CAREERS 360

PRACTICE Series

Maharashtra HSC

Maths Sample Paper 2025

Std. XII

MATHEMATICS (40)

Specimen Question Bank

Chapter 1 : Mathematatial Logic

(2 - Marks)

1. Write the truth values of the following statments.
   - i) 2 is a rational number and it is the only even prime number.
   - ii) $\exists\ x \in N$ such that $x + 3 > 5$
   - iii) 3 + 2i is a real number or it is a complex number.
   - iv) It is false that New Delhi is not a capital of India.
   - v) The cube roots of unity are in G.P.
2. If p, q, r are statments with truth value T, F T respectively, determine the truth values the of following :
   - i) $(p \vee r) \ \sim q$
   - ii) $(p \rightarrow q) \leftrightarrow r$
   - iii) $(p \leftrightarrow q) \wedge (q \leftrightarrow r)$
   - iv) $\sim (r \wedge \sim q) \vee (p \wedge \sim r)$
   - v) $(r \wedge q) \leftrightarrow \sim p$
3. Write the negations of the following statements.
   - i) He is rich and happy.
   - ii) If I beome a teacher, then I will open a school.
   - iii) $\forall\ x \in N,\ x + 5 > 8$
   - iv) A person is busy if and only if he is a doctor.
   - v) $\sim p \rightarrow (q \vee r)$
   - vi) All parents care for their children.
4. Prepare the truth table for each of the following statement pattern.
   - i) $p \wedge (q \rightarrow p)$
   - ii) $(\sim p \vee \sim q) \leftrightarrow (p \wedge q)$
   - iii) $(p \wedge \sim p) \vee q$
   - iv) $(p \leftrightarrow q) \vee (q \leftrightarrow p)$
   - v) $\sim q \rightarrow \sim p$
   - vi) $p \vee \sim p$

5. Write the following statments in symbolic form :

i) Manisha does not live in Mumbai.

ii) If a number n^2 is even, then n is even.

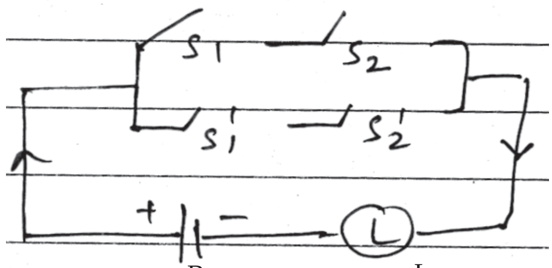
iii) Rohit is neither healthy nor wealthy.

iv) If ΔABC is right angled at B, then $AB^2 + BC^2 = AC^2$.

v) It is raining if and only if the weather is humid.

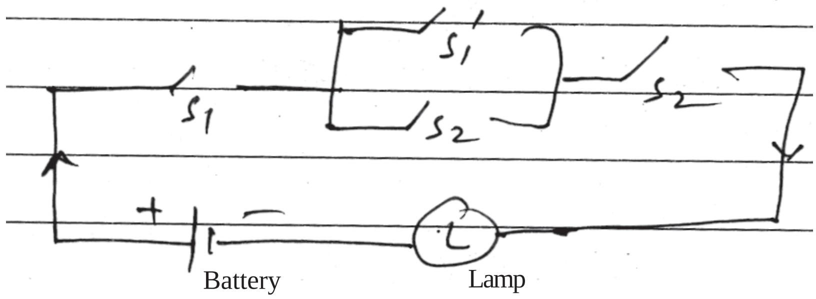
6. Express the given circuits in symbolic form.

i)



Battery Lamp

ii)



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7. If p : The earth is round.

q : The moon rotates around the earth.

and r : The sun is hot.

Write the following in verbal form.

i) $p \wedge q$ (ii) $p \leftrightarrow q$ (iii) $p \rightarrow (q \vee r)$

(iv) $(\sim p \wedge q) \vee r$ (v) $q \rightarrow r$

8. If A = {4, 5, 7, 9}, determine the truth value of each of the following.

quantified statements.

i) $\exists\, x \in A$, such that $x + 2 = 7$

ii) $\forall\, x \in A$, $x + 3 < 10$

iii) $\exists\, x \in A$, such $x + 5 > 9$

iv) $\exists\, x \in A$, such that x is even.

v) $\forall\, x \in A$, $2x < 17$

9. Write duals of the following statements.

i) $(p \vee q) \vee r$

ii) $(p \vee q) \wedge T$

iii) $\sim (p \vee q) \wedge [p \vee \sim (q \wedge \sim r)]$

iv) Sohan and Kavita can not read french.

