need to use the chain rule.
1. Differentiate $\cos \sqrt{x}$ with respect to $\sqrt{x}$:

$\frac{d}{d(\sqrt{x})}(\cos \sqrt{x}) = -\sin \sqrt{x}$

2. Now, differentiate $\sqrt{x}$ with respect to x :

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Step 2: Combine the results
By the chain rule, we multiply these results:

$\frac{dy}{dx} = -\sin \sqrt{x} \times \frac{1}{2\sqrt{x}}$
Final Answer:
The value of $\frac{dy}{dx}$ is:

$\frac{-\sin \sqrt{x}}{2\sqrt{x}}$

v) The rate of change of the area of a circle with respect to its radius r at $r = 3$ cm is ______.

Solution:

The area A of a circle is given by the formula:

$A = \pi r^2$
We are asked to find the rate of change of the area with respect to the radius r, which means we need to compute $\frac{dA}{dr}$, the derivative of A with respect to r.

Step 1: Differentiate the area with respect to r

$\frac{dA}{dr} = \frac{d}{dr}\left(\pi r^2\right) = 2\pi r$
Step 2: Substitute $r = 3$ cm
Now, substitute $r = 3$ cm into the expression for $\frac{dA}{dr}$:

$\frac{dA}{dr} = 2\pi \times 3 = 6\pi \text{ cm}^2/\text{cm}$
Final Answer:
The rate of change of the area of the circle with respect to its radius at $r = 3$ cm is:

$6\pi \text{ cm}^2/\text{cm}$

vi) The numbers of arbitrary constants present in the particular solution of a differential equation of third order are _____.

Solution:

The particular solution of a differential equation contains no arbitrary constants, regardless of the order of the differential equation. Arbitrary constants are only present in the general solution, where the solution includes constants determined by initial or boundary conditions.

Therefore, for a particular solution of a third-order differential equation, the number of arbitrary constants is 0.

vii) A vector whose initial and terminal points coincide, is called

Solution:

A vector whose initial and terminal points coincide is called a zero vector (or null vector).

A zero vector has:
- Magnitude equal to zero.
- No specific direction.

The zero vector is denoted as:

$\vec{0}$
This represents a vector with all its components equal to zero.

3 i) Find the value of determinant $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

Solution:

Step 1: Use the formula for a 2×2 determinant
The formula for a 2×2 determinant is:

$$\text{Determinant } = (a \cdot d) - (b \cdot c)$$

where the matrix is:

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
Step 2: Identify the values of a, b, c, and d
For our matrix:

$a = \cos\theta, \quad b = -\sin\theta, \quad c = \sin\theta, \quad d = \cos\theta$

Step 3: Apply the determinant formula

Now, apply the formula:

$$\begin{aligned} \text{Determinant} &= (\cos\theta \cdot \cos\theta) - (-\sin\theta \cdot \sin\theta) \\ &= \cos^2\theta + \sin^2\theta \end{aligned}$$

Step 4: Use the Pythagorean identity

Using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$, we get:

1

ii) Find equation of line joining $(1,2)$ and $(3,6)$ using determinants.

Solution:

Step 1: Use the formula for the equation of a line passing through two points

The formula for the equation of a line passing through points (x_1, y_1) and (x_2, y_2) is:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Step 2: Substitute the given points

For the points $(1,2)$ and $(3,6)$, we substitute $(x_1, y_1) = (1,2)$ and $(x_2, y_2) = (3,6)$:

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Step 3: Expand the determinant

Now, expand the determinant:

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = x\begin{vmatrix} 2 & 1 \\ 6 & 1 \end{vmatrix} - y\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}$$

Step 4: Compute the 2×2 determinants

1. $\begin{vmatrix} 2 & 1 \\ 6 & 1 \end{vmatrix} = (2)(1) - (1)(6) = 2 - 6 = -4$
2. $\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = (1)(1) - (1)(3) = 1 - 3 = -2$
3. $\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = (1)(6) - (2)(3) = 6 - 6 = 0$

Step 5: Substitute back into the equation

Substituting the values back into the expansion:

$x(-4) - y(-2) + 1(0) = 0$

$-4x + 2y = 0$

Step 6: Rearrange the equation

Solving for y :

$$2y = 4x \quad \Rightarrow \quad y = 2x$$

Final Answer:

The equation of the line passing through the points $(1, 2)$ and $(3, 6)$ is:

$$y \overset{\downarrow}{=} 2x$$

iii) The radius of a circle is increasing uniformly at the rate of $3 \mathrm{~cm}/\mathrm{s}$. Find tt rate at which the area of the circle is increasing when the radius is 10 cm .

Solution:

Step 1: Area of a circle

The formula for the area of a circle is:

$$A = \pi r^2$$

Step 2: Differentiate with respect to time t

We want to find the rate of change of the area with respect to time, i.e., $\frac{dA}{dt}$. Using the chain rule, we differentiate the area formula:

$$\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right) = 2\pi r \frac{dr}{dt}$$

Step 3: Substitute the known values

We know that $\frac{dr}{dt} = 3 \mathrm{~cm}/\mathrm{s}$ and the radius $r = 10 \mathrm{~cm}$. Substituting these values into the equation:

$$\frac{dA}{dt} = 2\pi\left(\overset{\neg}{\downarrow}\right)(3) = 60\pi \mathrm{~cm}^2/\mathrm{s}$$

iv) Prove that the logarithmic function is increasing on $(0, \infty)$.

Solution:

Step 1: Find the derivative of $f(x) = \ln(x)$

To determine whether a function is increasing, we need to examine its derivative. The derivative of $\ln(x)$ is:

$$f'(x) = \frac{d}{dx}\ln(x) = \frac{1}{x}$$

Step 2: Analyze the derivative

For $x > 0$ (since the logarithmic function is only defined for positive x):

- $f'(x) = \frac{1}{x}$
- Since $x > 0, \frac{1}{x} > 0$ for all $x > 0$.

Step 3: Conclusion

Because $f'(x) > 0$ for all $x > 0$, the function $f(x) = \ln(x)$ is increasing on the interval $(0, \infty)$.

Thus, the logarithmic function is increasing on $(0, \infty)$.

v) Evaluate $\int(2x - 3\cos x + e^x)dx$

Solution:

Step 1: Break the integral into parts
We can split the given integral into three simpler integrals:

$$\int(2x - 3\cos x + e^x)dx = \int 2x dx - \int 3\cos x dx + \int e^x dx$$

Step 2: Evaluate each integral
1. First Integral: $\int 2x dx$

Using the power rule for integrals:

$$\int 2x dx = x^2 + C_1$$

2. Second Integral: $\int 3\cos x dx$

The integral of $\cos x$ is $\sin x$, so:

$$\int 3\cos x dx = 3\sin x + C_2$$

3. Third Integral: $\int e^x dx$

The integral of e^x is e^x, so:

$$\int e^x dx = e^x + C_3$$

Step 3: Combine the results
Now, combine all three integrals:

$$\int(2x - 3\cos x + e^x)dx = x^2 - 3\sin x + e^x + C$$

where C is the constant of integration, combining C_1, C_2, and C_3.

vi) Evaluate $\int \frac{\sin x}{1+\cos x} dx$

Solution:

Step 1: Substitution
We will use the substitution method. Let:

$$u = 1 + \cos x$$

Now, differentiate u with respect to x :

$$\frac{du}{dx} = -\sin x$$

Thus:

$$du = -\sin x dx$$

This allows us to rewrite the integral as:

$I=\int \frac{\sin x}{1+\cos x} d x=-\int \frac{d u}{u}$
Step 2: Integrate
The integral of $\frac{1}{u}$ is $\ln |u|$:

$-\int \frac{1}{u} d u=-\ln |u|+C$

Step 3: Substitute back $u=1+\cos x$
Substitute $u=1+\cos x$ back into the expression:

$I=-\ln |1+\cos x|+C$
Final Answer:

$\int \frac{\sin x}{1+\cos x} d x=-\ln |1+\cos x|+C$

vii) Verify that the function $y=e^x+1$ is a solution of the differential equation $y''-y'=0$.

Solution:

Step 1: Find the first derivative y'
The function is $y=e^x+1$. The derivative of e^x is e^x, and the derivative of a constant (1) is 0 .
Therefore, the first derivative is:

$y'=e^x$
Step 2: Find the second derivative y''
The derivative of $y'=e^x$ is:

$y''=e^x$
Step 3: Substitute into the differential equation
Now, substitute y' and y'' into the given differential equation $y''-y'=0$:

$e^x-e^x=0$
This simplifies to:

$0=0$

viii) Find the position vector of the mid point of the vector joining the point: $\mathrm{P}(2,3,4)$ and $\mathrm{Q}(4,1,-2)$.

Solution:

Step 1: Formula for the midpoint
The midpoint M of two points $P\left(x_1, y_1, z_1\right)$ and $Q\left(x_2, y_2, z_2\right)$ is given by:

$M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
Step 2: Apply the formula
For the points $P(2,3,4)$ and $Q(4,1,-2)$, the coordinates of the midpoint are:

$M=\left(\frac{2+4}{2}, \frac{3+1}{2}, \frac{4+(-2)}{2}\right)$
Simplifying:

$M = \left(\frac{6}{2}, \frac{4}{2}, \frac{2}{2}\right) = (3, 2, 1)$
Final Answer:
The position vector of the midpoint is $\vec{M} = (2, 2, 1)$.

ix) Find the position vector of the mid point of the vector joining the point: $\mathrm{P}(2, 3, 4)$ **and** $\mathrm{Q}(4, 1, -2)$.

Solution:

Step 1: Formula for the midpoint
The midpoint M of two points $P\left(x_1, y_1, z_1\right)$ and $Q\left(x_2, y_2, z_2\right)$ is given by:

$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
Step 2: Apply the formula
For the points $P(2, 3, 4)$ and $Q(4, 1, -2)$, the coordinates of the midpoint are:

$M = \left(\frac{2+4}{2}, \frac{3+1}{2}, \frac{4+(-2)}{2}\right)$
Simplifying:

$M = \left(\frac{6}{2}, \frac{4}{2}, \frac{2}{2}\right) = (3, 2, 1)$
Final Answer:
The position vector of the midpoint is $\vec{M} = (3, 2, 1)$.

x) Evaluate the product $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

Solution:

Step 1: Expand the expression
We use the distributive property of the dot product:

$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}) = 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$
Step 2: Simplify each term
- First term: $3\vec{a} \cdot 2\vec{a} = 6\vec{a} \cdot \vec{a} = 6|\vec{a}|^2$
- Second term: $3\vec{a} \cdot 7\vec{b} = 21\vec{a} \cdot \vec{b}$
- Third term: $-5\vec{b} \cdot 2\vec{a} = -10\vec{b} \cdot \vec{a} = -10\vec{a} \cdot \vec{b}$
- Fourth term: $-5\vec{b} \cdot 7\vec{b} = -35\vec{b} \cdot \vec{b} = -35|\vec{b}|^2$

Step 3: Combine the terms
Now, we combine the terms:

$6|\vec{a}|^2 + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$
Simplifying further:

$6|\vec{a}|^2 + (21 - 10)\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$
$6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$

SECTION-B

4. Prove that the relation R in the set $\{1,2,3\}$ given by $R=\{(1,2),(2,1)\}$ is symmetric but neither reflexive nor transitive.

Solution:

Let's analyze the relation $R=\{(1,2),(2,1)\}$ on the set $\{1,2,3\}$ to prove that it is symmetric but neither reflexive nor transitive.
1. Reflexive

A relation is reflexive if for every element a in the set, (a, a) is in the relation. The set is $\{1,2,3\}$, so we need the pairs $(1,1),(2,2),(3,3)$ to be in R.
- However, none of the pairs $(1,1),(2,2),(3,3)$ are present in R.
- Therefore, R is not reflexive.
2. Symmetric

A relation is symmetric if for every pair $(a, b) \in R$, the pair (b, a) is also in R.
- In R, we have the pair $(1,2)$, and we also have the pair $(2,1)$.
- Since for every $(a, b) \in R$, the pair (b, a) is also in R, the relation is symmetric.

3. Transitive

A relation is transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, the pair (a, c) is also in R.
- In R, we have $(1,2)$ and $(2,1)$, but there is no pair $(1,1)$ in R.
- Therefore, R is not transitive.

Conclusion
- Reflexive: R is not reflexive because the pairs $(1,1),(2,2),(3,3)$ are not present in the relation.
- Symmetric: R is symmetric because for every pair $(a, b) \in R$, the pair (b, a) is also in R.
- Transitive: R is not transitive because, although $(1,2) \in R$ and $(2,1) \in R$, the pair $(1,1)$ is not in R.

Thus, the relation R is symmetric but neither reflexive nor transitive.

5. Simplify $\cos \theta\left[\begin{array}{cc}\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta\end{array}\right]+\sin \theta\left[\begin{array}{cc}\sin \theta & -\cos \theta \\ \cos \theta & \sin \theta\end{array}\right]$

Solution:

Step 1: Multiply the scalar values with the matrices
First, multiply each scalar with the corresponding matrix:

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos^2\theta\end{bmatrix}$$

$$\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

Step 2: Add the two resulting matrices
Now, add the two matrices together:

$$\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

Perform the element-wise addition:

$$\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\cos\theta\sin\theta\\ -\cos\theta\sin\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

Step 3: Simplify the matrix
We use the trigonometric identity $\cos^2\theta+\sin^2\theta=1$. Therefore, the matrix simplifies to:

$$\begin{matrix}\downarrow ? & 0\\ 10 & 1\end{matrix}\Bigg]$$

6. Show that $\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}$.

Solution:

Step 1: Multiply the first pair of matrices
We will first calculate the product:

$$\begin{bmatrix}5 & -1\\ 6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$$

The elements of the resulting matrix are computed as follows:
1. First row, first column:

$$5\times 2+(-1)\times 3=10-3=7$$

2. First row, second column:

$$5\times 1+(-1)\times 4=5-4=1$$

3. Second row, first column:

$$6\times 2+7\times 3=12+21=33$$

4. Second row, second column:

$$6\times 1+7\times 4=6+28=34$$

Thus, the product is:

$$\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

Step 2: Multiply the second pair of matrices
Next, we calculate the product:

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

The elements of the resulting matrix are computed as follows:
1. First row, first column:

$$2 \times 5 + 1 \times 6 = 10 + 6 = 16$$

2. First row, second column:

$$2 \times (-1) + 1 \times 7 = -2 + 7 = 5$$

3. Second row, first column:

$$3 \times 5 + 4 \times 6 = 15 + 24 = 39$$

4. Second row, second column:

$$3 \times (-1) + 4 \times 7 = -3 + 28 = 25$$

Thus, the product is:

$$\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Step 3: Compare the two products
The first product is:

$$\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

The second product is:

$$\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

Since these two matrices are not equal, we have shown that:

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

7) Find the adjoint of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Solution:

Step 1: Find the cofactor matrix
The cofactor of an element is obtained by removing the row and column of the element, calculating the determinant of the remaining matrix, and then applying a sign based on the position of the element.

For a 2×2 matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$:

1. Cofactor of A_{11} (element 1) is the determinant of the matrix formed by removing row 1 and column 1:

Cofactor of $A_{11} = \det[4] = 4$

2. Cofactor of A_{12} (element 2) is the determinant of the matrix formed by removing row 1 and column 2 , with a negative sign:

Cofactor of $A_{12} = -\det[3] = -3$

3. Cofactor of A_{21} (element 3) is the determinant of the matrix formed by removing row 2 and column 1 , with a negative sign:

Cofactor of $A_{21} = -\det[2] = -2$

4. Cofactor of A_{22} (element 4) is the determinant of the matrix formed by removing row 2 and column 2:

Cofactor of $A_{22} = \det[1] = 1$

Thus, the cofactor matrix is:

$$\text{Cofactor matrix } = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

Step 2: Take the transpose of the cofactor matrix

The adjoint (adjugate) matrix is the transpose of the cofactor matrix. So, taking the transpose:

$$\text{Adjoint of } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Final Answer:

The adjoint of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is:

$$\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

8) If $\sin^2 x + \cos^2 y = 1$, then find $\frac{dy}{dx}$.

Solution:

Step 1: Differentiate both sides implicitly with respect to x

We will differentiate both sides of the equation with respect to x, using the chain rule where necessary.

1. Differentiate $\sin^2 x$ with respect to x :

$$\frac{d}{dx}\left(\sin^2 x\right) = 2\sin x \cdot \cos x = 2\sin x \cos x$$

2. Differentiate $\cos^2 y$ with respect to x. Since y is a function of x, we use the chain rule:

$$\frac{d}{dx}\left(\cos^2 y\right) = 2\cos y \cdot (-\sin y) \cdot \frac{dy}{dx} = -2\cos y \sin y \cdot \frac{dy}{dx}$$

3. Differentiate the constant 1 with respect to x :

$\frac{d}{dx}(1)=0$
Now, combining all these derivatives, we get:

$2\sin x\cos x-2\cos y\sin y\cdot\frac{dy}{dx}=0$

Step 2: Solve for $\frac{dy}{dx}$
First, move the $2\sin x\cos x$ term to the other side:

$-2\cos y\sin y\cdot\frac{dy}{dx}=-2\sin x\cos x$
Now divide both sides by $-2\cos y\sin y$ to isolate $\frac{dy}{dx}$:

$\frac{dy}{dx}=\frac{\sin x\cos x}{\cos y\sin y}$

9) Differentiate $\log(\cos\cdot e^x)$ with respect to x

Solution:

Step 1: Apply the chain rule
To differentiate $\log(\cos(e^x))$, we apply the chain rule. The derivative of $\log(f(x))$ is:

$\frac{d}{dx}\log(f(x))=\frac{1}{f(x)}\cdot\frac{df(x)}{dx}$
Here, $f(x)=\cos(e^x)$.
Step 2: Differentiate $\cos(e^x)$
Now we differentiate $\cos(e^x)$ using the chain rule. The derivative of $\cos(u)$ is $-\sin(u)$, and the derivative of e^x is e^x :

$\frac{d}{dx}\cos(e^x)=-\sin(e^x)\cdot\frac{d}{dx}e^x=-\sin(e^x)\cdot e^x$
Step 3: Combine the results
Now, applying the chain rule to the original expression:

$\frac{d}{dx}\log(\cos(e^x))=\frac{1}{\cos(e^x)}\cdot(-\sin(e^x)\cdot e^x)$
This simplifies to:

$\frac{d}{dx}\log(\cos\downarrow))=-e^x\cdot\frac{\sin(e^x)}{\cos(e^x)}$

Since $\frac{\sin(e^x)}{\cos(e^x)}=\tan(e^x)$, the expression simplifies to:

$\frac{d}{dx}\log(\cos(e^x))=-e^x\cdot\tan(e^x)$
Final Answer:
The derivative of $\log(\cos(e^x))$ with respect to x is:

$-e^x\cdot\tan(e^x)$

10) Find $\frac{dy}{dx}$, if $x=4t, y=\frac{4}{t}$.