v) $(\sim p \wedge \sim q) \equiv \sim (p \vee q)$

vi) $(p \wedge T) \vee (F \wedge \sim q)$

10. Prove the following results ; using truth tables.

i) $p \rightarrow q \equiv \sim p \vee q$

ii) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

iii) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

iv) $p \wedge q \equiv \sim (p \rightarrow \sim q)$

v) $\sim (p \rightarrow q) \equiv p \wedge \sim q$

3 Marks

1. State the converse, inverse and contrapositive of the following conditional statement.

i) If the teacher is absent, then the students are happy.

ii) If $2 + 3 < 7$ then $7 + 3 > 2$

iii) If f (x) is differentiable function then it is continuous

iv) $[p \wedge (p \rightarrow q)] \rightarrow q$

v) A family becomes literate if the woman in it are literate.

2. Prepare the truth table of the following statement patterns.

i) $(p \wedge q) \rightarrow (\sim p \vee \sim q)$

ii) $[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$

iii) $(\sim p \wedge q) \leftrightarrow (p \rightarrow \sim q)$

iv) $(p \leftrightarrow r) \wedge (q \leftrightarrow p)$

3. Using truth tables. Prove the following logical equivalenes.

i) $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii) $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

iii) $\sim (\sim p \wedge q) \wedge (p \vee q) \equiv p$

4. Using truth tables, examine whether each of the following statement patterns is a tautology or a contradiction or a contingency.

i) $[(p \to q) \wedge p] \to q$

ii) $(p \to \sim p) \wedge (\sim p \to p)$

iii) $(\sim p \wedge q) \wedge (q \to p)$

iv) $(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$

v) $(p \vee q) \wedge (p \vee r)$

5. Using the rules of negation, write the negation of the following.

i) $p \wedge \sim (q \wedge r)$

ii) $(\sim p \wedge q) \vee (p \wedge \sim q)$

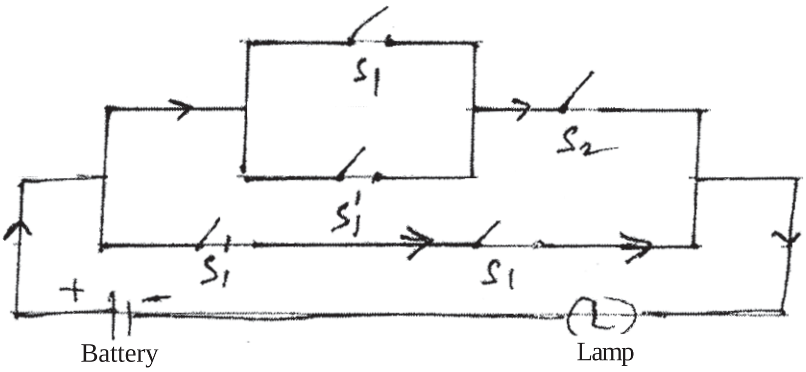
iii) $(p \to q) \wedge r$

iv) If $10 > 5$ and $5 < 8$ then $8 < 7$

v) It is false that the sky is not blue.

6. Express the following circuits in symbolic form and write input output table.

i)



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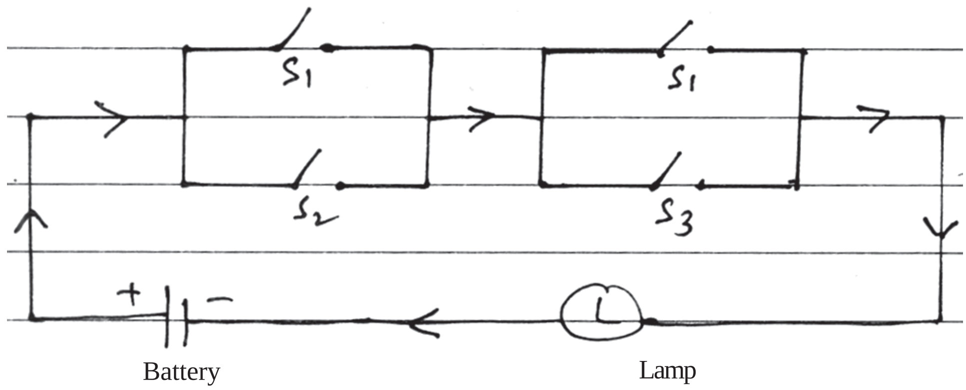
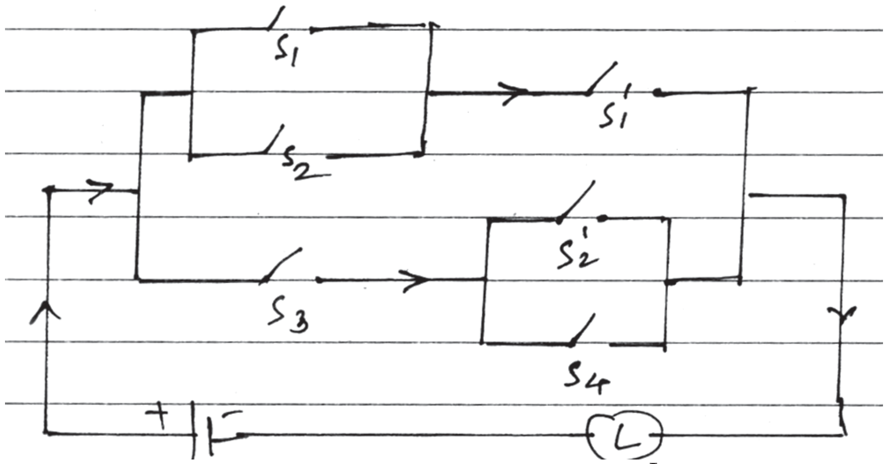