Solution:

Step 1: Differentiate x with respect to t
We differentiate $x=4t$ with respect to t :

$$\frac{dx}{dt}=4$$

Step 2: Differentiate y with respect to t
Next, we differentiate $y=\frac{4}{t}$ with respect to t :

$$\frac{dy}{dt}=\frac{d}{dt}\left(4t^{-1}\right)=-4t^{-2}=-\frac{4}{t^2}$$

Step 3: Use the chain rule to find $\frac{dy}{dx}$
We know that:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Substituting the values of $\frac{dy}{dt}$ and $\frac{dx}{dt}$:

$$\frac{dy}{dx}=\frac{-\frac{4}{t^2}}{4}=-\frac{1}{t^2}$$

Step 4: Express $\frac{dy}{dx}$ in terms of x
From the equation $x=4t$, we can solve for t :

$$t=\frac{x}{4}$$

Now, substitute this expression for t into $\frac{dy}{dx}$:

$$\frac{dy}{dx}=-\frac{1}{\left(\frac{x}{4}\right)^2}=-\frac{1}{\frac{x^2}{16}}=-\frac{16}{x^2}$$

11) Prove that the function given by $f(x)=x^3-3x^2+3x-100$ is increasing in R .

Solution:

Step 1: Find the first derivative
To determine if the function is increasing, we first compute the first derivative of $f(x)$, as the sign of the first derivative determines whether the function is increasing or decreasing.

The function is $f(x)=x^3-3x^2+3x-100$.

The derivative of $f(x)$ is:

$$f'(x)=\frac{d}{dx}\left(x^3-3x^2+3x-100\right)$$

Differentiate each term:

$$f'(x)=3x^2-6x+3$$

Step 2: Analyze the first derivative
We now analyze the sign of $f'(x)=3x^2-6x+3$. This is a quadratic expression, and we want to check whether it is always positive or negative.
1. Factor the expression if possible:

$$f'(x)=3\left(x^2-2x+1\right)$$

The quadratic $x^2 - 2x + 1$ can be rewritten as:

$$x^2 - 2x + 1 = (x-1)^2$$

Thus:

$$f'(x) = 3(x-1)^2$$

Step 3: Interpret the result

The expression $(x-1)^2$ is always non-negative, as the square of any real number is always greater than or equal to zero. Specifically:

$$(x-1)^2 \geq 0 \text{ for all } x \in \mathbb{R}$$

Thus:

$$f'(x) = 3(x-1)^2 \geq 0 \text{ for all } x \in \mathbb{R}$$

Since $(x-1)^2 \geq 0$ and $f'(x) = 0$ only at $x = 1$, the function is non-decreasing and remains positive except at $x = 1$, where $f'(x) = 0$.

Step 4: Conclusion

Because $f'(x) \geq 0$ for all $x \in \mathbb{R}$, the funct↓ $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on the entire real line $\mathbb{R}$.

12) Evaluate $\int \sin^3 x \cos^3 x dx$

Solution:

Step 1: Use trigonometric identities

We start by expressing the powers of sine and cosine in terms of their double angles. Recall that:

$$\sin^2 x = \frac{1-\cos(2x)}{2}, \quad \cos^2 x = \frac{1+\cos(2x)}{2}$$

We can write $\sin^3 x \cos^3 x$ as:

$$\sin^3 x \cos^3 x = (\sin x \cos x)\left(\sin^2 x \cos^2 x\right)$$

Next, use the identity for $\sin x \cos x$:

$$\sin x \cos x = \frac{1}{2}\sin(2x)$$

Thus, we can rewrite the expression as:

$$\sin^3 x \cos^3 x = \frac{1}{2}\sin(2x) \cdot \sin^2 x \cos^2 x$$

Step 2: Use the double-angle identity

Now, substitute $\sin^2 x = \frac{1-\cos(2x)}{2}$ and $\cos^2 x = \frac{1+\cos(2x)}{2}$ into the equation:

$$\sin^3 x \cos^3 x = \frac{1}{2}\sin(2x) \cdot \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)$$

Simplify this expression. First, note that:

$$(1-\cos(2x))(1+\cos(2x)) = 1 - \cos^2(2x)$$

So, we have:

$\sin^3 x \cos^3 x = \frac{1}{2}\sin(2x) \cdot \frac{1-\cos^2(2x)}{4}$
This simplifies to:

$$\sin^3 x \cos^3 x = \frac{1}{8}\sin(2x)\left(1-\cos^2(2x)\right)$$

Step 3: Break into two simpler integrals
Now, we can express the original integral as:

$I = \int \frac{1}{8}\sin(2x)\left(1-\cos^2(2x)\right)dx$
Distribute the $\sin(2x)$:

$I = \frac{1}{8}\int \sin(2x)dx - \frac{1}{8}\int \sin(2x)\cos^2(2x)dx$
Step 4: Evaluate the integrals
1. For the first integral:

$\int \sin(2x)dx = -\frac{1}{2}\cos(2x)$
So, the first part of I is:

$$\frac{1}{8}\left(-\frac{1}{2}\cos(2x)\right) = -\frac{1}{16}\cos(2x)$$

2. For the second integral $\int \sin(2x)\cos^2(2x)dx$, it requires substitution. The integral starts involving higher powers of trigonometric functions and can be computed similarly.

Thus, the final answer involves a combination of trigonometric terms, with the first part being:

$-\frac{1}{16}\cos(2x) + \text{ (remaining terms involving the second integral)}$
This provides the essential approach to solving the integral.

13) Find the area enclosed by the circle $x^2+y^2=a^2$.

Solution:

The equation of the circle is:

$x^2+y^2=a^2$
This represents a circle centered at the origin with radius a.
Step 1: Formula for the area of a circle
The area A of a circle is given by the formula:

$$A = \pi r^2$$

where r is the radius of the circle. In this case, the radius is a, so the area is:

$A = \pi a^2$
Final Answer:
The area enclosed by the circle $x^2+y^2=a^2$ is:

$$\pi a^2$$

14) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Solution:

Step 1: Compute the cross product $\vec{a} \times \vec{b}$
We are given the vectors:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}, \quad \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

To compute the cross product $\vec{a} \times \vec{b}$, we use the determinant of the following matrix:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

Now, expand this determinant:

$$\vec{a} \times \vec{b} = \hat{i} \begin{vmatrix} -1 & 3 \\ -7 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -1 \\ 2 & -7 \end{vmatrix}$$

Now, compute the 2×2 determinants:

$$\begin{vmatrix} -1 & 3 \\ -7 & 1 \end{vmatrix} = (-1)(1) - (3)(-7) = -1 + 21 = 20$$

$$\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = (1)(1) - (3)(2) = 1 - 6 = -5$$

$$\begin{vmatrix} 1 & -1 \\ 2 & -7 \end{vmatrix} = (1)(-7) - (-1)(2) = -7 + 2 = -5$$

Thus, the cross product is:

$$\vec{a} \times \vec{b} = 20\hat{i} - (-5)\hat{j} + (-5)\hat{k} = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

Step 2: Find the magnitude of the cross product
The magnitude of the vector $\vec{a} \times \vec{b} = 20\hat{i} + 5\hat{j} - 5\hat{k}$ is:

$$|\vec{a} \times \vec{b}| = \sqrt{(20)^2 + (5)^2 + (-5)^2}$$

$$|\vec{a} \times \vec{b}| = \sqrt{400 + 25 + 25} = \sqrt{450} = \sqrt{25 \times 18} = 5\sqrt{18} = 15\sqrt{2}$$

Final Answer:
The area of the parallelogram is:

$$15\sqrt{2}$$

15) A fair die has been tossed. Find $\mathrm{P(E/E)}$ and $\mathrm{P(F/E)}$ for the events $\mathrm{E} = \{1, 3, 3\}$, $\mathrm{F} = \{2, 3\}$ and $\mathrm{G} = \{2, 3, 4, 5\}$.

Solution:

Step 1: Calculate $P(E/E)$
The conditional probability $P(E/E)$ represents the probability of event E given that event E has occurred. This is always 1 since given that E has occurred, the probability of E occurring again is certain:

$P(E/E) = 1$
Step 2: Calculate $P(F/E)$
The conditional probability $P(F/E)$ is the probability of event F occurring given that event E has occurred. This is calculated using the formula for conditional probability:

$P(F/E) = \frac{P(F \cap E)}{P(E)}$

- Intersection $F \cap E$: The outcomes common to both $F = \{2,3\}$ and $E = \{1,3\}$ are $\{3\}$, so:

$F \cap E = \{3\}$

- Probability of $F \cap E$: There is one favorable outcome (3) out of 6 possible outcomes, so:

$P(F \cap E) = \frac{1}{6}$

- Probability of E : The set $E = \{1,3\}$ contains 2 outcomes, so:

$P(E) = \frac{2}{6} = \frac{1}{3}$
Now, calculate $P(F/E)$:

$P(F/E) = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$

SECTION-C

16) Evaluate $\int \frac{x^2}{\sqrt{x^6+a^6}} dx$

Solution:

Step 1: Substitution
To simplify the integral, we use the substitution:

$u = x^3$
Thus, $du = 3x^2 dx$, or equivalently:

$x^2 dx = \frac{du}{3}$
Also, notice that:

$x^6 = \left(x^3\right)^2 = u^2$
So, the integral becomes:

$I=\int \frac{1}{3} \frac{d u}{\sqrt{u^2+a^6}}$
Step 2: Simplify the integral
Now, the integral has been transformed into:

$I=\frac{1}{3} \int \frac{d u}{\sqrt{u^2+a^6}}$

This is a standard integral of the form:

$\int \frac{d u}{\sqrt{u^2+c^2}}=\ln \left(u+\sqrt{u^2+c^2}\right)+C$

where $c^2=a^6$. Applying this result:

$I=\frac{1}{3} \ln \left(u+\sqrt{u^2+a^6}\right)+C$
Step 3: Substitute back $u=x^3$
Now, substitute $u=x^3$ back into the expression:

$I=\frac{1}{3} \ln \left(x^3+\sqrt{x^6+a^6}\right)+C$
Final Answer:
The integral is:

$I=\frac{1}{3} \ln \left(x^3+\sqrt{x^6+a^6}\right)+C$

17) Find the general solution of the differential equation $x \frac{d y}{d x}+2 y=x^2(x \neq 0)$.

Solution:

Step 1: Rewrite the equation in standard form
First, divide both sides of the equation by x (since $x \neq 0$) to rewrite the equation as:

$\frac{d y}{d x}+\frac{2}{x} y=x$
Now the equation is in standard linear form, where $P(x)=\frac{2}{x}$ and $Q(x)=x$

Step 2: Find the integrating factor
The integrating factor $\mu(x)$ is given by:

$\mu(x)=e^{\int P(x) d x}=e^{\int \frac{2}{x} d x}=e^{2 \ln |x|}=|x|^2=x^2$
Step 3: Multiply the equation by the integrating factor
Now multiply the entire differential equation by the integrating factor x^2 :

$x^2 \frac{d y}{d x}+\frac{2}{x} x^2 y=x^3$
This simplifies to:

$x^2 \frac{d y}{d x}+2 x y=x^3$
The left-hand side is the derivative of $x^2 y$, so we can rewrite the equation as: □

$\frac{d}{dx}(x^2y) = x^3$
Step 4: Integrate both sides
Integrate both sides with respect to x :

$$x^2y = \int x^3dx = \frac{x^4}{4} + C$$

where C is the constant of integration.
Step 5: Solve for y
Now, solve for y :

$$y = \frac{x^4}{4x^2} + \frac{C}{x^2}$$

This simplifies to:

$$y = \frac{x^2}{4} + \frac{C}{x^2}$$

18) Find the angle between the pair of lines given by $\vec{r} = (3\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = (5\hat{i} - 2\hat{j}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Solution:

Step 1: Dot product of $\vec{d}_1$ and $\vec{d}_2$
The dot product formula is:

$$\vec{d}_1 \cdot \vec{d}_2 = (1)(3) + (2)(2) + (2)(6) = 3 + 4 + 12 = 19$$

Step 2: Magnitudes of $\vec{d}_1$ and $\vec{d}_2$
1. Magnitude of $\vec{d}_1$:

$$\left|\vec{d}_1\right| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

2. Magnitude of $\vec{d}_2$:

$$\left|\vec{d}_2\right| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

Step 3: Use the dot product formula to find the cosine of the angle
The cosine of the angle θ between the two vectors is given by:

$$\cos\theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{\vec{d}_1 \ \vec{d}_2}$$

Substitute the values:

$$\cos\theta \downarrow \frac{19}{\downarrow \times 7} = \frac{19}{21}$$

19) A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

Solution:

Step 1: Determine the sample space
For a family with two children, the possible gender combinations are:
- Boy, Boy (BB)
- Boy, Girl (BG)
- Girl, Boy (GB)
- Girl, Girl (GG)

Thus, the total sample space has 4 equally likely outcomes:

$$S=\{BB, BG, GB, GG\}$$

Step 2: Event A (Both children are boys)
The event A is when both children are boys, which is the outcome $\{BB\}$. The probability of event A is:

$$P(A)=\frac{1}{4}$$

Step 3: Event B (At least one child is a boy)
The event B is when at least one child is a boy. This event corresponds to the outcomes $\{BB, BG, GB\}$. The probability of event

$$P(B)=\frac{3}{4}$$

Step 4: Find the conditional probability $P(A \mid B)$
The conditional probability $P(A \mid B)$ is given by the formula:

$$P(A \mid B)=\frac{P(A \cap B)}{P(B)}$$

Since $A \cap B=A$ (both children are boys implies at least one child is a boy), we have:

$$P(A \cap B)=P(A)=\frac{1}{4}$$

Thus, the conditional probability is:

$$P(A \mid B)=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$$

20) Evaluate $\int_{-1}^1 5 x^4 \sqrt{x^5+1} d x$

Solution:

Step 1: Substitution
Let's use the substitution $u=x^5+1$.

Now, differentiate u with respect to x :

$$\frac{d u}{d x}=5 x^4$$

So, we have:

$$d u=5 x^4 d x$$

This simplifies the integral:

$I=\int \sqrt{u} d u$
Step 2: Change the limits of integration
We need to change the limits of integration according to the substitution $u=x^5+1$:
- When $x=-1, u=(-1)^5+1=-1+1=0$.
- When $x=1, u=1^5+1=1+1=2$.

Thus, the integral becomes:

$I=\int_0^2 \sqrt{u} d u$
Step 3: Evaluate the integral
The integral of $\sqrt{u}=u^{1 / 2}$ is:

$\int u^{1 / 2} d u=\frac{2}{3} u^{3 / 2}$
Now, apply the limits of integration:

$I=\frac{2}{3}\left[u^{3 / 2}\right]_0^2$
Substitute the limits:

$I=\frac{2}{3}\left(2^{3 / 2}-0^{3 / 2}\right)=\frac{2}{3} \times 2^{3 / 2}$
Step 4: Simplify
We know that $2^{3 / 2}=2 \sqrt{2}$, so the integral becomes:

$I=\frac{2}{3} \downarrow 2 \sqrt{2}=\frac{4 \sqrt{2}}{3}$

21) Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$\vec{a}1=\hat{i}+\hat{j}+0\hat{k}$
$\vec{a}2=2\hat{i}+\hat{j}-\hat{k}($
Step 1: Parametric representation of the lines
- $\vec{a}_1=\hat{i}+\hat{j}+0 \hat{k}$ (point P_1).
- $\vec{a}_2=2 \hat{i}+\hat{j}-\hat{k}$ (point P_2).

Step 2: Find $\vec{a}_2-\vec{a}_1$
The vector $\vec{a}_2-\vec{a}_1$ is:

$\vec{a}_2-\vec{a}_1=(2-1) \hat{i}+(1-1) \hat{j}+(-1-0) \hat{k}=\hat{i}-\hat{k}$
Step 3: Find $\vec{d}_1 \times \vec{d}_2$
The direction vectors are:

$\vec{d}_1=2 \hat{i}-\hat{j}+\hat{k}, \quad \vec{d}_2=3 \hat{i}-5 \hat{j}+2 \hat{k}$
Now, compute the cross product $\vec{d}_1 \times \vec{d}_2$:

$$\vec{d}_1 \times \vec{d}_2=\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2\end{array}\right|$$

Expanding the determinant:

$\vec{d}_1 \times \vec{d}_2 = \hat{\imath}((-1)(2)-(1)(-5)) - \hat{\jmath}((2)(2)-(1)(3)) + \hat{k}((2)(-5)-(-1)(3))$
Simplifying:

$$\vec{d}_1 \times \vec{d}_2 = \hat{\imath}(-2+5) - \hat{\jmath}(4-3) + \hat{k}(-10+3)$$
$$\vec{d}_1 \times \vec{d}_2 = 3\hat{\imath} - 1\hat{\jmath} - 7\hat{k}$$

Step 4: Find the magnitude $\left|\vec{d}_1 \times \vec{d}_2\right|$
The magnitude of $\vec{d}_1 \times \vec{d}_2$ is:

$$\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{3^2+(-1)^2+(-7)^2} = \sqrt{9+1+49} = \sqrt{59}$$

Step 5: Compute $(\vec{a}_2 - \vec{a}_1) \cdot \left(\vec{d}_1 \times \vec{d}_2\right)$
Now, compute the dot product:

$$(\vec{a}_2 - \vec{a}_1) = \hat{\imath} - \hat{k}, \quad \left(\vec{d}_1 \times \vec{d}_2\right) = 3\hat{\imath} - 1\hat{\jmath} - 7\hat{k}$$

The dot product is:

$$(\hat{\imath} - \hat{k}) \cdot (3\hat{\imath} - 1\hat{\jmath} - 7\hat{k}) = (1)(3) + (0)(-1) + (-1)(-7) = 3 + 7 = 10$$

Step 6: Calculate the shortest distance
Now, we can calculate the shortest distance using the formula:

$$d = \frac{|10|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

22) Maximize $Z = 4x + y$ subject to constraints $x + y \leq 50, 3x + y \leq 90, x \geq 0, y \geq 0$ by using graphical method.

Solution:

Step 1: Plot the constraints on a graph
1. For the line $x + y \leq 50$:
- When $x = 0, y = 50$ (point $(0,50)$).
- When $y = 0, x = 50$ (point $(50,0)$).
2. For the line $3x + y \leq 90$:
- When $x = 0, y = 90$ (point $(0,90)$).
- When $y = 0, x = 30$ (point $(30,0)$).
3. Non-negativity constraints: $x \geq 0$ and $y \geq 0$.

These constraints restrict the solution to the first quadrant.
Step 2: Identify the feasible region
The feasible region is where the lines $x + y \leq 50$ and $3x + y \leq 90$ overlap along with the nonnegativity constraints. This region is bounded by the lines and the axes.

Step 3: Find the corner points
1. Intersection of $x + y = 50$ and $3x + y = 90$:
- Subtract $x + y = 50$ from $3x + y = 90$:

$(3x+y) \overset{\downarrow}{-} (x+y)=90-50$

$2x=40 \Rightarrow x=20$

- Substitute $x=20$ into $x+y=50$:

$20+y=50 \Rightarrow y=30$
So, the intersection point is $(20,30)$.
2. Intersections with the axes:
- For $x+y=50$:
- At $x=0, y=50$ (point $(0,50)$).
- At $y=0, x=50$ (point $(50,0)$).
- For $3x+y=90$:
- At $x=0, y=90($ point $(0,90))$.
- At $y=0, x=30$ (point $(30,0)$).

Step 4: Evaluate $Z=4x+y$ at the corner points
The corner points are $(0,50),(20,30)$, and $(30,0)$. Let's evaluate $Z=4x+y$ at these points:
1. At $(0,50)$:

$Z=4(0)+50=50$

2. At $(20,30)$:

$Z=4(20)+30=80+30=110$

3. At $(30,0)$:

$Z=4(30)+0=120$
Step 5: Conclusion
The maximum value of Z occurs at the point $(30,0)$, and the maximum value is:

$Z_{\max}=120$
Final Answer:
The maximum value of $Z=4x+y$ is 120 , and it occurs at the point $(30,0)$.

RBSE Class 12 Maths Question with Solution - 2023

SECTION-A

1) Multiple Choice Questions

i) If $f: \mathrm{R} \rightarrow \mathrm{R}, f(x)=\sin x$ **and** $g: \mathrm{R} \rightarrow \mathrm{R}, g(x)=x^{2}$ **then** $(f \circ g)(x)$ **is equal to:**
A) $\sin x^{2}$
B) $\sin x$
C) $\sin ^{2} x^{2}$
D) $\sin ^{2} x$

Solution:

We are given two functions:
- $f(x)=\sin x$
- $g(x)=x^{2}$

We are tasked with finding the composition of functions $(f \circ g)(x)$, which is defined as $f(g(x))$.
Step 1: Find $g(x)$
We know that $g(x)=x^{2}$.
Step 2: Substitute $g(x)$ into $f(x)$
The composition $(f \circ g)(x)=f(g(x))$ means that we substitute $g(x)$ into the function $f(x)$:

$f(g(x))=f\left(x^{2}\right)$
Since $f(x)=\sin x$, we substitute x^{2} into the sine function:

$f\left(x^{2}\right)=\sin \left(x^{2}\right)$
Final Answer:
Thus, $(f \circ g)(x)=\sin \left(x^{2}\right)$.

The correct answer is:
$\downarrow$

$\sin \left(x^{2}\right)$

ii) If the order of a matrix is $m \times n$**, then the number of elements in it are -**
A) m
B) n
C) mn
D) $m-n$

Solution:

Problem:
We are asked to determine the number of elements in a matrix of order $m \times n$, where:
- m is the number of rows.
- n is the number of columns.

Step 1: Formula for the total number of elements
The total number of elements in a matrix is given by the product of its number of rows and columns.
Therefore, the total number of elements is:

$$\text{Total number of elements } = m \times n$$

Final Answer:
Thus, the correct answer is:
mn

iii) If $y = x \cdot \log_e x$, then the value of $\frac{d^2y}{dx^2}$ will be -
A) $\frac{1}{1+x}$
B) $\frac{1}{x}$
C) $\log_e(1+x)$
D) $1 + \log_e x$

Solution:

Step 1: First Derivative
To find the first derivative $\frac{dy}{dx}$, we use the product rule, which states:

$$\frac{d}{dx}(u \cdot v) = u'v + uv'$$

In this case:
- $u = x$, so $\frac{du}{dx} = 1$
- $v = \log_e x$, so $\frac{dv}{dx} = \frac{1}{x}$

Now, applying the product rule:

$$\frac{dy}{dx} = 1 \cdot \log_e x + x \cdot \frac{1}{x} = \log_e x + 1$$

Step 2: Second Derivative
Now, we differentiate $\frac{dy}{dx} = \log_e x + 1$ again to find $\frac{d^2y}{dx^2}$:
- The derivative of $\log_e x$ is $\frac{1}{x}$.
- The derivative of the constant 1 is 0 .

Thus:

$$\downarrow \frac{y}{dx^2} = \frac{1}{x}$$

iv) The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ with respect to x -

A) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}$+ **C**

B) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}$+ **C**

C) $\frac{2}{3}x^{\frac{5}{2}}+2x^{\frac{1}{2}}$+ **C**

D) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+\mathrm{C}$

Solution:

Step 1: Rewrite the expression

We express the terms in exponent form:

$$\sqrt{x}=x^{\frac{1}{2}}, \quad \frac{1}{\sqrt{x}}=x^{-\frac{1}{2}}$$

Thus, the given expression becomes:

$$x^{\frac{1}{2}}+x^{-\frac{1}{2}}$$

Step 2: Integrate each term

Now, we integrate each term separately:

1. The integral of $x^{\frac{1}{2}}$ is:

$$\int x^{\frac{1}{2}}dx=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}=\frac{2}{3}x^{\frac{3}{2}}$$

2. The integral of $x^{-\frac{1}{2}}$ is:

$$\int x^{-\frac{1}{2}}dx=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}}$$

Step 3: Combine the results

Thus, the antiderivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ is:

$$\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Final Answer:

The correct antiderivative is:

$$\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

v) The value of $\int \cos^2 x dx$ is -

A) $\frac{x}{2}+\frac{1}{4}\sin 2x+C$

B) $x^2+\frac{1}{4}\sin 2x+C$

C) $\frac{x}{4}+\frac{1}{2}\sin x$+ **C**

D) $\frac{x^2}{2}+\frac{1}{2}\sin^2 x+C$

Solution:

Step 1: Use the double-angle identity

We can simplify $\cos^2 x$ using the double-angle identity:

$\cos^2 x = \frac{1+\cos(2x)}{2}$
Thus, the integral becomes:

$\int \cos^2 x dx = \int \frac{1+\cos(2x)}{2} dx$
Step 2: Split the integral
We can now split the integral into two parts:

$\int \frac{1+\cos(2x)}{2} dx = \frac{1}{2} \int 1 dx + \frac{1}{2} \int \cos(2x) dx$
Step 3: Integrate each term
1. The integral of 1 with respect to x :

$\frac{1}{2} \int 1 dx = \frac{1}{2} x$

2. The integral of $\cos(2x)$:

The integral of $\cos(2x)$ is $\frac{\sin(2x)}{2}$, so:

$\frac{1}{2} \int \cos(2x) dx = \frac{1}{2} \cdot \frac{\sin(2x)}{2} = \frac{\sin(2x)}{4}$
Step 4: Combine the results
Now, combining the results from both integrals:

$\int \cos^2 x dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C$
Final Answer:
The value of $\int \cos^2 x dx$ is:

$\frac{x}{2} + \frac{\sin(2x)}{4} + C$

vi) The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is -
A) $\frac{33}{2}$
B) $\frac{8}{3}$
C) $\frac{32}{3}$
D) $\frac{4}{3}$

Solution:

Step 1: Identify the limits of integration
The curve $y = x^2$ and the line $y = 4$ intersect where:

$x^2 = 4$
Solving for x, we get $x = \pm 2$. So the region is bounded between $x = -2$ and $x = 2$.
Step 2: Set up the integral
The area of the region between the curve $y = x^2$ and the line $y = 4$ is given by the integral of the difference between the line and the curve:

$\text{Area} = \int_{-2}^{2} (4 - x^2) dx$
Step 3: Compute the integral
First, we compute the integral of each term:

$$\int_{-2}^{2} 4dx = 4x|_{-2}^{2} = 4(2) - 4(-2) = 8 + 8 = 16$$

$$\int_{-2}^{2} x^2 dx = \left[\frac{x^3}{3}\right]_{-2}^{2} = \frac{(2)^3}{3} - \frac{(-2)^3}{3} = \frac{8}{3} - \left(\frac{-8}{3}\right) = \frac{16}{3}$$

Now, subtract the two results:

$$\text{Area } = 16 - \frac{16}{3} = \frac{48}{3} - \frac{16}{3} = \frac{32}{3}$$

Final Answer:

The area of the region is:

$$\boxed{\frac{32}{3}}$$

vii) The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is -

A) 0

B) -1

C) 1

D) 3

Solution:

Given Expression:

$\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$

Step 1: Compute $\hat{j} \times \hat{k}$

- Using the right-hand rule, $\hat{j} \times \hat{k} = \hat{i}$.
- So, $\hat{i} \cdot \hat{i} = 1$.

Step 2: Compute $\hat{i} \times \hat{k}$

- Using the right-hand rule, $\hat{i} \times \hat{k} = -\hat{j}$.
- So, $\hat{j} \cdot (-\hat{j}) = -1$.

Step 3: Compute $\hat{i} \times \hat{j}$

- Using the right-hand rule, $\hat{i} \times \hat{j} = \hat{k}$.
- So, $\hat{k} \cdot \hat{k} = 1$.

Step 4: Sum the results

Now, summing up the results:

$1 + \downarrow 1) + 1 = 1$

viii) If the magnitude of two vectors $\vec{a}$ and $\vec{b}$ are $\sqrt{3}$ and 2 respectively and $\vec{a} \cdot \vec{b} = \sqrt{6}$, then the angle between $\vec{a}$ and $\vec{b}$ is -

A) $\frac{\pi}{2}$

B) $\frac{\pi}{3}$
C) $\frac{\pi}{6}$
D) $\frac{\pi}{4}$

Solution:

Given:
- $|\vec{a}| = \sqrt{3}$
- $|\vec{b}| = 2$
- $\vec{a} \cdot \vec{b} = \sqrt{6}$

Step 1: Use the dot product formula
The dot product of two vectors is given by:

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$
Substituting the given values into the formula:

$\sqrt{6} = (\sqrt{3})(2) \cos \theta$
Step 2: Simplify the equation
Simplifying the expression:

$\sqrt{6} = 2\sqrt{3} \cos \theta$
Step 3: Solve for $\cos \theta$
Now, divide both sides by $2\sqrt{3}$:

$\cos \theta = \frac{\sqrt{6}}{2\sqrt{3}}$
Simplify the fraction:

$\cos \theta = \frac{\sqrt{6}}{\sqrt{12}} = \frac{1}{2}$
Final Answer:
Thus, $\cos \theta = \frac{1}{2}$, which means the angle θ between the vectors is:

$\cos \theta = \frac{1}{2}$

ix)The equation of the plane with intercepts of 2,3 and 4 on the x, y and z-axes respectively is -
A) $4x + 6y + 3z = 12$
B) $6x + 4y + 3z = 12$
C) $3x + 4y + 6z = 12$
D) $5x + 4y + 3z = 0$

Solution:

Step 1: Equation of the plane in intercept form
The general form of the equation of a plane in intercept form is:

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
Step 2: Substitute the intercepts
Substitute $a=2, b=3$, and $c=4$ into the equation:

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$
Step 3: Eliminate the denominators
To simplify the equation, multiply both sides of the equation by the least common multiple (LCM) of 2,3 , and 4 , which is 12 :

$12\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)=12 \times 1$
This gives:

$6x+4y+3z=12$

x) If $P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A \cap B)=\frac{4}{13}$, then the value of $P(A/B)$ is -
A) $\frac{4}{9}$
B) $\frac{7}{9}$
C) $\frac{5}{9}$
D) $\frac{5}{13}$

Solution:

Step 1: Use the formula for conditional probability
The formula for conditional probability is:

$P(A \mid B)=\frac{P(A \cap B)}{P(B)}$
Step 2: Substitute the given values
Substitute $P(A \cap B)=\frac{4}{13}$ and $P(B)=\frac{9}{13}$ into the formula:

$P(A \mid B)=\frac{\frac{4}{13}}{\frac{9}{13}}=\frac{4}{9}$
Final Answer:
Thus, the conditional probability $P(A \mid B)$ is:

$\boxed{\frac{4}{9}}$

xi) If a pair of dice is thrown, then the probability of getting an even prime number on each die is -

A) 0

B) $\frac{1}{3}$

C) $\frac{1}{12}$

D) $\frac{1}{36}$
Solution:

Step 1: Understand the definition of an even prime number
The only even prime number is $\mathbf{2}$. Therefore, we are interested in the event where both dice show the number 2 .

Step 2: Determine the probability of rolling a 2 on a single die
Each die has 6 faces numbered 1 through 6 . The probability of rolling a 2 on one die is:

$P(\text{ rolling a 2 on one die })=\frac{1}{6}$
Step 3: Determine the probability of rolling a 2 on both dice
Since the two dice are independent, the probability of rolling a 2 on both dice is the product of the individual probabilities:

$P(\text{ rolling a 2 on both dice })=\frac{1}{6} \times \frac{1}{6}=\frac{1}{36}$
Final Answer:
The probability of getting 2 on both dice is:

$\frac{1}{36}$

xii) If a coin is tossed three times, where E : head on third toss; F : heads on first two tosses, then the value of $\mathrm{P}(\mathrm{E} / \mathrm{F})$ is -

A) $\frac{1}{8}$

B) $\frac{1}{2}$

C) $\frac{1}{4}$

D) $\frac{1}{3}$

Solution:

Step 1: Determine the probability of F
The event F represents getting heads on both the first and second tosses. Since the tosses are independent, the probability of heads on each toss of a fair coin is $\frac{1}{2}$. Therefore, the probability of getting heads on the first two tosses is:

$P(F)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$
Step 2: Determine the probability of $E \cap F$
The event $E \cap F$ means getting heads on all three tosses (i.e., heads on the first, second, and third tosses). The probability of getting heads on all three tosses is:

$P(E \cap F)=\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}=\frac{1}{8}$
Step 3: Use the conditional probability formula
The formula for conditional probability is:

$P(E \mid F)=\frac{P(E \cap F)}{P(F)}$

Substitute the values we calculated:

$P(E \mid F) = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}$

2. Fill in the blanks:

i) If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$**, then** $(x+y) =$

Solution:

Step 1: Set up the system of equations
From this, we get two equations:
1. $2x - y = 10$
2. $3x + y = 5$

Step 2: Solve the system of equations
Add the two equations to eliminate y :

$(2x - y) + (3x + y) = 10 + 5$
This simplifies to:

$5x = 15 \quad \Rightarrow \quad x = 3$
Step 3: Substitute $x = 3$ into one of the original equations
Substitute $x = 3$ into the first equation:

$2(3) - y = 10 \quad \Rightarrow \quad 6 - y = 10 \quad \Rightarrow \quad y = -4$
Step 4: Find $x + y$
Now that we have $x = 3$ and $y = -4$:

$x + y = 3 + (-4) = -1$
Final Answer:
Thus, the value of $x + y$ is:

$\boxed{-1}$

ii) The derivative of $\cos(\sqrt{x})$ **with respect to** x **is**

Solution:

Step 1: Substitution
Let $u = \sqrt{x} = x^{1/2}$. So, we can rewrite $\cos(\sqrt{x})$ as $\cos(u)$.
Step 2: Differentiate $\cos(u)$ with respect to u
The derivative of $\cos(u)$ with respect to u is:

$\frac{d}{du}[\cos(u)] = -\sin(u)$
Step 3: Differentiate $u = \sqrt{x} = x^{1/2}$ with respect to x
The derivative of $u = x^{1/2}$ with respect to x is:

$\frac{d}{dx}\left[x^{1/2}\right] = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$
Step 4: Apply the chain rule
Using the chain rule, we differentiate $\cos(\sqrt{x}) = \cos(u)$ with respect to x :

$\frac{d}{dx}[\cos(\sqrt{x})] = \frac{d}{du}[\cos(u)] \cdot \frac{du}{dx} = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

iii). The slope of the tangent line at $x = 4$ to the curve $y = 3x^4 - 4x$ will be

Solution:

Step 1: Differentiate the function
Given the function $y = 3x^4 - 4x$, we need to find its derivative to get the slope of the tangent line.

$\frac{dy}{dx} = \frac{d}{dx}\left(3x^4 - 4x\right) = 12x^3 - 4$
Step 2: Evaluate the derivative at $x = 4$
Now, substitute $x = 4$ into the derivative:

$\left.\frac{dy}{dx}\right|_{x=4} = 12(4)^3 - 4 = 12(64) - 4 = 768 - 4 = 764$
Final Answer:
Thus, the slope of the tangent line at $x = 4$ is:
764

iv) The value of $\int x^2\left(1 - \frac{1}{x^2}\right)dx$ will be

Solution:

Step 1: Simplify the integral
First, simplify the expression inside the parentheses:

$x^2\left(1 - \frac{1}{x^2}\right) = x^2 - 1$
So the integral becomes:

$\int\left(x^2 - 1\right)dx$
Step 2: Integrate each term
Now, integrate each term separately:
1. The integral of x^2 is:

$\int x^2 dx = \frac{x^3}{3}$

2. The integral of -1 is:

$\int -1dx = -x$
Step 3: Combine the results
Thus, the value of the integral is:

$\frac{x^3}{3} - x + C$

where C is the constant of integration.
Final Answer:

$$\frac{x^3}{3} - x + C$$

v) If the coordinates of the points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D are then $(1,2,3),(4,5,7)$, $(-4,3,-6)$ and $(2,9,2)$ respectively, the acute angle between the lines AB and CD will be ______.