iii) 

7. Construct the swiching circuits of the following statement patterns.

   i) [p Λ (q V r) V (~ p V s)

   ii) (p Λ q) V (~ P) V (p Λ ~ q)

   iii) [(p V q) Λ ~ p] V [r Λ (~ q V s)]

   iv) p Λ [q V (r Λ ~ p)] Λ s

8. Write the following compund statement in symbolic form and write their negations :

   i) Mahesh is fat but not lazy.

   ii) It is neither cold nor raining.

   iii) Some countries are digital and all people are technosavy.

   iv) If I drive fast and do not follow traffic rules, then I will meet with an accident.

9. If p, q are true statements and r, s are false statements, then find the truth values of the following compound statements.

   i) (~ p V q) → (s Λ ~ r)

   ii) [(p → q) → r] → s

   iii) (p ↔ s) Λ (p → q)

   iv) [p Λ (q V r)] V [s Λ ~ q]

   v) p Λ [q Λ (~ p Λ r) V ~ s] V ~ r

10. i) Write the contrapositive of the inverse of the statement "If two numbers are not equal, then their squares are not equal".

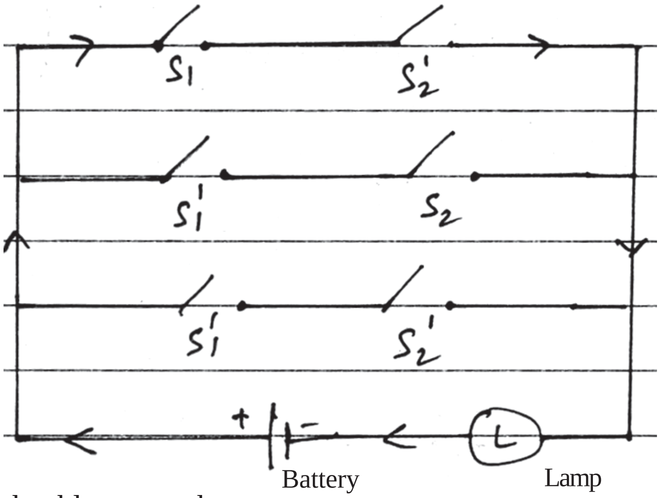
   ii) If (p Λ q) → r is false, then find the ruth value of the negation of the statement.

   (p v ~ r) → (q Λ ~ p)

   iii) Show that the dual of (p → ~ q) V q is a contradiction.

(D) for 4 marks

1. Simplify the following so that the new circuit has minimum number of switches. Also, draw the simplified circuit.



2. Without using truth table, prove that

   i) (~ p Λ q) V (~ p Λ ~ q) V (p Λ ~ q) ≡ ~ p V ~ q

   ii) [p Λ (q V r)] V [~ r Λ ~ q Λ p] ≡ p

   iii) p ↔ q ≡ ~ (p Λ ~ q) Λ ~ (q Λ ~ p)

3. Identify the pairs of following statements having same meaning.

   i) If a person is a social, then he is happy.

   ii) If a person is not social, then he is not happy.