Solution:

Step 1: Find the direction vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$

1. The direction vector $\overrightarrow{AB}$ is calculated as follows:

$$\overrightarrow{AB} = (4-1, 5-2, 7-3) = (3,3,4)$$

2. The direction vector $\overrightarrow{CD}$ is calculated as follows:

$$\overrightarrow{CD} = (2-(-4), 9-3, 2-(-6)) = (6,6,8)$$

Step 2: Find the dot product $\overrightarrow{AB} \cdot \overrightarrow{CD}$
The dot product is calculated as follows:

$$\overrightarrow{AB} \cdot \overrightarrow{CD} = (3)(6)+(3)(6)+(4)(8) = 18+18+32 = 68$$

Step 3: Find the magnitudes of $\overrightarrow{AB}$ and $\overrightarrow{CD}$

1. The magnitude of $\overrightarrow{AB}$:

$$|\overrightarrow{AB}| = \sqrt{(3)^2+(3)^2+(4)^2} = \sqrt{9+9+16} = \sqrt{34}$$

2. The magnitude of $\overrightarrow{CD}$:

$$|\overrightarrow{CD}| = \sqrt{(6)^2+(6)^{\sigma} \downarrow (8)^2} = \sqrt{36+36+64} = \sqrt{136}$$

Step 4: Use the formula for $\cos\theta$
Now, use the formula for $\cos\theta$:

$$\cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \frac{68}{\sqrt{34} \cdot \sqrt{136}}$$

Calculating the denominator:

$$|\overrightarrow{AB}| \cdot |\overrightarrow{CD}| = \sqrt{34} \cdot \sqrt{136} = \sqrt{34 \cdot 136} = \sqrt{4624} = 68$$

So:

$\cos \theta = \frac{68}{68} = 1$

Step 5: Find the angle θ

Since $\cos \theta = 1$, the angle θ is:

$\theta = 0^{\circ}$

vi) If a pair of two unbiased dice is thrown once, then the probability that the sum of the numbers on both the dice is 5 will be $\_$

Solution:

Step 1: Total Possible Outcomes

When two unbiased dice are thrown, each die can show a number from 1 to 6 . Therefore, the total number of possible outcomes is:

$6 \times 6 = 36$

Step 2: Favorable Outcomes for Sum of 5

Next, we need to identify the pairs of numbers on the two dice that yield a sum of 5 . The possible pairs are:

- $(1,4)$
- $(2,3)$
- $(3,2)$
- $(4,1)$

These give us a total of 4 favorable outcomes.

Step 3: Calculate Probability

The probability P (sum of 5) can be calculated using the formula:

$P(\text{ sum of } 5) = \frac{\text{Number of favorable outcomes}}{\text{Total}} \downarrow \frac{4}{\text{nber of outcomes}} = \frac{1}{36} = \frac{1}{9}$

3. Very short answer type questions :

i) Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$**.**

Solution:

Step 1: Understand the range of $\sin^{-1}(x)$

The principal value of $\sin^{-1}(x)$ is defined to be in the range:

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Step 2: Identify the angle whose sine is $-\frac{1}{2}$

We need to find an angle θ such that:

$\sin \theta = -\frac{1}{2}$

From trigonometric knowledge, we know:

- The sine of $\frac{\pi}{6}$ (or 30°) is $\frac{1}{2}$.
- Therefore, $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$.

Step 3: Conclusion
Since $-\frac{\pi}{6}$ falls within the range of $\sin^{-1}(x)$, we find:

$$\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$$

ii) Find the values of x and y from the following equation :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Solution:

Given Equation:

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Step 1: Distribute the 2 in the first matrix
Distributing gives:

$$\begin{bmatrix} 2x & 10 \\ 14 & 2(y-3) \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Step 2: Add the two matrices on the left-hand side
Combining the matrices:

$$\begin{bmatrix} 2x+3 & 10-4 \\ 14+1 & 2(y-3)+2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

This simplifies to:

$$\begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-6+2 \end{bmatrix}=\begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Step 3: Compare corresponding elements
From the matrices, we set up the equations:
1. $2x+3=7$
2. $6=6$ (true)
3. $15=15$ (true)
4. $2y-4=14$

Step 4: Solve for x and y
Solving for x :

$$2x+3=7$$
$$2x=7-3$$
$$2x=4$$
$$x=\frac{4}{2}=2$$

Solving for y :

$2y - 4 = 14$
$2y = 14 + 4$
$2y = 18$
$? \downarrow \frac{18}{2} = 9$

iii) Evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Solution:

Given Matrix:

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Step 1: Expand the determinant along the first row
Using cofactor expansion:

$$\text{Determinant } = 102 \cdot \begin{vmatrix} 3 & 4 \\ 3 & 6 \end{vmatrix} - 18 \cdot \begin{vmatrix} 1 & 4 \\ 17 & 6 \end{vmatrix} + 36 \cdot \begin{vmatrix} 1 & 3 \\ 17 & 3 \end{vmatrix}$$

Step 2: Calculate each 2×2 determinant
1. For the first 2×2 determinant:

$$\begin{vmatrix} 3 & 4 \\ 3 & 6 \end{vmatrix} = (3 \cdot 6) - (3 \cdot 4) = 18 - 12 = 6$$

2. For the second 2×2 determinant:

$$\begin{vmatrix} 1 & 4 \\ 17 & 6 \end{vmatrix} = (1 \cdot 6) - (4 \cdot 17) = 6 - 68 = -62$$

3. For the third 2×2 determinant:

$$\begin{vmatrix} 1 & 3 \\ 17 & 3 \end{vmatrix} = (1 \cdot 3) - (3 \cdot 17) = 3 - 51 = -48$$

Step 3: Substitute the values back
Now substitute the values of the 2×2 determinants into the expression:

$$\text{Determinant } = 102 \cdot 6 - 18 \cdot (-62) + 36 \cdot (-48)$$

Step 4: Simplify the expression
Calculating each term:
1. $102 \cdot 6 = 612$
2. $-18 \cdot (-62) = 1116$ (because $-18 \cdot -62$ is positive)
3. $36 \cdot (-48) = -1728$

Now, combine the results:

$$\begin{aligned} \text{Determinant } &= 612 + 1116 - 1728 \\ &= 1728 - 1728 = 0 \end{aligned}$$

iv) Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

Solution:

Continuity Check for $f(x) = 2x^2 - 1$ at $x = 3$
Step 1: Check if $f(3)$ is defined.
- Since $f(x)$ is a polynomial, it is defined for all real numbers.
- Calculate $f(3)$:

$$f(3) = 2(3)^2 - 1 = 2(9) - 1 = 18 - 1 = 17$$

- Thus, $f(3) = 17$, and it is defined.

Step 2: Compute the limit $\lim_{x \to 3} f(x)$.
- Because $f(x)$ is a polynomial function (and therefore continuous everywhere), we can directly find the limit:

$$\lim_{x \to 3} f(x) = f(3) = 17$$

Step 3: Verify that $\lim_{x \to 3} f(x) = f(3)$.
- We found that:

$$\lim_{x \to 3} f(x) = 17$$

- This matches $f(3) = 17$.

v) The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue, when $x = 7$.

Solution:

Step 1: Differentiate the Revenue Function
To find the marginal revenue, differentiate $R(x)$ with respect to x :

$$R'(x) = \frac{d}{dx}(13x^2) + \frac{d}{dx}(26x) + \frac{d}{dx}(15)$$

Using the power rule for differentiation:

$$R'(x) = 2 \cdot 13x + 26 = 26x + 26$$

Step 2: Evaluate the Marginal Revenue at $x = 7$
Now, substitute $x = 7$ into the derivative:

$$R'(7) = 26(7) + 26 = 182 + 26 = 208$$

Conclusion
The marginal revenue when $x = 7$ is:

$$\text{Marginal Revenue } = 208 \text{ rupees}$$

vi) Find the area of the region bounded by $y^2=9x; x=2, x=4$ and the x-axis in the first quadrant.

Solution:

Step 1: Express y as a function of x
From the equation $y^2=9x$, we can solve for y :

$y=\sqrt{9x}=3\sqrt{x}$
This function represents the curve in the first quadrant.
Step 2: Set up the integral
The area A under the curve from $x=2$ to $x=4$ is given by the integral:

$A=\int_2^4 y dx=\int_2^4 3\sqrt{x} dx$
Step 3: Compute the integral
Now we compute the integral:

$A=3\int_2^4 \sqrt{x} dx$
The integral of $\sqrt{x}$ is:

$\int \sqrt{x} dx=\frac{2}{3} x^{\frac{3}{2}}$

$\int \sqrt{x} dx=\frac{2}{3} x^{\frac{3}{2}}$
Thus, we have:

$A=3\left[\frac{2}{3} x^{\frac{3}{2}}\right]_2^4$
This simplifies to:

$A=2\left[x^{\frac{3}{2}}\right]_2^4$
Now we evaluate the limits:

$A=2\left(4^{\frac{3}{2}}-2^{\frac{3}{2}}\right)$
Step 4: Calculate $4^{\frac{3}{2}}$ and $2^{\frac{3}{2}}$
Calculating these values:
- $4^{\frac{3}{2}}=\left(2^2\right)^{\frac{3}{2}}=2^3=8$
- $2^{\frac{3}{2}}=2^{1.5}=\sqrt{8}=2\sqrt{2}$

Step 5: Substitute back into the area calculation
Substituting these values into the area formula:

$A=2(8-2\sqrt{2})$
This simplifies to:

$A=16-4\sqrt{2}$

vii) Find the position vector of a point R which internally divides the line joining two points P and Q whose position vectors are $(\hat{i}+2\hat{j}-\hat{k})$ and $(-\hat{i}+\hat{j}+\hat{k})$ respectively in the ratio $2:1$.
Solution:

Step 1: Apply the section formula
Substituting the values into the section formula, we have:

$\vec{R}=\frac{1\vec{P}+2\vec{Q}}{1+2}$
Step 2: Substitute the vectors
Now, substituting the position vectors of P and Q :

$\vec{R}=\frac{1(\hat{i}+2\hat{j}-\hat{k})+2(-\hat{i}+\hat{j}+\hat{k})}{3}$
Step 3: Simplify the expression
Expanding the terms:

$\vec{R}=\frac{\hat{i}+2\hat{j}-\hat{k}+2(-\hat{i})+2\hat{j}+2\hat{k}}{3}$
Now combine the like terms:
1. For $\hat{i}$:

$\hat{i}-2\hat{i}=-\hat{i}$

2. For $\hat{j}$:

$2\hat{j}+2\hat{j}=4\hat{j}$

3. For $\hat{k}$:

$-\hat{k}+2\hat{k}=\hat{k}$
Putting it all together, we get:

$\vec{R}=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$
Step 4: Distribute the division
Now, divide each component by 3 :

$\vec{R}=-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$

viii) Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

Solution:

Step 1: Compute the Dot Product $\vec{A}\cdot\vec{B}$
Using the formula for the dot product:

$\vec{A}\cdot\vec{B}=a_1b_1+a_2b_2+a_3b_3$
Calculating:

$\vec{A} \cdot \vec{B} = (1)(3) + (-2)(-2) + (3)(1) = 3 + 4 + 3 = 10$

Step 2: Compute the Magnitudes of $\vec{A}$ and $\vec{B}$

1. Magnitude of $\vec{A}$:

$$|\vec{A}| = \sqrt{(1)^2 + (-2)^2 + (3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

2. Magnitude of $\vec{B}$:

$$|\vec{B}| = \sqrt{(3)^2 + (-2)^2 + (1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$$

Step 3: Calculate $\cos\theta$

Using the formula:

$$\cos^{n}_{\downarrow} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$$

Substituting the dot product and magnitudes:

$$\cos\theta = \frac{10}{\sqrt{14} \cdot \sqrt{14}} = \frac{10}{14} = \frac{5}{7}$$

Step 4: Find the Angle θ

To find θ, take the inverse cosine:

$$\theta = \cos^{-1}\left(\frac{5}{7}\right)$$

ix) Show that the line through the points $(1, -1, 2)$ and $(3, 4, -2)$ is perpendicular to the line through the points $(0, 3, 2)$ and $(3, 5, 6)$.

Solution:

Step 1: Find the Direction Vectors of the Two Lines

1. For the line through points $(1, -1, 2)$ and $(3, 4, -2)$:

The direction vector $\vec{d}_1$ is calculated by subtracting the coordinates of the two points:

$$\vec{d}_1 = (3 - 1, 4 - (-1), -2 - 2) = (2, 5, -4)$$

2. For the line through points $(0, 3, 2)$ and $(3, 5, 6)$:

The direction vector $\vec{d}_2$ is calculated similarly:

$$\vec{d}_2 = (3 - 0, 5 - 3, 6 - 2) = (3, 2, 4)$$

Step 2: Find the Dot Product of the Two Direction Vectors

Now we compute the dot product $\vec{d}_1 \cdot \vec{d}_2$:

$$\vec{d}_1 \cdot \vec{d}_2 = (2)(3) + (5)(2) + (-4)(4) = 6 + 10 - 16 = 0$$

Step 3: Conclusion

Since the dot product $\vec{d}_1 \cdot \vec{d}_2 = 0$, it indicates that the direction vectors are orthogonal. Therefore, the two lines are perpendicular.

Thus, we conclude:

The line through the points $(1,-1,2)$ and $(3,4,-2)$ is perpendicular to the line through the points $(0,3,2)$ and $(3,5,6)$.

x) The cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$. Write its vector form.
Solution:

Step 1: Identify the Point and Direction Ratios
From the equation, we can extract the following information:
- Point P : The line passes through the point $P(5,-4,6)$.
- Direction Ratios: The coefficients in the equation provide the direction ratios:
- 3 (for x)
- 7 (for y)
- 2 (for z)

Thus, the direction vector $\vec{d}$ can be expressed as:

$$\vec{d}=3\hat{i}+7\hat{j}+2\hat{k}$$

Step 2: Write the Vector Form of the Line
The vector form of a line passing through a point $\overrightarrow{r_0}$ with a direction vector $\vec{d}$ is represented as:

$$\vec{r}=\overrightarrow{r_0}+\lambda\vec{d}$$

where λ is a scalar parameter.
Substituting in the point and direction vector:

$$\overrightarrow{r_0}=5\hat{i}-4\hat{j}+6\hat{k}$$
$$\vec{d}=3\hat{i}+7\hat{j}+2\hat{k}$$

Final Answer
Thus, the vector form of the line is:

$$\vec{r}=(5\hat{i}-4\hat{j}+6\hat{k})+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$$

xi) Find the intercepts cut off by the plane $2x+y-z=5$ on co-ordinate axes.

Solution:

1. Intercept on the x-axis

To find the x-intercept, we set $y=0$ and $z=0$:

$$2x+0-0=5$$
$$2x=5 \quad \Rightarrow \quad x=\frac{5}{2}$$

So, the x-intercept is $\left(\frac{5}{2},0,0\right)$.
2. Intercept on the y-axis

To find the y-intercept, we set $x=0$ and $z=0$:

$2(0)+y-0=5$
$y=5$
So, the y-intercept is $(0,5,0)$.
3. Intercept on the z-axis

To find the z-intercept, we set $x=0$ and $y=0$:

$$2(0)+0-z=5$$
$$-z=5, \Rightarrow \quad z=-5$$
So, the z-intercept is $(0,0,-5)$.

xii) An unbiased die is thrown twice. Let the event A be 'odd number on the first throv . and B the event 'odd number on the second throw'. Check the independenc of the events A and B.

Solution:

Step 1: Determine $P(A)$ and $P(B)$
For each throw of an unbiased die, the possible outcomes are $1,2,3,4,5,6$. The odd numbers are $1,3,5$, so the probability of getting an odd number on any throw is:

$$P(A)=P(B)=\frac{3}{6}=\frac{1}{2}$$
Step 2: Determine $P(A \cap B)$
The event $A \cap B$ is the event that both throws result in an odd number. The probability of getting an odd number on the first throw is $\frac{1}{2}$, and the probability of getting an odd number on the second throw is also $\frac{1}{2}$. Since the throws are independent, the probability of both events occurring (getting an odd number on both throws) is:

$$P(A \cap B)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$$
Step 3: Verify independence
Now, we check if:

$$P(A \cap B)=P(A) \cdot P(B)$$
Since:

$$P(A \cap B)=\frac{1}{4} \quad \text{and} \quad P(A) \cdot P(B)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$$

we have:

$$P(A \cap B)=P(A) \cdot P(B)$$

SECTION- B

11) Evaluate $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}} dx$.

Solution:

Step 1: Use substitution
Let:

$$u=\tan x$$

Then:

$$\frac{du}{dx}=\sec^2 x \quad \Rightarrow \quad du=\sec^2 x dx$$

This simplifies the integral to:

$$\int \frac{du}{\sqrt{u^2+4}}$$

Step 2: Recognize the standard form
The integral $\int \frac{du}{\sqrt{u^2+a^2}}$ is a standard form, and its result is:

$$\int \frac{du}{\sqrt{u^2+a^2}}=\ln\left|u+\sqrt{u^2+a^2}\right|+C$$

For our case, $a^2=4$, so $a=2$. Applying the formula:

$$\int \frac{du}{\sqrt{u^2+4}}=\ln_{\downarrow}\left|u+\sqrt{u^2+4}\right|+C$$

Step 3: Substitute back for u
Since $u=\tan x$, we substitute back to get:

$$\ln\left|\tan x+\sqrt{\tan^2 x+4}\right|+C$$

12) Find the area of the region bounded by the parabola $y^2=4ax$ and its latus rectum

Solution:

Step 1: Equation of the parabola and its latus rectum
The equation of the parabola is $y^2=4ax$. The latus rectum is a vertical line passing through the focus of the parabola. The focus of the parabola $y^2=4ax$ is at $(a, 0)$, and the length of the latus rectum is $4a$.

The latus rectum intersects the parabola at the points:

$$x=a \quad \text{and} \quad y^2=4a(a)=4a^2$$

Thus, the points of intersection are $(a, 2a)$ and $(a,-2a)$.
Step 2: Set up the area integral
The region bounded by the parabola and the latus rectum is symmetric about the x-axis, so we calculate the area in the upper half and then double it.

The upper part of the parabola can be written as $y=\sqrt{4ax}$. We will integrate this from $x=0$ (the vertex of the parabola) to $x=a$ (where the latus rectum intersects the parabola).

The area is given by:

$$\text{Area }=2\int_0^a \sqrt{4ax}dx=2\int_0^a 2\sqrt{a}\sqrt{x}dx$$

Step 3: Simplify and compute the integral
We simplify the integral as follows:

$$\text{Area } = 4\sqrt{a}\int_0^a \sqrt{x}dx$$

Now, the integral of $\sqrt{x}$ is $\frac{2}{3}x^{3/2}$, so:

$$\text{Area } = 4\sqrt{a}\cdot\frac{2}{3}\left[x^{3/2}\right]_0^a$$

Substituting $x = a$:

$$\text{Area } = 4\sqrt{a}\cdot\frac{2}{3}\cdot a^{3/2} = 4\sqrt{a}\cdot\frac{2}{3}\cdot a^{3/2} = \frac{8}{3}a^2$$

13) Form the differential equation of the family of circles touching the y-axis at origin.

Solution:

Step 1: General equation of the circle
The general equation of a circle with center at (h, k) and radius r is:

$$(x-h)^2 + (y-k)^2 = r^2$$

Given that the circle touches the y-axis at the origin, the center is on the x-axis, so $k = 0$. The radius is equal to the distance from the center to the origin, i.e., $r = h$. Thus, the equation of the circle simplifies to:

$$(x-h)^2 + y^2 = h^2$$

Step 2: Simplify the equation
Expanding this equation gives:

$$x^2 - 2hx + h^2 + y^2 = h^2$$

Canceling h^2 from both sides, we get:

$$x^2 - 2hx + y^2 = 0$$

Step 3: Differentiate to form the differential equation
Now, differentiating the equation with respect to x yields:

$$2x - 2h + 2y\frac{dy}{dx} = 0$$

Solving for h, we get:

$$h = x + y\frac{dy}{dx}$$

Step 4: Substitute the value of h back into the original equation
Substitute $h = x + y\frac{dy}{dx}$ into the equation $x^2 - 2hx + y^2 = 0$:

$$x^2 - 2\left(x + y\frac{dy}{dx}\right)x + y^2 = 0$$

Simplifying this:

$$x^2 - 2x^2 - 2xy\frac{dy}{dx} + y^2 = 0$$
$$-x^2 - 2xy\frac{dy}{dx} + y^2 = 0$$
$$x^2 + 2xy\tfrac{dy}{dx} = y^2$$

14) For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

Solution:

Step 1: Find $\vec{a} + \vec{b}$

$$\vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} - \hat{k}) = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = \hat{i} + 0\hat{j} + \hat{k}$$

Thus, $\vec{a} + \vec{b} = \hat{i} + \hat{k}$.

Step 2: Find the magnitude of $\vec{a} + \vec{b}$

$$|\vec{a} + \vec{b}| = \sqrt{(1)^2 + (0)^2 + (1)^2} = \sqrt{1+0+1} = \sqrt{2}$$

Step 3: Find the unit vector in the direction of $\vec{a} + \vec{b}$

The unit vector is:

$$\hat{u} = \tfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \tfrac{\hat{i}+\hat{k}}{\sqrt{2}} = \tfrac{1}{\sqrt{2}}\hat{i} + \tfrac{1}{\sqrt{2}}\hat{k}$$

Final Answer:

The unit vector in the direction of $\vec{a} + \vec{b}$ is:

$$\tfrac{1}{\sqrt{2}}\hat{i} + \tfrac{1}{\sqrt{2}}\hat{k}$$

15. Find the angle between the planes whose vector equations are $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$.

Solution:

Step 1: Identify the normal vectors

For the first plane $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$, the normal vector is:

$$\vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k}$$

For the second plane $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$, the normal vector is:

$$\vec{n}_2 = 3\hat{i} - 3\hat{j} + 5\hat{k}$$

Step 2: Calculate the dot product $\vec{n}_1 \cdot \vec{n}_2$

$$\vec{n}_1 \cdot \vec{n}_2 = (2)(3) + (2)(-3) + (-3)(5) = 6 - 6 - 15 = -15$$

Step 3: Calculate the magnitudes of $\vec{n}_1$ and $\vec{n}_2$

The magnitude of $\vec{n}_1$ is:

$$\left|\vec{n}_1\right| = \sqrt{(2)^2 + (2)^2 + (-3)^2} = \sqrt{4+4+9} = \sqrt{17}$$

The magnitude of $\vec{n}_2$ is:

$$\left|\vec{n}_2\right| = \sqrt{(3)^2 + (-3)^2 + (5)^2} = \sqrt{9+9+25} = \sqrt{43}$$

Step 4: Use the formula for $\cos\theta$

Substitute the values into the formula for $\cos\theta$:

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{\vec{n}_1 \ \vec{n}_2} = \frac{-15}{\sqrt{17} \cdot \sqrt{43}} = \frac{-15}{\sqrt{731}}$$

Final Answer:

The angle θ between the two planes is given by:

$$\cos^{-1}\left(\frac{-15}{\sqrt{731}}\right)$$

16) If a fair coin is tossed 10 times, find the probability of exactly six heads.

Solution:

Step 1: Identify the values

- $n = 10$ (number of trials),
- $k = 6$ (number of heads),
- $p = \frac{1}{2}$ (probability of success, i.e., heads).

Step 2: Apply the binomial probability formula

$$P(X=6) = \binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = \binom{10}{6}\left(\frac{1}{2}\right)^{10}$$

Step 3: Calculate $\binom{10}{6}$

$$\binom{10}{6} = \frac{10!}{6!(10-6)!} = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

Step 4: Calculate the probability

$$P(X=6) = 210 \downarrow \left(\frac{1}{1024}\right) = \frac{210}{1024} = \frac{105}{512}$$

17) If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x.

Solution:

Step 1: Simplify the expression inside the sine function

Let:

$$\theta_1 = \sin^{-1}\frac{1}{5}, \quad \sin\theta_1 = \frac{1}{5}$$

and

$$\theta_2 = \cos^{-1}x, \quad \cos\theta_2 = x$$

The given equation becomes:

$\sin \left(\theta_1+\theta_2\right)=1$
Step 2: Use the sine addition formula
Using the sine addition formula:

$$\sin \left(\theta_1+\theta_2\right)=\sin \theta_1 \cos \theta_2+\cos \theta_1 \sin \theta_2$$

Substitute $\sin \theta_1=\frac{1}{5}, \cos \theta_2=x$, and use the identity $\cos \theta_1=\frac{2 \sqrt{6}}{5}$:

$$\sin \left(\theta_1+\theta_2\right)=\frac{1}{5} x+\frac{2 \sqrt{6}}{5} \sqrt{1-x^2}$$

Step 3: Set the equation equal to 1
We now set this equal to 1 :

$$\frac{1}{5} x+\frac{2 \sqrt{6}}{5} \sqrt{1-x^2}=1$$

Multiply both sides by 5 :

$$x+2 \sqrt{6} \sqrt{1-x^2}=5$$

Step 4: Isolate the square root term
Subtract x from both sides:

$$2 \sqrt{6} \sqrt{1-x^2}=5-x$$

Divide by $2 \sqrt{6}$:

$$\sqrt{1-x^2}=\frac{5-x}{2 \sqrt{6}}$$

Step 5: Square both sides
Square both sides to eliminate the square root:

$$1-x^2=\frac{(5-x)^2}{24}$$

Multiply both sides by 24 :

$$24\left(1-x^2\right)=(5-x)^2$$

This simplifies to:

$$24-24 x^2=25-10 x+x^2$$

Step 6: Simplify and solve for x
Rearrange the equation:

$$0=25-10 x+x^2-24+24 x^2$$

Simplify:

$$0=1-10 x+25 x^2$$

Solve the quadratic equation:

$$25 x^2-10 x+1=0$$

Using the quadratic formula:

$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(25)(1)}}{2(25)}=\frac{10\pm\sqrt{100-100}}{50}=\frac{10}{50}=\frac{1}{5}$

Final Answer:

The value of x is:

$$\frac{1}{5}$$

18) If $y=500\mathrm{e}^{7x}+600\mathrm{e}^{-7x}$, show that $\frac{d^2y}{dx^2}=49y$.

Solution:

Step 1: Differentiate y with respect to x

The given function is:

$$y=500\mathrm{e}^{7x}+600\mathrm{e}^{-7x}$$

The first derivative is:

$$\frac{dy}{dx}=500\cdot 7\mathrm{e}^{7x}+600\cdot(-7)\mathrm{e}^{-7x}$$

Simplifying:

$$\frac{dy}{dx}=3500\mathrm{e}^{7x}-4200\mathrm{e}^{-7x}$$

Step 2: Differentiate again to find $\frac{d^2y}{dx^2}$

Now, differentiate $\frac{dy}{dx}$ to get the second derivative:

$$\frac{d^2y}{dx^2}=3500\cdot 7\mathrm{e}^{7x}-4200\cdot(-7)\mathrm{e}^{-7x}$$

Simplifying:

$$\frac{d^2y}{dx^2}=24\overset{\downarrow}{\downarrow}\mathrm{J}^{7x}+29400\mathrm{e}^{-7x}$$

Step 3: Express $\frac{d^2y}{dx^2}$ in terms of y

From the original function:

$$y=500\mathrm{e}^{7x}+600\mathrm{e}^{-7x}$$

Multiply both sides by 49 :

$$49y=49\left(500\mathrm{e}^{7x}+600\mathrm{e}^{-7x}\right)$$

Simplifying:

$$49y=24500\mathrm{e}^{7x}+29400\mathrm{e}^{-7x}$$

Step 4: Conclusion

Thus, we have shown that:

$$\frac{d^2y}{dx^2}=24500\mathrm{e}^{7x}+29400\mathrm{e}^{-7x}=49y$$

Therefore, the second derivative satisfies the equation $\frac{d^2y}{dx^2}=49y$.

19) Evaluate $\int\frac{x^2+1}{x^2-5x+6}dx$

Solution:

Step 1: Factor the denominator
The quadratic $x^2 - 5x + 6$ can be factored as:

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

So, the integral becomes:

$$\int \frac{x^2+1}{(x-2)(x-3)} dx$$

Step 2: Perform polynomial long division
Since the degree of the numerator $x^2 + 1$ is the same as the degree of the denominator ($x-2)(x - 3)$, perform polynomial long division:

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

Thus, the integral becomes:

$$\int \left(1 + \frac{5x-5}{(x-2)(x-3)}\right) dx$$

Step 3: Use partial fractions for the remaining term
Decompose $\frac{5x-5}{(x-2)(x-3)}$ into partial fractions:

$$\frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

Multiplying both sides by $(x - 2)(x - 3)$ and solving for A and B:

$$5x - 5 = A(x - 3) + B(x - 2)$$

Expanding and equating coefficients gives the system:

$$A + B = 5 \quad \text{and} \quad -3A - 2B = -5$$

Solving this, you get:

$$A = -5 \quad \text{and} \quad B = 10$$

Thus, the partial fraction decomposition is:

$$\frac{5x-5}{(x-2)(x-3)} = \frac{-5}{x-2} + \frac{10}{x-3}$$

Step 4: Substitute back into the integral
The integral becomes:

$$\int (1 + \frac{-5}{x-2} + \frac{10}{x-3}) dx$$

Step 5: Integrate each term
Now, integrate each term:
1. $\int 1 dx = x$
2. $\int \frac{-5}{x-2} dx = -5 \ln|x - 2|$
3. $\int \frac{10}{x-3} dx = 10 \ln|x - 3|$

Thus, the result is:

$x-5\ln|x-2|+10\ln|x-3|+C$

20) Find the area of a triangle having the points $\mathrm{A}(1,1,1), \mathrm{B}(1,2,3)$ **and** $\mathrm{C}(2,3,1)$ **as its vertices.**

Solution:

To find the area of a triangle with vertices at points $A(1,1,1)$, $B(1,2,3)$, and $C(2,3,1)$ in 3D space, we can use the following formula based on the cross product of two vectors formed by the vertices:

$$\text{Area of the triangle } = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$

Step 1: Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$

$$\overrightarrow{AB} = B - A = (1-1, 2-1, 3-1) = (0,1,2)$$

$$\overrightarrow{AC} = C - A = (2-1, 3-1, 1-1) = (1,2,0)$$

Step 2: Compute the cross product $\overrightarrow{AB} \times \overrightarrow{AC}$

The cross product of $\overrightarrow{AB} = (0,1,2)$ and $\overrightarrow{AC} = (1,2,0)$ is:

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$$

Expanding the determinant:

$$\begin{aligned} \overrightarrow{AB} \times \overrightarrow{AC} &= \hat{i}((1)(0)-(2)(2)) - \hat{j}((0)(0)-(2)(1)) + \hat{k}((0)(2)-(1)(1)) \\ &= \hat{i}(0-4) - \hat{j}(0-2) + \hat{k}(0-1) \end{aligned}$$

Thus, $\overrightarrow{AB} \times \overrightarrow{AC} = (-4, 2, -1)$.

Step 3: Find the magnitude of the cross product

The magnitude of $\overrightarrow{AB} \times \overrightarrow{AC}$ is:

$$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-4)^2 + (2)^2 + (-1)^2} = \sqrt{16+4+1} = \sqrt{21}$$

Step 4: Calculate the area of the triangle

The area of the triangle is:

$$\text{Area } = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \times \sqrt{21} = \frac{\sqrt{21}}{2}$$

Final Answer:

The area of the triangle is:

$\frac{\sqrt{21}}{2}$

SECTION-C

21) Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x dx$.

Solution:

Your solution is mostly correct, but there is a small correction in the limits of integration and the final expression. Let's walk through the solution step-by-step.

We are tasked with evaluating the integral:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x dx$$

Step 1: Use the identity for $\sin^2 x$
We use the identity:

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

Thus, the integral becomes:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-\cos(2x)}{2} dx$$

Step 2: Split the integral
Now, split the integral into two parts:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-\cos(2x)}{2} dx = \downarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1dx - \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(2x) dx$$

Step 3: Evaluate the first integral
The first integral is straightforward:

$$\frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1dx = \frac{1}{2}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$$

Step 4: Evaluate the second integral
The second integral involves $\cos(2x)$:

$$\frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(2x) dx$$

The antiderivative of $\cos(2x)$ is $\frac{\sin(2x)}{2}$, so:

$$\frac{1}{2}\left[\frac{\sin(2x)}{2}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{4}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right)$$

Since $\sin\left(\frac{\pi}{2}\right) = 1$ and $\sin\left(-\frac{\pi}{2}\right) = -1$, we have:

$$\frac{1}{4} \cdot (1 - (-1)) = \frac{1}{4} \cdot 2 = \frac{1}{2}$$

Step 5: Subtract the results
Now, subtract the result of the second integral from the first:

$$\frac{\pi}{4} - \frac{1}{2}$$

Thus, the final result is:

$\frac{\pi}{4}-\frac{1}{2}$

22) Find the general solution of the differential equation $ydx-\left(x+2y^2\right)dy=0$.

Solution:

Step 1: Rearrange the equation
Given the equation:

$ydx=\left(x+2y^2\right)dy$
Divide both sides by y (assuming $y\neq 0$):

$dx=\frac{x+2y^2}{y}dy$
Step 2: Separate the variables
Rearrange to separate the variables:

$\frac{dx}{x}=(2y+1)dy$
Step 3: Integrate both sides
Integrate both sides:

$\int\frac{dx}{x}=\int(2y+1)dy$
The integrals are:

$\ln\mid x_1\downarrow y^2+y+C$

where C is the constant of integration.
Step 4: Solve for x
Exponentiate both sides to solve for x :

$x=e^{y^2+y+C}$
This can be rewritten as:

$x=Ae^{y^2+y}$

where $A=e^C$ is a new constant.
Final Answer:
The general solution of the differential equation is:

$x=Ae^{y^2+y}$

23)

Maximize

$$Z=5x+3y$$

Subject to the constraints:

$$3x + 5y \leq 15$$
$$5x + 2y \leq 10$$
$$x \geq 0, \quad y \geq 0$$

(using the graphical method).

Solution:

Step 1: Plot the constraints
1. First constraint: $3x + 5y \leq 15$

The line $3x + 5y = 15$ has intercepts:
- $x = 0 \Rightarrow y = 3$
- $y = 0 \Rightarrow x = 5$

The line passes through points $(5, 0)$ and $(0, 3)$. The region $3x + 5y \leq 15$ lies below this line.
2. Second constraint: $5x + 2y \leq 10$

The line $5x + 2y = 10$ has intercepts:
- $x = 0 \Rightarrow y = 5$
- $y = 0 \Rightarrow x = 2$

The line passes through points $(2, 0)$ and $(0, 5)$. The region $5x + 2y \leq 10$ lies below this line.
3. Non-negative constraints: $x \geq 0$ and $y \geq 0$
$\downarrow$

These constraints restrict the feasible region to the first quadrant.

Step 2: Identify the corner points of the feasible region
The feasible region is bounded by the lines $3x + 5y = 15$ and $5x + 2y = 10$, along with the non-negative constraints.

To find the intersection of $3x + 5y = 15$ and $5x + 2y = 10$, solve the system of equations:

$$3x + 5y = 15$$
$$5x + 2y = 10$$

- Multiply the first equation by 5 and the second equation by 3 :

$$15x + 25y = 75$$
$$15x + 6y = 30$$

- Subtract the second equation from the first:

$$(15x + 25y) - (15x + 6y) = 75 - 30$$
This simplifies to:

$$19y = 45 \quad \Rightarrow \quad y = \frac{45}{19}$$

- Substitute $y = \frac{45}{19}$ into $3x + 5y = 15$:

$3x+5\times\frac{45}{19}=15 \quad \Rightarrow \quad 3x+\frac{225}{19}=15$
Simplifying: □

$$3x=15-\frac{225}{19}=\frac{285}{19}-\frac{225}{19}=\frac{60}{19}$$
$$x=\frac{20}{19}$$

Thus, the point of intersection is $\left(\frac{20}{19}, \frac{45}{19}\right)$.
- The intercepts on the axes are:
- From $3x+5y=15$, the points are $(5,0)$ and $(0,3)$.
- From $5x+2y=10$, the points are $(2,0)$ and $(0,5)$.