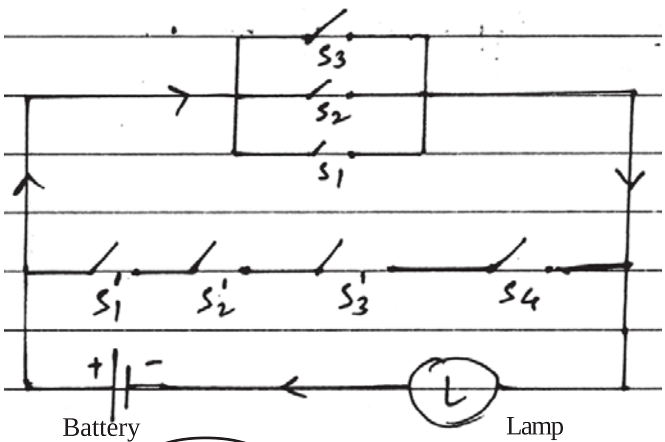
   iii) If a person is unhapyy, then he is not social.

   iv) If a person is happy, then he is social.

4. Write the following statement in four different ways, conveying the same meaning.

   "If you drive over 80 km per hour, then you will get a fine."

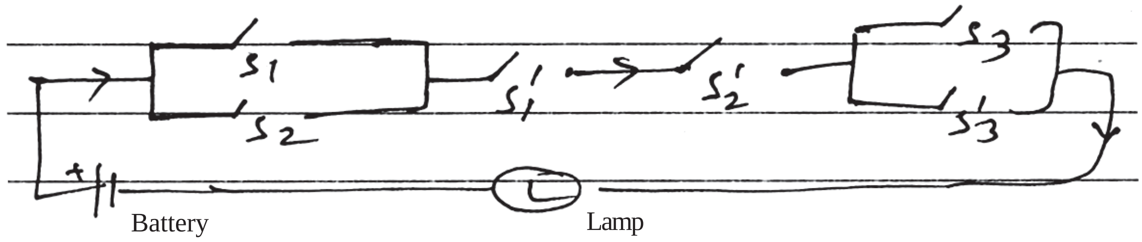
5. Show that, the following, circuit can be simplified and reconstructed so as to reduce its number of switches from 7 to 4.



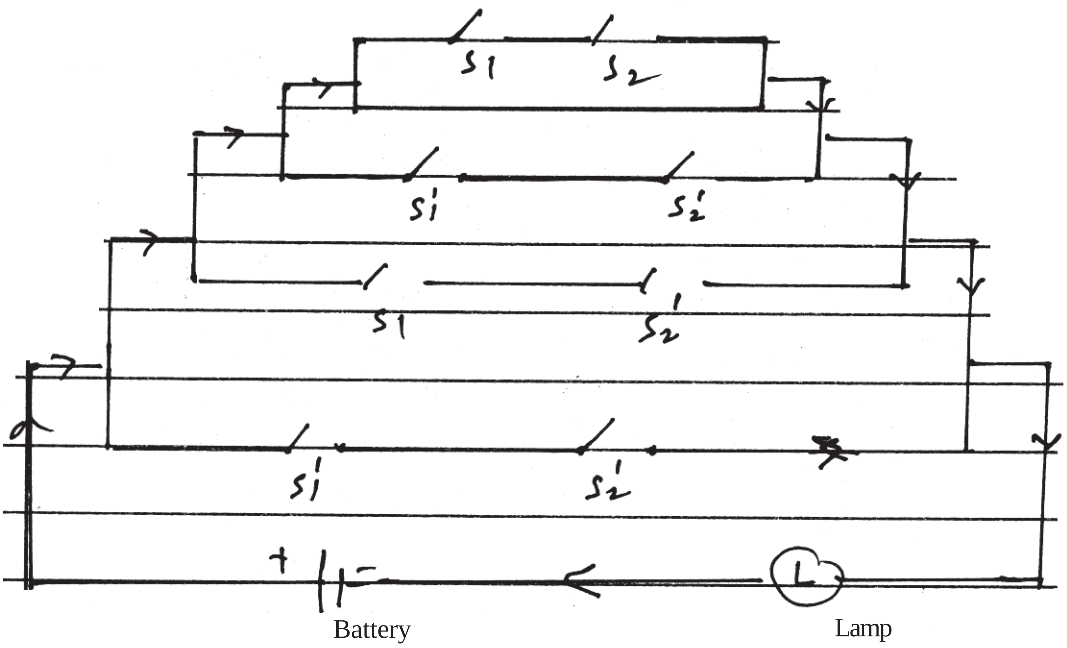
6. State the dual of the following statement by applying the principle of duality. Also, prove that both sides of the dual are equivalent.