Step 3: Evaluate the objective function
Now, evaluate the objective function $Z=5x+3y$ at each corner point.
1. At $(5,0)$:

$$Z=5(5)+3(0)=25$$

2. At $(0,3)$:

$$Z=5(0)+3(3)=9$$

3. At $(2,0)$:

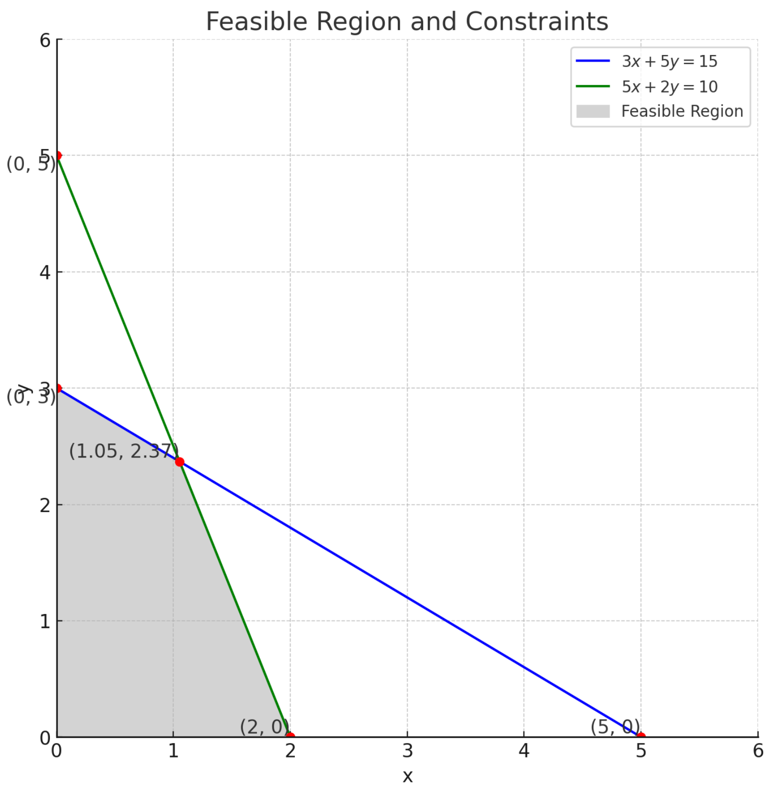
$$Z=5(2)+3(0)=10$$

4. At $(0,5)$:

$$Z=5(0)+3(5)=15$$

5. At $\left(\frac{20}{19}, \frac{45}{19}\right)$:

$Z=5\times\frac{20}{19}+3\times\frac{45}{19}=\frac{100}{19}+\frac{135}{19}=\frac{235}{19}\approx 12.37$
Step 4: Conclusion
The maximum value of $Z=25$ occurs at the point $(5,0)$.
Thus, the maximum value of Z is 25 .


Feasible Region and Constraints
$3x + 5y = 15$
$5x + 2y = 10$
Feasible Region
(0, 5)
(0, 3)
(1.05, 2.37)
(2, 0)
(5, 0)
x
y
0
1
2
3
4
5
6

RBSE Class 12 Maths Question with Solution - 2022

SECTION-A

1. Multiple Choice Questions :

i) If $f : R \rightarrow R$ be defined as $f(x) = x^4$, then the function
(a) f is one-one and onto
(c) f is one-one but not onto
(b) f is many-one-onto
(d) f is neither one-one nor onto

Solution:

ii) The function $f(x) = x^4$ is neither one-one nor onto, so the correct answer is (d).

ii) The value of $2\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ is :
(a) $\frac{\pi}{2}$
(b) $\frac{2\pi}{3}$
(c) $\frac{3\pi}{2}$
(d) $\frac{5\pi}{6}$

Solution:

Step 1: Find $\sin^{-1}\left(\frac{1}{2}\right)$.
We know that $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ because $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
Step 2: Find $\cos^{-1}\left(\frac{1}{2}\right)$.
Similarly, $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ because $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.
Step 3: Substitute and calculate.

$$2\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = 2 \times \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Thus, the value is $\frac{2\pi}{3}$, and the correct answer is:
(b) $\frac{2\pi}{3}$.

iii) If $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ ad $B = \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$ are two matrices, then AB will be

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$
Solution:

To find AB, multiply the matrices $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$:

$$AB = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, the correct answer is (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

iv) In the equation $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, the value of x is

(a) 3
(b) 4
(c) 5
(d) 2

Solution:

From the given system of equations:

$$x+y+z=9, \quad x+z=5, \quad y+z=7$$

subtract the second equation from the first: $(x+y+z)-(x+z)=9-5$, which gives $y=4$

Now, substitute $y=4$ into $y+z=7$, so $4+z=7$, which gives $z=3$.

Finally, substitute $z=3$ into $x+z=5$, so $x+3=5$, which gives $x=2$.

Thus, the value of x is (d) 2 .

v) If $3x+2y=\sin x$, then the $\frac{dy}{dx}$ is
(a) $\frac{\cos x+3}{2}$
(b) $\frac{\cos x-2}{3}$
(c) $\frac{\cos x-3}{2}$
(d) $\frac{\cos x+2}{3}$
Solution:

Step 1: Differentiate both sides with respect to x.

$$\frac{d}{dx}(3x+2y)=\frac{d}{dx}(\sin x)$$
This gives:

$3+2\frac{dy}{dx}=\cos x$

Step 2: Solve for $\frac{dy}{dx}$.

$$2\frac{dy}{dx}=\cos x-3$$
$$\frac{dy}{dx}=\frac{\cos x-3}{2}$$

Thus, the correct answer is (c) $\frac{\cos x-3}{2}$.

vi) If $\begin{vmatrix}3 & 3\\ x & 1\end{vmatrix}=\begin{vmatrix}-3 & x\\ 1 & 1\end{vmatrix}$, then value of x is

(a) 2

(b) 3

(c) -3

(d) -2

Solution:

Step 1: Evaluate both determinants.

The determinant on the left-hand side is:

$$\begin{vmatrix}3 & 3\\ x & 1\end{vmatrix}=(3)(1)-(3)(x)=3-3x$$

The determinant on the right-hand side is:

$$\begin{vmatrix}-3 & x\\ 1 & 1\end{vmatrix}=(-3)(1)-(x)(1)=-3-x$$

Step 2: Set the two determinants equal.

$$3-3x=-3-x$$

Step 3: Solve for x.

$$3+3=3x-x$$
$$6=2x$$
$$x=3$$

Thus, the value of x is (b) 3 .

vii) The second order derivative of $x^3\log x$ w.r. to x is

(a) $x(5+6\log x)$

(c) $x(6+5\log x)$

(b) $x^2(5+6\log x)$

(d) $x^2(6+5\log x)$

Solution:

The second-order derivative of $x^3\log x$ with respect to x is $(c)x(6+5\log x)$.

viii) The integration of the function $\frac{x}{e^{x^2}}$, with respect to x is

(a) $\frac{1}{2e^{x^2}}+C$

(b) $\frac{2}{e^{x^2}} + C$
(c) $-\frac{2}{e^{x^2}} + C$
(d) $-\frac{1}{2e^{x^2}} + \mathrm{C}$

Solution:

The integration of the function $\frac{x}{e^{x^2}}$ with respect to x is:

$\int \frac{x}{e^{x^2}} dx$
We can use substitution. Let $u = x^2$, so $du = 2x dx$, or $\frac{du}{2} = x dx$.
Thus, the integral becomes:

$\int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{du}{e^u} = -\frac{1}{2} e^{-u} + C$
Substituting $u = x^2$ back in, we get:

$-\frac{1}{2} e^{-x^2} + C = -\frac{1}{2e^{x^2}} + C$
Thus, the correct answer is:

$(d) - \frac{1}{2e^{x^2}} + C$
ix) The degree of differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^3 - y\frac{dy}{dx} = 0$ is

(a) 3
(b) 1
(c) 0
(d) 2

Solution:

The given differential equation is:

$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^3 - y\frac{dy}{dx} = 0$
Step 1: Check the order of the differential equation.
The highest derivative present is $\frac{d^2y}{dx^2}$, which makes the order of the differential equation 2 .
Step 2: Check the degree of the highest derivative.
The degree of the differential equation is the exponent of the highest-order derivative, provided the equation is polynomial in derivatives. In this equation, $\frac{d^2y}{dx^2}$ appears to the power of 1 (it is not raised to any higher power or involved in any non-polynomial function).

Thus, the degree of the differential equation is 1.

The correct answer is:
$(b) 1$
x) The magnitude of the vector $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$ is

(a) 3

(b) 1
(c) -1
(d) 2

Solution:

Step 1: Use the formula for the magnitude of a vector.
The magnitude of a vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ is given by:

$$|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$$

Step 2: Apply the formula.
In this case, $a = \frac{1}{\sqrt{3}}, b = \frac{1}{\sqrt{3}}, c = -\frac{1}{\sqrt{3}}$. So:

$$|\vec{v}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$$

$$|\vec{v}| = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \sqrt{1} = 1$$

Thus, the magnitude of the vector is 1 , and the correct answer is:
(b) 1

xi) The unit vector in the direction of the vector $\vec{a} = -2\hat{i} + 3\hat{j} - \hat{k}$ is
(a) $\frac{2\hat{i}}{\sqrt{14}} - \frac{3\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$
(b) $\frac{2\hat{\mathrm{i}}}{\sqrt{14}} - \frac{3\hat{\mathrm{j}}}{\sqrt{14}} - \frac{\hat{\mathrm{k}}}{\sqrt{14}}$
(c) $\frac{2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$
(d) $\frac{-2\hat{\mathrm{i}}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} - \frac{\hat{k}}{\sqrt{14}}$

Solution:

Step 1: Find the magnitude of $\vec{a}$.
The magnitude of a vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is given by:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

For $\vec{a} = -2\hat{i} + 3\hat{j} - \hat{k}$, we have:

$$|\vec{a}| = \sqrt{(-2)^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

Step 2: Find the unit vector.
The unit vector in the direction of $\vec{a}$ is:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{-2\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{14}}$$

This gives:

$\hat{a}=\frac{-2}{\sqrt{14}}\hat{i}+\frac{3}{\sqrt{14}}\hat{j}-\frac{1}{\sqrt{14}}\hat{k}$
Thus, the unit vector is:
(d) $\frac{-2\hat{i}}{\sqrt{1}} \downarrow \frac{3\hat{j}}{\sqrt{14}}-\frac{\hat{k}}{\sqrt{14}}$

xii) If a die is thrown twice, then the probability of getting the sum of the numbers equal to 7 , will be
(a) $\frac{5}{36}$
(b) $\frac{1}{12}$
(c) $\frac{1}{6}$
(d) $\frac{1}{9}$
Solution:

When a die is thrown twice, the total number of possible outcomes is $6 \times 6=36$ since there are 6 possible outcomes for each throw.

To get a sum of 7 , the following pairs of numbers on the dice are possible:
- $(1,6)$
- $(2,5)$
- $(3,4)$
- $(4,3)$
- $(5,2)$
- $(6,1)$

There are 6 favorable outcomes.

Thus, the probability of getting a sum of 7 is:

$\frac{6}{36}=\frac{1}{6}$
The correct answer is:

$\downarrow, c) \frac{1}{6}$

2. Fill in the blanks :

i) If $\mathrm{f}(x)=27 x^3$ and $\mathrm{g}(x)=x^{1/3}$, then $\mathrm{gof}(x)=$

Solution:

We are given two functions $f(x)=27 x^3$ and $g(x)=x^{1/3}$. We need to find $g \circ f(x)$, which means $g(f(x))$.

Step 1: Substitute $f(x)$ into $g(x)$.

$g(f(x))=g\left(27 x^3\right)$
Step 2: Apply the function $g(x)=x^{1/3}$.

$g\left(27 x^3\right)=\left(27 x^3\right)^{1/3}$
Step 3: Simplify the expression.

$\left(27x^3\right)^{1/3} = 27^{1/3} \cdot \left(x^3\right)^{1/3} = 3 \cdot x = 3x$
Thus, $g \circ f(x) = 3x$.

The final answer is:
$3x$

ii) The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is

Solution:

We are asked to find the principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.
Step 1: Recall the range of $\cos^{-1}(x)$
The principal value of $\cos^{-1}(x)$ lies in the interval $[0, \pi]$.
Step 2: Solve for θ
We need to find $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$, which means $\cos(\theta) = \frac{-1}{\sqrt{2}}$.
We know that $\cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$.
Step 3: Principal value
Since $\frac{3\pi}{4}$ lies within the range $[0, \pi]$, the principal value is:

$\boxed{\frac{3\pi}{4}}$

iii) If $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, then $A - B =$
Solution:

We are given the matrices:

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$$
To find $A - B$, we subtract the corresponding elements of B from A.

$$A - B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1-2 & 3-4 \\ -2-3 & 5-2 \end{bmatrix}$$
Simplifying the entries:

$$A - B = \begin{bmatrix} -1 & -1 \\ -5 & 3 \end{bmatrix}$$
Thus, the result is:

$$\begin{bmatrix} -1 & -1 \\ -5 & 3 \end{bmatrix}$$
iv) If $3x + 2y = \cos y$, then $\frac{dy}{dx} =$

Solution:

Step 1: Differentiate both sides of the equation with respect to x.
Differentiating both sides:

$$\frac{d}{dx}(3x+2y)=\frac{d}{dx}(\cos y)$$

This gives:

$$3+2\frac{dy}{dx}=-\sin y\cdot\frac{dy}{dx}$$

Step 2: Solve for $\frac{dy}{dx}$.
Rearranging the equation:

$$2\frac{dy}{dx}+\sin y\cdot\frac{dy}{dx}=-3$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx}(2+\sin y)=-3$$

Finally, solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx}=\frac{-3}{2+\sin y}$$

v) The value of $\int_0^1\frac{dx}{1+x^2}$ is

Solution:

The given integral is:

$$I=\int_0^1\frac{dx}{1+x^2}$$

Step 1: Recognize the standard integral form.
The integral $\int\frac{dx}{1+x^2}$ is the standard form of $\tan^{-1}(x)$, whose derivative is $\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$

Step 2: Apply the formula.
Thus, we can write:

$$I=\left[\tan^{-1}(x)\right]_0^1$$

Step 3: Evaluate the limits.

$$I=\tan^{-1}(1)-\tan^{-1}(0)$$

We know that $\tan^{-1}(1)=\frac{\pi}{4}$ and $\tan^{-1}(0)=0$, so:

$$I=\frac{\pi}{\downarrow}-0=\frac{\pi}{4}$$

vi) The vector joining the points $\mathrm{A}(1,2,2)$ and $\mathrm{B}(2,3,1)$ directed from A to B is

Solution:

The vector joining the points $A(1,2,2)$ and $B(2,3,1)$, directed from A to B, is given by subtracting the coordinates of point A from the coordinates of point B.

The formula for the vector from point $A\left(x_1,y_1,z_1\right)$ to point $B\left(x_2,y_2,z_2\right)$ is:

$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
Substituting the coordinates of $A(1,2,2)$ and $B(2,3,1)$:

$\overrightarrow{AB} = (2-1, 3-2, 1-2) = (1,1,-1)$
Thus, the vector directed from A to B is:

$(1,1,-1)$

3. Very short answer-type questions :

i) Show that the function f : N $\rightarrow$ N, given by f$(x) = 2x$ is not onto.

Solution:

To show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x$ is not onto, we need to demonstrate that there is at least one element in the codomain (which is $\mathbb{N}$) that is not the image of any element in the domain $\mathbb{N}$ under f.

Step 1: Define the function
The function $f(x) = 2x$ takes any natural number x and maps it to $2x$. This means that for any $x \in \mathbb{N}$, the output of the function is always an even number because $2x$ is always even for any natural number x.

Step 2: Identify an element in $\mathbb{N}$ that cannot be reached
The codomain of the function is the set of natural numbers, $\mathbb{N} = \{1,2,3,4,5,\ldots\}$. However, the function $f(x) = 2x$ only produces even numbers like $2,4,6,8,\ldots$.

For example, the number $1 \in \mathbb{N}$ is not in the image of f, because there is no $x \in \mathbb{N}$ such that $f(x) = 1$ (since $2x$ can never be odd). Similarly, other odd numbers like $3,5,7,\ldots$ are also not in the image of f.

Step 3: Conclusion
Since not every natural number (specifically, odd numbers like $1,3,5,\ldots$) is the image of some $x \in \mathbb{N}$ under f, the function is not onto (surjective).

Thus, $f(x) = 2x$ is not onto.

ii) Find the value of $3\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Solution:

Step 1: Find the value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
We know that $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$, since $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.
Step 2: Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

We know that $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$, since $\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$.

Step 3: Substitute these values into the expression

Now substitute the values into the given expression:

$$3\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=3\times\frac{\pi}{6}+\frac{\pi}{3}$$

Step 4: Simplify the expression

$$3\times\frac{\pi}{6}=\frac{\pi}{2},\quad \frac{\pi}{3}=\frac{\pi}{3}$$

Now add them together:

$$\frac{\pi}{2}+\frac{\pi}{3}=\frac{3\pi}{6}+\frac{2\pi}{6}=\frac{5\pi}{6}$$

Final Answer:

The value of the expression is:

$$\frac{5\pi}{6}$$

iii) Write the identity matrix of 3×3 order.

Solution:

The identity matrix of order 3×3 is:

$$I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

This matrix has 1 s along the main diagonal and 0 s elsewhere.

iv) For which value of x, the value of determinant $\begin{vmatrix}3&2\\5&x\end{vmatrix}$ will be zero?

Solution:

The determinant of the given matrix is:

$$\begin{vmatrix}3&2\\5&x\end{vmatrix}=(3)(x)-(2)(5)$$

We want this determinant to be zero:

$$3x-10=0$$

Step 1: Solve for x

$$3x=10$$
$$x=\frac{10}{3}$$

Thus, the value of x that makes the determinant zero is:

$\boxed{\frac{10}{3}}$

v) Find the minor of the element 6 in the determinant $\begin{vmatrix} 1 & 3 & 2 \\ 8 & 6 & 3 \\ 9 & 5 & 4 \end{vmatrix}$.

Solution:

To find the minor of the element 6 in the determinant:

$$\begin{vmatrix} 1 & 3 & 2 \\ 8 & 6 & 3 \\ 9 & 5 & 4 \end{vmatrix}$$

we first identify the position of the element 6 . It is located in the second row and second column. To calculate its minor, we remove the second row and second column from the matrix and compute the determinant of the remaining 2×2 matrix.

The remaining matrix is:

$$\begin{bmatrix} 1 & 2 \\ 9 & 4 \end{bmatrix}$$

Now, we compute the determinant of this 2×2 matrix:

$$\text{Determinant } = (1)(4) - (2)(9) = 4 - 18 = -14$$

Thus, the minor of the element 6 is:

$$-14$$

vi) Show that the function $\mathrm{f}(x) = x^2$, is continuous at $x = 0$.

Solution:

To show that the function $f(x) = x^2$ is continuous at $x = 0$, we need to check the definition of continuity at a point. A function $f(x)$ is continuous at $x = c$ if:

$$\lim_{x \to c} f(x) = f(c)$$

In this case, $c = 0$, so we need to show that:

$$\lim_{x \to 0} f(x) = f(0)$$

Step 1: Find $f(0)$

The function is $f(x) = x^2$, so:

$$f(0) = 0^2 = 0$$

Step 2: Find $\lim_{x \to 0} f(x)$

The limit of $f(x) = x^2$ as $x \to 0$ is:

$$\lim_{x \to 0} x^2 = 0^2 = 0$$

Step 3: Compare the limit and the function value

We found that:

$\lim_{x \to 0} f(x) = 0 \quad \text{and} \quad f(0) = 0$
Since $\lim_{x \to 0} f(x) = f(0)$, the function $f(x) = x^2$ is continuous at $x = 0$. Thus, $f(x) = x^2$ is continuous at $x = 0$.

3. Very short answer-type questions :

i) Show that the function f : $\mathrm{N} \rightarrow \mathrm{N}$, given by $\mathrm{f}(x) = 2x$ is not onto.

Solution:

To show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x$ is not onto, we need to demonstrate that there is at least one element in the codomain (which is $\mathbb{N}$) that is not the image of any element in the domain $\mathbb{N}$ under f.

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The function $f(x) = 2x$ takes any natural number x and maps it to $2x$. This means that for any $x \in \mathbb{N}$, the output of the function is always an even number because $2x$ is always even for any natural number x.

Step 2: Identify an element in $\mathbb{N}$ that cannot be reached
The codomain of the function is the set of natural numbers, $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$. However, the function $f(x) = 2x$ only produces even numbers like $2, 4, 6, 8, \ldots$.

For example, the number $1 \in \mathbb{N}$ is not in the image of f, because there is no $x \in \mathbb{N}$ such that $f(x) = 1$ (since $2x$ can never be odd). Similarly, other odd numbers like $3, 5, 7, \ldots$ are also not in the image of f.

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Since not every natural number (specifically, odd numbers like $1, 3, 5, \ldots$) is the image of some $x \in \mathbb{N}$ under f, the function is not onto (surjective).

Thus, $f(x) = 2x$ is not onto.

ii) Find the value of $3 \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Solution:

Step 1: Find the value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
We know that $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$, since $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.
Step 2: Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
We know that $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$, since $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$.
Step 3: Substitute these values into the expression
Now substitute the values into the given expression:

$3 \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 3 \times \frac{\pi}{6} + \frac{\pi}{3}$
Step 4: Simplify the expression

$3 \times \frac{\pi}{6} = \frac{\pi}{2}, \quad \frac{\pi}{3} = \frac{\pi}{3}$

Now add them together:

$\frac{\pi}{2} + \frac{\pi}{3} = \frac{3\pi}{6} + \frac{2\pi}{6} = \frac{5\pi}{6}$

Final Answer:

The value of the expression is:

$\frac{5\pi}{6}$

iii) Write the identity matrix of 3×3 order.

Solution:

The identity matrix of order 3×3 is:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix has 1 s along the main diagonal and 0 s elsewhere.

iv) For which value of x, the value of determinant $\begin{vmatrix} 3 & 2 \\ 5 & x \end{vmatrix}$ will be zero?

Solution:

The determinant of the given matrix is:

$$\begin{vmatrix} 3 & 2 \\ 5 & x \end{vmatrix} = (3)(x) - (2)(5)$$

We want this determinant to be zero:

$$3x - 10 = 0$$

Step 1: Solve for x

$$3x = 10$$

$$x = \frac{10}{3}$$

Thus, the value of x that makes the determinant zero is:

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Solution:

To find the minor of the element 6 in the determinant:

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we first identify the position of the element 6 . It is located in the second row and second column. To calculate its minor, we remove the second row and second column from the matrix and compute the determinant of the remaining 2×2 matrix.

The remaining matrix is:

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Now, we compute the determinant of this 2×2 matrix:

$$\text{Determinant } = (1)(4) - (2)(9) = 4 - 18 = -14$$

Thus, the minor of the element 6 is:

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vi) Show that the function $\mathrm{f}(x) = x^2$, is continuous at $x = 0$.

Solution:

To show that the function $f(x) = x^2$ is continuous at $x = 0$, we need to check the definition of continuity at a point. A function $f(x)$ is continuous at $x = c$ if:

$$\lim_{x \to c} f(x) = f(c)$$

In this case, $c = 0$, so we need to show that:

$$\lim_{x \to 0} f(x) = f(0)$$

Step 1: Find $f(0)$

The function is $f(x) = x^2$, so:

$$f(0) = 0^2 = 0$$

Step 2: Find $\lim_{x \to 0} f(x)$

The limit of $f(x) = x^2$ as $x \to 0$ is:

$$\lim_{x \to 0} x^2 = 0^2 = 0$$

Step 3: Compare the limit and the function value

We found that:

$$\lim_{x \to 0} f(x) = 0 \quad \text{and} \quad f(0) = 0$$

Since $\lim_{x \to 0} f(x) = f(0)$, the function $f(x) = x^2$ is continuous at $x = 0$. Thus, $f(x) = x^2$ is continuous at $x = 0$.

vii) Find $\int \sqrt[3]{x^4} dx$

Solution:

The integral $\int \sqrt[3]{x^4} dx$ can be rewritten as:

$\int x^{4/3} dx$

Now, apply the power rule for integration:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$

For $n = \frac{4}{3}$:

$\int x^{4/3} dx = \frac{x^{(4/3)+1}}{(4/3)+1} + C = \frac{x^{7/3}}{7/3} + C = \frac{3}{7} x^{7/3} + C$

Thus, the solution is:

$\frac{3}{7} x^{7/3} + C$

viii) Find the general solution of the differential equation $(1 + x^2) dy = (1 + y^2) \mathrm{d}x$.

Solution:

The given differential equation is:

$(1 + x^2) dy = (1 + y^2) dx$

Step 1: Rearrange the equation.

Rearranging the terms to separate variables:

$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

Step 2: Integrate both sides.

We now integrate both sides:

$\int \frac{1}{1+y^2} dy = \int \frac{1}{1+x^2} dx$

We know that:

$\int \frac{1}{1+y^2} dy = \tan^{-1}(y)$ and $\int \frac{1}{1+x^2} dx = \tan^{-1}(x)$

So, we have:

$\tan^{-1}(y) = \tan^{-1}(x) + C$

Step 3: Solve for the general solution.

Taking the tangent of both sides:

$y = \tan\left(\tan^{-1}(x) + C\right)$

Thus, the general solution is:

$y = \tan\left(\tan^{-1}(x) + C\right)$

ix) Find the equation of a curve if the slope to the tangent at the point (x, y) of the curve is $\frac{3x^2}{y^2}$.

Solution:

We are given that the slope of the tangent to the curve at the point (x, y) is $\frac{3x^2}{y^2}$. This means:

$\frac{dy}{dx} = \frac{3x^2}{y^2}$
We need to find the equation of the curve.
Step 1: Rearrange the differential equation.
Rearrange to separate the variables:

$y^2 dy = 3x^2 dx$
Step 2: Integrate both sides.
Integrate both sides:

$\int y^2 dy = \int 3x^2 dx$
The integrals are:

$\frac{y^3}{3} = x^3 + C$

Step 3: Multiply both sides by 3 to simplify.

$y^3 = 3x^3 + C'$

where $C' = 3C$.

Thus, the equation of the curve is:

$y^3 = 3x^3 + C'$

x) Find the unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\overrightarrow{\mathrm{b}} = 2\hat{\mathrm{i}} + \hat{\mathrm{j}} + 3\hat{\mathrm{k}}$.

Solution:

Step 1: Find the sum of the vectors.

$\vec{a} + \vec{b} = (2\hat{\imath} + 2\hat{\imath}) + (2\hat{\jmath} + \hat{\jmath}) + (-5\hat{k} + 3\hat{k})$
Simplifying:

$\vec{a} + \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$
Step 2: Find the magnitude of the sum vector.
The magnitude of $\vec{a} + \vec{b}$ is:

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 3^2 + (-2)^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$
Step 3: Find the unit vector.
The unit vector in the direction of $\vec{a} + \vec{b}$ is:

$\hat{u} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{4\hat{\imath}+3\hat{\jmath}-2\hat{k}}{\sqrt{29}}$
Thus, the unit vector is:

$\frac{4}{\sqrt{29}}\hat{\imath} + \frac{3}{\sqrt{29}}\hat{\jmath} - \frac{2}{\sqrt{29}}\hat{k}$

xi) Find the vector components of the vector with initial point $(2, 1)$ and terminal point $(-5, 7)$.

Solution:

To find the vector components of a vector given its initial point $P(2,1)$ and terminal point $Q(-5,7)$, we use the formula for the vector from point $P\left(x_1, y_1\right)$ to point $Q\left(x_2, y_2\right)$:

$\overrightarrow{PQ}=\left(x_2-x_1, y_2-y_1\right)$
Substitute the coordinates of $P(2,1)$ and $Q(-5,7)$:

$\overrightarrow{PQ}=(-5-2,7-1)=(-7,6)$
Thus, the vector components of the vector are:

$(-7,6)$
xii) If a die is thrown once, then find the probability that the number appeared on the die is a multiple of 2 .

Solution:

When a die is thrown, the possible outcomes are $\{1,2,3,4,5,6\}$.
The multiples of 2 from these outcomes are $\{2,4,6\}$.
Step 1: Total number of possible outcomes
There are 6 possible outcomes when a die is thrown.
Step 2: Number of favorable outcomes
The favorable outcomes (multiples of 2) are $\{2,4,6\}$, so there are 3 favorable outcomes.
Step 3: Calculate the probability
The probability is given by:

$P(\text{ multiple of } 2)=\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}=\frac{3}{6}=\frac{1}{2}$
Thus, the probability is:

$\frac{1}{2}$

SECTION-B

4. Considering $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ given by $\mathrm{f}(x)=2 x+3$, prove that f is invertible.

Solution:

To prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x)=2 x+3$ is invertible, we need to demonstrate two things:
1. f is one-one (injective): This means that for any x_1 and x_2, if $f\left(x_1\right)=f\left(x_2\right)$, then $x_1=x_2$
2. f is onto (surjective): This means that for any $y \in \mathbb{R}$, there exists an $x \in \mathbb{R}$ such that $f(x)=y$.

Step 1: Prove $f(x)=2 x+3$ is one-one
Let $f\left(x_1\right)=f\left(x_2\right)$

$2 x_1+3=2 x_2+3$
Subtract 3 from both sides:

$2x_1 = 2x_2$
Divide by 2 :

$x_1 = x_2$
Thus, f is injective (one-one).

Step 2: Prove $f(x) = 2x + 3$ is onto
We need to show that for any $y \in \mathbb{R}$, there exists an $x \in \mathbb{R}$ such that $f(x) = y$.

Let $y = f(x) = 2x + 3$. Solve for x :

$y = 2x + 3$
Subtract 3 from both sides:

$y - 3 = 2x$
Divide by 2 :

$x = \frac{y-3}{2}$
Since $x \in \mathbb{R}$ for any $y \in \mathbb{R}$, the function is surjective (onto).
Step 3: Find the inverse function
To find the inverse function, solve for x in terms of y :

$$y = 2x + 3$$
$$\downarrow \frac{y-3}{2}$$

So, the inverse function is:

$f^{-1}(y) = \frac{y-3}{2}$
Thus, f is invertible and its inverse is $f^{-1}(y) = \frac{y-3}{2}$.

5. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$, then verify that $(AB)' = B'A'$.

Solution:

Step 1: Compute AB
Since A is a 3×1 matrix and B is a 1×3 matrix, their product AB will result in a 3×3 matrix.

$$AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & -6 \end{bmatrix} = \begin{bmatrix} -2 \times 1 & -2 \times 3 & -2 \times (-6) \\ 4 \times 1 & 4 \times 3 & 4 \times (-6) \\ 5 \times 1 & 5 \times 3 & 5 \times (-6) \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$$

Step 2: Find $(AB)'$
The transpose of AB is obtained by swapping rows with columns:

$$(AB)' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

Step 3: Compute $B'A'$

Next, we compute $B'A'$, where B' is the transpose of B and A' is the transpose of A.

$$B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}, \quad A' = \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

Now, compute the product $B'A'$, which results in a 3×3 matrix:

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \times \begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 \times (-2) & 1 \times 4 & 1 \times 5 \\ 3 \times (-2) & 3 \times 4 & 3 \times 5 \\ -6 \times (-2) & -6 \times 4 & -6 \times 5 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

Step 4: Conclusion

We observe that:

$$(AB)' = B'A'$$

Thus, the identity $(AB)' = B'A'$ is verified.

6. If $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ then find X and Y.

Solution:

Step 1: Add equations (1) and (2).

Add the two equations:

$$(X + Y) + (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Simplifying the left side:

$$X + X = 2X$$

Now add the matrices on the right side:

$$2X = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

Step 2: Solve for X.

Divide both sides by 2 :

$$X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

So, $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Step 3: Subtract equation (2) from equation (1).
Now subtract equation (2) from equation (1):

$$(X+Y)-(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$$

Simplifying the left side:

$$Y+Y=2Y$$

Now subtract the matrices on the right side:

$$2Y=\begin{bmatrix}7-3 & 0-0\\ 2-0 & 5-3\end{bmatrix}=\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}$$

Step 4: Solve for Y.
Divide both sides by 2 :

$$Y=\frac{1}{2}\begin{bmatrix}4 & 0\\ 2 & 2\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

So, $Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$

Final Answer:
The matrices X and Y are:

$$X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}, \quad Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}.$$

7. If $\begin{vmatrix}2 & 3\\ y & x\end{vmatrix}=3, \begin{vmatrix}x & y\\ 4 & 2\end{vmatrix}=5$, then find the values of x and y.

Solution:

Step 1: Expand both determinants.
For the first determinant (1):

$$\begin{vmatrix}2 & 3\\ y & x\end{vmatrix}=(2)(x)-(3)(y)=2x-3y$$

So the equation becomes:

$$2x-3y=3$$

For the second determinant (2):

$$\begin{vmatrix}x & y\\ 4 & 2\end{vmatrix}=(x)(2)-(y)(4)=2x-4y$$

So the equation becomes:

$$2x-4y=5$$

Step 2: Solve the system of equations.
We now have the system of equations:

$2x-3y=3$
$2x-4y=5$
Subtract equation (3) from equation (4):

$(2x-4y)-(2x-3y)=5-3$
Simplifying:

$$-4y+3y=2$$
$$-y=2$$
$$y=-2$$

Step 3: Substitute $y=-2$ into one of the original equations.
Substitute $y=-2$ into equation (3):

$$2x-3(-2)=3$$
$$2x+6=3$$
$$2x=3-6$$
$$2x=-3$$
$$x=-\frac{3}{2}$$

Final Answer:
The values of x and y are:

$x=-\frac{3}{2}, \quad y=-2$
8. Prove that the points $\mathrm{A}(\mathrm{a}, \mathrm{b}+\mathrm{c}), \mathrm{B}(\mathrm{b}, \mathrm{c}+\mathrm{a})$ and $\mathrm{C}(\mathrm{c}, \mathrm{a}+\mathrm{b})$ are collinear.

Solution:

To prove that the points $A(a, b+c), B(b, c+a)$, and $C(c, a+b)$ are collinear, we can use the condition that three points are collinear if the area of the triangle formed by them is zero.

The area of the triangle formed by three points $A\left(x_1, y_1\right), B\left(x_2, y_2\right)$, and $C\left(x_3, y_3\right)$ is given by:

$\text{Area }=\frac{1}{2}\left|x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right|$
If the area is zero, the points are collinear.
Step 1: Apply the formula
Let the coordinates of points A, B, and C be:
- $A(a, b+c)$
- $B(b, c+a)$
- $C(c, a+b)$

The area of the triangle formed by these points is:

$\text{Area }=\frac{1}{2}|a((c+a)-(a+b))+b((a+b)-(b+c))+c((b+c)-(c+a))|$

Step 2: Simplify each term
Simplify the terms inside the absolute value:
- First term: $(c+a)-(a+b)=c-b$

- Second term: $(a+b)-(b+c)=a-c$
- Third term: $(b+c)-(c+a)=b-a$

Substitute these into the formula:

$$\text{Area } =\frac{1}{2}|a(c-b)+b(a-c)+c(b-a)|$$

Step 3: Expand and simplify
Now expand the terms:

$$\text{Area } =\frac{1}{2}(ac-ab+ba-bc+cb-ca)$$

Group the like terms:

$$\text{Area } =\frac{1}{2}(ac-ca+ab-ab+bc-bc)$$

All terms cancel out:

$$\text{Area } =\frac{1}{2}\times 0=0$$

Step 4: Conclusion
Since the area of the triangle is zero, the points $A(a, b+c), B(b, c+a)$, and $C(c, a+b)$ are collinear.

9. If $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right); 0<x<1$, then find $\frac{dy}{dx}$.

Solution:

Step 1: Differentiate both sides with respect to x
Using the chain rule for the derivative of the inverse sine function, we know that:

$$\frac{d}{dx}\left(\sin^{-1}(u)\right)=\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

Here, $u=\frac{2x}{1+x^2}$, so we first compute the derivative of u with respect to x.
Step 2: Differentiate $u=\frac{2x}{1+x^2}$
Using the quotient rule:

$$\frac{du}{dx}=\frac{(1+x^2) \cdot 2-2x \cdot 2x}{(1+x^2)^2}$$

Simplifying the numerator:

$$\frac{du}{dx}=\frac{2\left(1+x^2\right)-4x^2}{(1+x^2)^2}=\frac{2+2x^2-4x^2}{(1+x^2)^2}=\frac{2-2x^2}{(1+x^2)^2}$$

$$\frac{du}{dx}=\frac{2\left(1-x^2\right)}{(1+x^2)^2}$$

Step 3: Use the chain rule
Now, applying the derivative of the inverse sine function:

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{du}{dx}$$

First, simplify the square inside the square root:

$\left(\frac{2x}{1+x^2}\right)^2 = \frac{4x^2}{(1+x^2)^2}$

Thus, the expression becomes:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{4x^2}{\left(1+x^2\right)^2}}} \cdot \frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

Step 4: Simplify the square root

Simplify the square root term:

$$1 - \frac{4x^2}{(1+x^2)^2} = \frac{\left(1+x^2\right)^2-4x^2}{(1+x^2)^2} = \frac{\left(1+x^2-2x\right)\left(1+x^2+2x\right)}{(1+x^2)^2} = \frac{\left(1-x^2\right)^2}{(1+x^2)^2}$$

Thus:

$$\sqrt{1-\frac{4x^2}{(1+x^2)^2}} = \frac{1-x^2}{1+x^2}$$

Step 5: Final expression for $\frac{dy}{dx}$

Now, substitute back:

$$\frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \cdot \frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

Simplifying:

$$\frac{dy}{dx} = \frac{1+x^2}{1-x^2} \cdot \frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

10) If $y = 3\cos x - 2\sin x$, then prove that $\frac{d^2y}{dx^2} + y = 0$.