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

7. Simplify the following circuit and reconstruct an alternative circuit having minimum switches :



8. Write the following circuit symbolically and construct its switching table. What conclusion would you draw from the table ?



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Chapter 2 : Matries

(2 - Marks)

1. Find the adjoint of the following matrices.

   i) $\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$ ii) $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}$ iii) $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$

2. Find the inverse of the following Matrices using elementary row transformations.

   i) $\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ iii) $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

   iv) $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ v) $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

3. Find the inverse of the following Matrices using elementary column transformations.

   i) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ iii) $\begin{bmatrix} \operatorname{cosec}\theta & \cot\theta \\ -\cot\theta & \operatorname{cosec}\theta \end{bmatrix}$

   iv) $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$ v) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

4. Find the inverse of the following Matrices using adjoint method.

   i) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ iii) $\begin{bmatrix} \sin\theta & 1 \\ 0 & \cos\theta \end{bmatrix}$

   iv) $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ v) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix}$

5. Express the following equations in matrix form and solve them by using.

a) reduction method b) Inversion method.

i) $x + y = 2, \quad 3x + 2y = 5$

ii) $2x + y = 5, \quad 3x + 5y = -3$

iii) $x + 3y = 4, \quad 4x - y = 3$

iv) $2x - y = -2, \quad 3x + 4y = 3$

v) $4x + 3y = 1, \quad 2x + y = 1$

6. i) Find the matrix X such that AX = B, where $A = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

ii) Find the matrix X such that AX = I where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

iii) If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$, then find the matrix X such taht AXB = C

iv) If $A = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then find the matrix X such that $A^{-1}X = B$

7. i) If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ then find matrix $(AB)^{-1}$.

ii) If $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$, then find adj (adj A)

iii) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ and $A^{-1} = KA$, then find the value of K.

8. i) If $A = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$ find $A'B'$.

ii) If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & -2 \end{bmatrix}$ verity that $(AB)' = B'A'$.

iii) If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ show that $A^2 - 5A - 14I = 0$

iv) If $f(x) = x^2 - 2x - 3$, find $f(A)$ when $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

9. If A is invertible matrix of order 3 and $|A| = 5$, then find the value of $|\text{adj } A|$.

10. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 6 & 19 \end{bmatrix}$

verify that $\text{adj}(AB) = (\text{adj } B)(\text{adj } A)$

3 Marks

1. i) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$ and hence find A^{-1}.

ii) If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, show that $A^{-1} = \frac{1}{6}(A - 5I)$

iii) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$, then find $(AB)^{-1}$

iv) If $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ then prove that $A^2 = I$. Hence show that $A^{-1} = A$

2. Find adj A, if A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

3. Nina and Meena want to buy pens and books. Nina wants 2 pens and 5 books while Meena wants 6 pens and 8 books. They both go to a shop and buy them. When the shopkeeper gives them the pens and the books. Nina pays him Rs. 110 and Meena pays Rs. 190. Find the prices of one pen and one book using matrices.

4. If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

then find the matrix X such that XA = B.

5. Find the adjoint of the matrix

A = $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify that A (adj A) = (adj A) A = | A | I

6. Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$ using adjoint method.

7. If A and B are two invertible matrices of the same order, then prove that $(AB)^{-1} = B^{-1}A^{-1}$

8. If A = $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, show that adj A = A

9. If A = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 4 \\ 0 & -1 & 2 \end{bmatrix}$ with usual notations verify that

i) $a_{31} C_{31} + a_{32} C_{32} + a_{33} C_{33} = |A|$

ii) $a_{21} C_{31} + a_{22} C_{32} + a_{23} C_{33} = 0$, Where C_{ij} the co factor of a_{ij}

10. Using elementary transformations show that the inverse of the matrix.

$\begin{bmatrix} a & -b & 0 \\ b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is $\begin{bmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{bmatrix}$ if $a^2 + b^2 = 1$

4 Marks

1. Find the inverse of the following matrices using

(a) elementary row transformation. (b) elementary column transformation.

i) $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 3 & 1 \\ 3 & 2 & 3 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & 0 & 1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

iii) $\begin{bmatrix} 3 & 1 & 2 \\ 5 & 2 & 4 \\ -2 & 3 & 9 \end{bmatrix}$ iv) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

v) $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ 2 & -1 & -4 \end{bmatrix}$ vii) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

2 i) If the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and

$B^{-1} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ then compute $(AB)^{-1}$.

ii) For the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$,

verify that $A^3 - 6A^2 + 9A - 4I = 0$. Hence find A^{-1}.

iii) Find the matrix A such that

$$A \begin{bmatrix} 2 & -1 & 0 \\ -5 & 4 & 1 \\ 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 5 & 6 \\ 7 & -5 & 4 \\ -7 & 7 & 4 \end{bmatrix}$$

3. Express the following equations in matrix form and solve them by (a) Reduction method (b) Inversion method

i) $2x - y + z = 1$, $x + 2y + 3z = 8$, $3x + y - 4z = 1$

ii) $x + y + z = 3$, $3x - 2y + 3z = 4$, $5x + 5y + z = 11$

iii) $x + y + z = 3$, $2x - y + z = 2$, $x - 2y + 3z = 2$

iv) $2x + 3y + 3z = 5$, $x - 2y + z = -4$, $3x - y - 2z = 3$

v) $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

4. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs. 60. The cost of 2 kg onion, 4kg wheat and 6 kg rice is Rs. 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs. 70. Find cost of each item per kg by matrix method.

5. If $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & 2 \\ 3 & -4 & -1 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of linear equations.

$2x - y + 3z = 13$, $x + 3y + 2z = 1$, $3x - 4y - z = 8$

6. A salesman has the following record of sales during the past three months for three items A, B and C which have the different rates of commission.

Months	Sales of Units			Total commission
	A	B	C	(in Rs.)
Janaury	90	100	20	800
February	130	50	40	900
March	60	100	30	850

Find out the rates of commission on items A, B and C

7. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ show that $(f(x))^{-1} = f(-x)$.

8. Ramesh buyes half a dozen pencils, 2 erasers and 2 sharpeners from a shop and pays Rs. 14 from the same shop, Suresh buys 15 pencils, 5 erasers and 3 sharpeners and pays Rs. 35, whereas their friend Mahesh, who accompanied them to the shop, buys as a token 1 pencil, 1 eraser and 1 sharpener for the payment of Rs. 3. Find the price of each item at the shop, by using matrices.

9. Three cricket fans, nick named as Soni, Moni and Dhoni, went to play for a country match. Their individual scores being x , y and z respectively. Find x, y, z using inversion method from the following data :

i) the sum of their scores is a centrury.

ii) if we subtract the sum of Soni and Moni’s score from twice of Dhoni’s score it is still a half centuary.

iii) four times Moni's score minus Soni's score equal to Dhoni's score.

10. Solve the following equations by using Reduction method.

$$\log_e^x + e^y + z^2 = 3$$

$$\log_e^x + 2\,e^y + 3\,z^2 = 6$$

$$2\,\log_e^x + 3\,e^y + 4\,z^2 = 1$$

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Chapter 3 : Trigonometric Functions

(2 - Marks)

1. Find the general solution of
   $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$.
2. Find the number of ordered pairs (*x*, y) satisfying y = 2 sin x and $y = 5x^2 + 2x + 3$
3. Find the number of solutions of the equation $1 + \sin x \, . \, \sin^2\left(\frac{x}{2}\right) = 0$ in $[-\pi, \pi]$
4. If $2\cos^2 x + 3\sin x - 3 = 0$, $0° \le x \le 180°$ then find the value of *x*.
5. If $\tan 2x = \tan\left(\frac{x}{2}\right)$, then find the value *x*.
6. Find the general solution of $4\sin^2 x - 3 = 0$
7. Find the principal solution of $\sqrt{3}\sec x + 2 = 0$
8. Find the general solution of $\cos x = -\frac{1}{2}$
9. Find the polar co-ordinates of point whose carlesian co-ordiantes are $(1,\sqrt{3})$
10. Find cartesian co-ordinates of the point whose polar co-ordinates are $(2, \pi/4)$
11. Show that ac cos B - bc cos A = $a^2 - b^2$.

3 Marks

1. Two adjacent sides of a cyclic quadrilateral are 2 and 5. The angle between them is 60^0. If the third side is 3, then find the fourth side.
2. Find the number of values of *x* in $[\,0,\ 2\pi\,]$ satisfying the equation $3\cos x - 10\cos x + 7 = 0$
3. If in Δ ABC, b = $\sqrt{3}$, c = 1 and B – C = 90^0, then find the angle A.
4. Find the value of *x* if $\sin^{-1}(1/3) + \sin^{-1}\left(\frac{2}{3}\right) = \sin^{-1}x$.
5. Find '*x*' if $\sec^2 2x = 1 - \tan 2x$.
6. Solve the equation $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$
7. The general solution of $\tan\left(\frac{2x}{3}\right) = \sqrt{3}$

8. Find the principal value of cosec $(x) = 2$

9. In right angled triangle Δ ABC, right angled at C. Show that $\tan A + \tan B = \dfrac{c^2}{ab}$

10) In Δ ABC

$\sin (A/2) \sin (C/2) = \sin (B/2)$ and '2s' is the perimeter of the triangle then find 's'.