Solution:

Step 1: Find the first derivative of y

Differentiate y with respect to x :

$$\frac{dy}{dx} = \frac{d}{dx}(3\cos x - 2\sin x)$$

Using the derivative rules $\frac{d}{dx}(\cos x) = -\sin x$ and $\frac{d}{dx}(\sin x) = \cos x$:

$$\frac{dy}{dx} = -3\sin x - 2\cos x$$

Step 2: Find the second derivative of y

Differentiate $\frac{dy}{dx}$ with respect to x again:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-3\sin x - 2\cos x)$$

Using the same derivative rules:

$$\frac{d^2y}{dx^2} = -3\cos x + 2\sin x$$

Step 3: Add $\frac{d^2y}{dx^2}$ and y

Now, we add $\frac{d^2y}{dx^2}$ and y :

$\frac{d^2y}{dx^2} + y = (-3\cos x + 2\sin x) + (3\cos x - 2\sin x)$
Simplifying the expression:

$\frac{d^2y}{dx^2} + y = -3\cos x + 2\sin x + 3\cos x - 2\sin x = 0$
Thus, we have shown that:

$\frac{d^2y}{dx^2} + y = 0$
11) Find the antiderivative $\mathrm{F}(x)$ of f defined by $\mathrm{f}(x) = 5x^4 - 5$, where $\mathrm{F}(0) = 2$.

Solution:

Step 1: Find the general antiderivative
To find the antiderivative $F(x)$, we integrate $f(x)$:

$F(x) = \int \left(5x^4 - 5\right) dx$
Integrating each term separately:

$F(x) = \int 5x^4 dx - \int 5 dx$
Using the power rule for integration:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
We get:

$F(x) = 5 \cdot \frac{x^5}{5} - 5x + C$
Simplifying:

$F(x) = x^5 - 5x + C$

Step 2: Use the initial condition $F(0) = 2$
We are given that $F(0) = 2$. Substituting $x = 0$ into the expression for $F(x)$:

$F(0) = 0^5 - 5(0) + C = C$
So, $C = 2$.
Step 3: Write the final antiderivative
The antiderivative is:

Thus, the solt
$x) = x^5 - 5x + 2$
$x) = x^5 - 5x + 2$

12) Find $\int \frac{\cos^2 x}{1+\sin x} dx$

Solution:

We are tasked with finding the integral:

$I=\int \frac{\cos ^2 x}{1+\sin x} d x$
Step 1: Simplify using substitution
We will use the substitution $u=\sin x$, which gives $d u=\cos x d x$.

Also, $\cos ^2 x=1-\sin ^2 x=1-u^2$.

Thus, rewrite the integral:

$I=\int \frac{\left(1-u^2\right)}{1+u} \cdot \frac{d u}{\cos x}$ wherewe
13) Find the integration factor of the differential equation $x \frac{d y}{d x}-y=2 x^2$.

Solution:

The given differential equation is:

$x \frac{d y}{d x}-y=2 x^2$
We need to find the integrating factor to solve this differential equation. First, let's rewrite the equation in standard linear form:

$\frac{d y}{d x}-\frac{y}{x}=2 x$
This is a first-order linear differential equation of the form:

$\frac{d y}{d x}+P(x) y=Q(x)$

where $P(x)=-\frac{1}{x}$ and $Q(x)=2 x$.

Step 1: Find the integrating factor
The integrating factor (IF) for a first-order linear differential equation is given by:

$\mu(x)=e^{\int P(x) d x}$
Here, $P(x)=-\frac{1}{x}$, so:

$\mu(x)=e^{\int-\frac{1}{x} d x}=e^{-\ln |x|}=\frac{1}{x}$
Step 2: Conclusion
The integrating factor for the given differential equation is:

$\boxed{\frac{1}{x}}$
14) Find $|\vec{a}-\vec{b}|$; if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2,|\vec{b}|=3$ and $\vec{a} \cdot \vec{b}=4$.

Solution:

We are asked to find $|\vec{a}-\vec{b}|$ given the following information:
- $|\vec{a}|=2$
- $|\vec{b}|=3$
- $\vec{a} \cdot \vec{b}=4$

Step 1: Use the formula for the magnitude of the difference of two vectors
The magnitude of the difference between two vectors is given by:

$$|\vec{a}-\vec{b}|=\sqrt{|\vec{a}|^2+|\vec{b}|^2-2(\vec{a}\cdot\vec{b})}$$

Step 2: Substitute the given values
We know that $|\vec{a}|=2, |\vec{b}|=3$, and $\vec{a}\cdot\vec{b}=4$. Substituting these values into the formula:

$$|\vec{a}-\vec{b}|=\sqrt{2^2+3^2-2(4)}$$
$$|\vec{a}-\vec{b}|=\sqrt{4+9-8}$$
$$|\vec{a}-\vec{b}|=\sqrt{5}$$

Step 3: Final answer
Thus, the magnitude of $\vec{a}-\vec{b}$ is:

$$\downarrow\sqrt{5}$$

15) If a die is thrown three times, then find the probability of getting an odd number on the die in each throw.

Solution:

When a die is thrown, the possible outcomes are $\{1,2,3,4,5,6\}$. The odd numbers from these outcomes are $\{1,3,5\}$, so there are 3 favorable outcomes (odd numbers) out of 6 possible outcomes.

Step 1: Probability of getting an odd number in one throw
The probability of getting an odd number on one throw is:

$$P(\text{ odd number })=\frac{3}{6}=\frac{1}{2}$$

Step 2: Probability of getting an odd number on three consecutive throws
Since each throw is independent, the probability of getting an odd number on all three throws is:

$$P(\text{ odd on all 3 throws })=\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3=\frac{1}{8}$$

Final Answer:
The probability of getting an odd number on the die in each of the three throws is:

We are given the following probabilities:

$$P(A)=\frac{5}{11}, \quad P(B)=\frac{6}{11}, \quad P(A\cap B)=\frac{4}{11}$$

We are asked to find $P(A/B)$, which represents the conditional probability of A given B. The formula for conditional probability is:

$$P(A/B)=\frac{P(A\cap B)}{P(B)}$$

Step 1: Substitute the given values

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{6}{11}}$

Step 2: Simplify the expression

$P(A/B) = \frac{4}{11} \times \frac{11}{6} = \frac{4}{6} = \frac{2}{3}$

The final answer for the conditional probability $P(A/B)$ is:

$$P(A/B) = \frac{2}{3}$$

The expression $\frac{1}{8}$ seems unrelated to this question about conditional probability.

To find $P(A/B)$, we use the formula for conditional probability:

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

Solution:

We are given the following probabilities:

$P(A) = \frac{5}{11}, \quad P(B) = \frac{6}{11}, \quad P(A \cap B) = \frac{4}{11}$

We are asked to find $P(A/B)$, which represents the conditional probability of A given B. The formula for conditional probability is:

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

Step 1: Substitute the given values

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{6}{11}}$

Step 2: Simplify the expression

$P(A/B) = \frac{4}{11} \times \frac{11}{6} = \frac{4}{6} = \frac{2}{3}$

Final Answer:

The conditional probability $P(A/B)$ is:

$\frac{2}{3}$

SECTION-C

17) Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$.

Solution:

Step 1: Use the formula for the sum of inverse tangents
We use the formula for the sum of two inverse tangents:

$$\tan^{-1} a+\tan^{-1} b=\tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

Here, $a=\frac{1}{2}$ and $b=\frac{2}{11}$.
Step 2: Apply the formula
Substitute $a=\frac{1}{2}$ and $b=\frac{2}{11}$ into the formula:

$$\tan^{-1} \frac{1}{2}+\tan^{-1} \frac{2}{11}=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2} \cdot \frac{2}{11}}\right)$$

Step 3: Simplify the expression
First, simplify the numerator:

$$\frac{\frac{1}{2}+\frac{2}{11}}{=} \frac{11+4}{22}=\frac{15}{22}$$

Next, simplify the denominator:

$$1-\frac{1}{2} \cdot \frac{2}{\downdownarrows}=1-\frac{1}{11}=\frac{10}{11}$$

Thus, the expression becomes:

$$\tan^{-1}\left(\frac{\frac{15}{22}}{\frac{10}{11}}\right)$$

Step 4: Simplify further
Now simplify the fraction:

$$\frac{\frac{15}{22}}{\frac{10}{11}}=\frac{15}{22} \cdot \frac{11}{10}=\frac{15 \cdot 11}{22 \cdot 10}=\frac{165}{220}=\frac{3}{4}$$

Step 5: Final result
We have:

$$\tan^{-1} \frac{1}{2}+\tan^{-1} \frac{2}{11}=\tan^{-1} \frac{3}{4}$$

Thus, the given equation is true, and we have proved that:

$$\tan^{-1} \frac{1}{2}+\tan^{-1} \frac{2}{11}=\tan^{-1} \frac{3}{4}$$

18) If x and y are connected parametrically by the equations $x=\mathrm{a}\left(\cos \mathrm{t}+\log \tan \frac{\mathrm{t}}{2}\right)$, $\mathrm{y}=\mathrm{a} \sin \mathrm{t}$, then without eliminating the parameters, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

Solution:

We are given the parametric equations:

$$x=a\left(\cos t+\log \tan \frac{t}{2}\right)$$

and

$$y=a \sin t$$

We need to find $\frac{dy}{dx}$ without eliminating the parameter t.
Step 1: Find $\frac{dx}{dt}$

Differentiate x with respect to t :

$$\frac{dx}{dt}=a\left(\frac{d}{dt}\cos t+\frac{d}{dt}\log\tan\frac{t}{2}\right)$$

The derivative of $\cos t$ is $-\sin t$, and the derivative of $\log\tan\frac{t}{2}$ is $\frac{1}{\sin t}$ (this is a known derivative).

So,

$$\frac{dx}{dt}=a\left(-\sin t+\frac{1}{\sin t}\right)$$

Step 2: Find $\frac{dy}{dt}$

Differentiate $y=a\sin t$ with respect to t :

$$\frac{dy}{dt}=a\cos t$$

Step 3: Find $\frac{dy}{dx}$

Now, using the chain rule, we know:

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Substitute the values for $\frac{dy}{dt}$ and $\frac{dx}{dt}$:

$$\frac{dy}{dx}=\frac{a\cos t}{a\left(-\sin t+\frac{1}{\sin t}\right)}$$

Simplifying by canceling a :

$$\frac{dy}{dx}=\frac{\cos t}{-\sin t+\frac{1}{\sin t}}$$

19) Find $\int\frac{x+1}{x^2+4x+5}dx$.

Solution:

Step 1: Completing the square for the denominator

We start by completing the square for the quadratic expression in the denominator:

$$x^2+4x+5=(x+2)^2+1$$

Thus, the integral becomes:

$$I=\int\frac{x+1}{(x+2)^2+1}dx$$

Step 2: Use substitution

Let's make the substitution $u=x+2$, which implies $du=dx$ and $x=u-2$. The integral becomes:

$$I=\int\frac{(u-1)}{u^2+1}du$$

Step 3: Split the integral

Now split the integral into two parts:

$$I=\int\frac{u}{u^2+1}du-\int\frac{1}{u^2+1}du$$

Step 4: Solve each integral

- The first integral:

$\int \frac{u}{u^2+1} du$
This is a simple substitution. Let $v = u^2 + 1$, so $dv = 2udu$. The integral simplifies to:

$\frac{1}{2} \int \frac{dv}{v} = \frac{1}{2} \ln \left|u^2+1\right|$
Substitute $u = x + 2$ back:

$\frac{1}{2} \ln \left(x^2 + 4x + 5\right)$

- The second integral:

$\int \frac{1}{u^2+1} du = \tan^{-1}(u)$
Substitute $u = x + 2$:

$\tan^{-1}(x+2)$

Step 5: Combine the results
Combining the results of both integrals, we get:

$I = \frac{1}{2} \ln \left(x^2 + 4x + 5\right) - \tan^{-1}(x+2) + C$
Thus, the solution to the integral is:

$I = \frac{1}{2} \ln \left(x^2 + 4x + 5\right) - \tan^{-1}(x+2) + C$
20) If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then find the value of $|\vec{a} \times \vec{b}|$.

Solution:

Step 1: Use the formula for the magnitude of the cross-product
The magnitude of the cross product $|\vec{a} \times \vec{b}|$ is given by the formula:

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$

where θ is the angle between $\vec{a}$ and $\vec{b}$.
Step 2: Use the dot product to find $\cos \theta$
The dot product formula is:

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$
Substituting the given values:

$12 = 10 \times 2 \times \cos \theta$
Simplifying:

$$12 = 20 \cos \theta$$
$$\cos \theta = \frac{12}{20} = \frac{3}{5}$$

Step 3: Find $\sin \theta$ using the Pythagorean identity
We know that:

$\sin^2\theta + \cos^2\theta = 1$
Substitute $\cos\theta = \frac{3}{5}$:

$$\sin^2\theta + \left(\frac{3}{5}\right)^2 = 1$$
$$\sin^2\theta + \frac{9}{25} = 1$$
$$\sin^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$$
$$\sin\theta = \frac{4}{5}$$

Step 4: Calculate $|\vec{a} \times \vec{b}|$
Now, substitute the values of $|\vec{a}|$, $|\vec{b}|$, and $\sin\theta$ into the formula for the magnitude of the cross product:

$|\vec{a} \times \vec{b}| = 10 \times 2 \times \frac{4}{5} = 20 \times \frac{4}{5} = 16$
Final Answer:
The value of $|\vec{a} \times \vec{b}|$ is:
16
21) Find the value of $\int_{-1}^{1} 5x^4\sqrt{x^5+1}dx$.

Solution:

Step 1: Analyze the symmetry of the integrand
The integrand is $5x^4\sqrt{x^5+1}$. First, let's check whether the integrand is an even or odd function.
- For even functions, $f(-x) = f(x)$, and the integral from -1 to 1 is twice the integral from 0 to 1.
- For odd functions, $f(-x) = -f(x)$, and the integral from -1 to 1 is zero.

Now, let's substitute $-x$ into the integrand:

$f(-x) = 5(-x)^4\sqrt{(-x)^5+1} = 5x^4\sqrt{-x^5+1}$
We see that:

$f(-x) \neq f(x) \quad \text{and} \quad f(-x) \neq -f(x)$
Thus, the function is neither even nor odd.
Step 2: Substitution attempt
We can try to simplify the integrand using substitution. Let's use the substitution:

$u = x^5 + 1$

Then, differentiate both sides:

$du = 5x^4dx$
This allows us to rewrite the integral as:

$I=\int_{x=-1}^{x=1}\sqrt{u}du$

At the limits $x=-1$, we get $u=(-1)^5+1=0$. At $x=1$, we get $u=1^5+1=2$. So, the limits of integration change accordingly:

$I=\int_0^2\sqrt{u}du$

Step 3: Solve the integral

We now evaluate the integral:

$I=\int_0^2 u^{1/2}du$

Using the power rule for integration:

$I=\frac{u^{3/2}}{\frac{3}{2}}\Big|_0^2=\frac{2}{3}u^{3/2}\Big|_0^2$

Substitute the limits:

$I=\frac{2}{3}\left(2^{3/2}-0^{3/2}\right)$

Simplify:

$I=\frac{2}{3}\times 2^{3/2}=\frac{2}{3}\times 2\sqrt{2}=\frac{4\sqrt{2}}{3}$

Final Answer:

The value of the integral is:

$\frac{4\sqrt{2}}{3}$

22) Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2,(x\neq 0)$.

Solution:

The given differential equation is:

$x\frac{dy}{dx}+2y=x^2$

This is a first-order linear differential equation. We can solve it using the method for solving linear differential equations, which involves finding an integrating factor.

Step 1: Rewrite the equation in standard form

Rewrite the equation as:

$\frac{dy}{dx}+\frac{2}{x}y=x$

This is in the standard linear form:

$\frac{dy}{dx}+P(x)y=Q(x)$

where $P(x)=\frac{2}{x}$ and $Q(x)=x$.

Step 2: Find the integrating factor

The integrating factor (IF) is given by:

$$\mu(x)=e^{\int P(x) d x}=e^{\int \frac{2}{x} d x}$$

$\mu(x)=e^{2 \ln |x|}=|x|^2=x^2 \quad(\text { since } x \neq 0)$

Step 3: Multiply the equation by the integrating factor

Now multiply the entire differential equation by $\mu(x)=x^2$:

$x^2 \frac{d y}{d x}+x^2 \cdot \frac{2}{x} y=x^2 \cdot x$

Simplify:

$x^2 \frac{d y}{d x}+2 x y=x^3$

Step 4: Recognize the left side as a product derivative

The left-hand side of the equation is the derivative of $x^2 y$:

$\frac{d}{d x}\left(x^2 y\right)=x^3$

Thus, the equation becomes:

$\frac{d}{d x}\left(x^2 y\right)=x^3$

Step 5: Integrate both sides

Now integrate both sides with respect to x :

$x^2 y=\int x^3 d x$

The integral of x^3 is:

$x^2 y=\frac{x^4}{4}+C$

Step 6: Solve for y

Solve for y by dividing both sides by x^2 :

$y=\frac{x^4}{4 x^2}+\frac{C}{x^2}$

Simplify:

$y=\frac{x^2}{4}+\frac{C}{x^2}$

23) If three cards are drawn successively without replacement from a pack of 52 well shuffled cards, then find the probability that first two cards are aces and the third card drawn is a king.

Solution:

Step 1: Probability of drawing the first ace

There are 4 aces in a deck of 52 cards, so the probability of drawing an ace on the first draw is:

$P(\text { First Ace })=\frac{4}{52}=\frac{1}{13}$

Step 2: Probability of drawing the second ace

After the first ace has been drawn, there are 3 aces left in a deck of 51 cards. So, the probability of drawing a second ace on the second draw is:

$P(\text { Second Ace })=\frac{3}{51}=\frac{1}{17}$

Step 3: Probability of drawing a king

After two aces have been drawn, there are still 4 kings in the deck, but only 50 cards remain. So, the probability of drawing a king on the third draw is:

$P(\text{ King }) = \frac{4}{50} = \frac{2}{25}$

Step 4: Total probability

The probability of drawing two aces followed by a king is the product of these individual probabilities:

$$P(\text{ Two Aces and a King }) = \frac{1}{13} \times \frac{1}{17} \times \frac{2}{25}$$

Simplifying the product:

$$P(\text{ Two Aces and a King }) = \frac{1 \times 1 \times 2}{13 \times 17 \times 25} = \frac{2}{5525}$$

Final Answer:

The probability that the first two cards are aces and the third card is a king is:

$$\frac{2}{5525}$$