4 Marks

1. In Δ ABC prove that $\dfrac{b - c}{a} = \dfrac{\tan (B/2) - \tan (C/2)}{\tan (B/2) + \tan (C/2)}$

2. In Δ ABC , $a \cos^2 (C/2) + c \cdot \cos^2 (A/2) = \dfrac{3b}{2}$ then prove that a, b and c are in A.P.

3. In Δ ABC, if a^2, b^2, c^2 are in A.P. then prove that cot A , cot B, cot C are in A.P.

4. Find the value of the expression $\tan \left\{ \dfrac{1}{2} \left[\sin^{-1} \left(\dfrac{2x}{1 + x^2} \right) + \cos^{-1} \left(\dfrac{1 - y^2}{1 + y^2} \right) \right] \right\}$

   Where $x > 0$, $y > 0$ Such that $xy < 1$

5. In Δ ABC if $\angle C = \dfrac{\pi}{2}$ then prove that $\sin (A - B) = \dfrac{a^2 - b^2}{a^2 + b^2}$

6. Show that $\tan^{-1} \left(\dfrac{\sqrt{1 + x} - \sqrt{1 - x}}{\sqrt{1 + x} + \sqrt{1 - x}} \right) = \dfrac{\pi}{4} - \dfrac{1}{2} \cos^{-1}x,$

   for $\dfrac{-1}{\sqrt{2}} < x < 1.$

7. Prove the following $\sin^{-1}\left(\dfrac{3}{5}\right) + \cos^{-1}\left(\dfrac{12}{13}\right) = \sin^{-1}\left(\dfrac{56}{65}\right)$

8. If $\tan^{-1}\left(\dfrac{x - 1}{x - 2}\right) + \cot^{-1}\left(\dfrac{x + 2}{x + 1}\right) = \dfrac{\pi}{4}$, then find x.

9. Prove that $\sin^{-1}\left(\dfrac{-1}{2}\right) + \cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right) = \cos^{-1}\left(\dfrac{-1}{2}\right)$

10. Show that $\sin^{-1}\left(\dfrac{8}{17}\right) + \sin^{-1}\left(\dfrac{3}{5}\right) = \sin^{-1}\left(\dfrac{77}{85}\right)$

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Chapter 4 : Pairs of Straight lines

(2 - Marks)

1. Find the condition that the lines joining origin to the points of intersection of the line $y = mx + c$ and the curve $x^2 + y^2 = a^2$ will mutually perpendicular.
2. Find the distance between pair of parallel lines given by $x^2 + 2xy + y^2 - 8ax - 9a^2 = 0$
3. The lines represented by $x^2 + \lambda xy + 2y^2 = 0$ and the lines represented by $(1 + \lambda) x^2 - 8xy + y^2 = 0$ are equally inclined, then find λ.
4. Show that the equations $(y - mx)^2 = a^2 (1 + m^2)$ and $(y - nx)^2 = a^2 (1 + n^2)$ form a rhombus.
5. For what value of 'k' the sum of the slopes of the lines given by $3x^2 + kxy - y^2 = 0$, is zero.
6. Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of lines.
7. Find the equations of angle bisectors between the lines $3x + 4y - 7 = 0$ and $12x + 5y + 17 = 0$
8. If the angle between the pair of straight lines represented by the equation. $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ is $\tan^{-1} (1/3)$ where 'λ' is non negative real number, then find 'λ'.
9. The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lie in which quadrant ?
10. The slopes of the lines represented by $x^2 + 2hxy + 2y^2 = 0$ are in the ratio 1 : 2 then find 'h'.

3 Marks

1. Find the joint equation of pair of lines through the origin which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$
2. Find the joint equation of the pair of lines which bisects anlges between the lines given by $x^2 + 3xy + 2y^2 = 0$
3. ΔOAB is formed by the lines $x^2 - 4xy + y^2 = 0$ and the line AB. The equation of the line AB is $2x + 3y - 1 = 0$. Find the equation of the median of the triangle drawn from the origin.

4. Show that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = \sqrt{6}$ form an equilateral. Also find its area.

5. If the lines represented by the equation $2x^2 - 3xy + y^2 = 0$ makes angle α and β with X - axis, find the value of $\cot^2\alpha + \cot^2\beta$.

6. Two lines are given by $(x - 2y)^2 + k(x - 2y) = 0$ then find the value of k, so that the distance between them is 3.

7. Find the difference between slopes of the lines represented by equation

   $x^2 (\sec^2 \theta - \sin^2 \theta) - 2xy \tan \theta + y^2 \sin^2 \theta = 0$

8. Find the condition of slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is the square of the other.

9. Find the number of lines that are parallel to $2x + 6y + 7 = 0$ and have intercept of length 10 between the co-ordinate axes.

10. If the lines $px^2 - qxy - y^2 = 0$ makes the angles α and β with X - axis then find the value of $\tan(\alpha + \beta)$

4 Marks

1. Find the condition that the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie among diameters of a circle and divide the circle into four sectors such that the area of one of the sector is thrice the area of the another sector.

2. Prove that the product the lengths of perpendicular form $P(x_1, y_1)$ to the line representd by $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}$$

3. Find the equation of the bisectors of the angles between the lines.

   $(\sqrt{a} + \sqrt{c})\ x^2 + 2\sqrt{d}\ xy + (\sqrt{b} + \sqrt{c})\ y^2 = 0$

4. Find the measure of the acute angle between the lines.

   $(a^2 - 3b^2)\ x^2 + 8\ ab\ xy + (b^2 - 3a^2)\ y^2 = 0$

5. Find the condition that the equation $ax^2 + by^2 + cx + cy = 0$ may represents a pair of lines.

6. Show that the equation $(x - 3)^2 + (x - 3)(y - 4) - 2(y - 4)^2 = 0$ reprsents a pair of lines also find the acute angle between them.

7. Find the joint eqation of pair of lines passing through the origin and making an angle of 30^o with the lines $x + y = 5$.

8. Find the combined equation of the lines, throught the origin forming an equilateral triangle with the line $x + y = \sqrt{3}$

9. Find the condition that the equation $hxy - gx - fy + c = 0$ represents a pair of lines.

10. Find 'k' if sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product.

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Chapter 5 : Vectors

(2 - Marks)

1. Find vector $\bar{c}$ if $|\bar{c}| = 3\sqrt{6}$ and $\bar{c}$ is directed along the angle bisectors of the vectors $\overline{OA} = 7\hat{i} - 4\hat{j} - 4\hat{k}$ and $\overline{OB} = -2\hat{i} - \hat{j} + 2\hat{k}$.
2. If $\bar{a}$ and $\bar{b}$ are non collinear vectors then find the value of x for which vectors $\bar{\alpha} = (x-2)\,\bar{a} + \bar{b}$ and $\bar{\beta} = (3+2x)\,\bar{a} - 2\,\bar{b}$ are collinear.
3. Let $\bar{a}, \bar{b}, \bar{c}$ are three non-zero vectors such that any two of them are non-collinear. If $\bar{a} + 2\,\bar{b}$ is collinear with $\bar{c}$ and $\bar{b} + 3\,\bar{c}$ is collinear with $\bar{a}$ then prove that $\bar{a} + 2\,\bar{b} + 6\,\bar{c} = \bar{O}$
4. $\overline{DA} = \bar{a}$; $\overline{AB} = \bar{b}$; $\overline{CB} = k\,\bar{a}$; k > 0 and X, Y are mid points of DB and AC repectively such that $|\bar{a}| = 17$ and $|\overline{XY}| = 4$. Find value of k.
5. If $\bar{a} + \lambda\,\bar{b} + 3\,\bar{c}$; $-2\,\bar{a} + 3\,\bar{b} - 4\,\bar{c}$; $\bar{a} - 3\,\bar{b} + 5\,\bar{c}$ are coplanar then find the value of λ.
6. Find the value of $$\frac{\bar{a}\,.\,(\bar{b}\times\bar{c})}{\bar{b}\,.\,(\bar{c}\times\bar{a})} + \frac{\bar{b}\,.\,(\bar{c}\times\bar{a})}{\bar{c}\,.\,(\bar{a}\times\bar{b})} + \frac{\bar{c}\,.\,(\bar{a}\times\bar{b})}{\bar{a}\,.\,(\bar{b}\times\bar{c})}$$
7. In ΔABC, M is mid-point of side BC. If $\angle BAM = \theta$ then using vector method prove that $$\cos\theta = \frac{\sin C + \sin B \cos A}{\sqrt{\sin^2 B + \sin^2 C + 2\sin B \sin C \cos A}}$$
8. $\bar{a}, \bar{b}, \bar{c}$ represent three concurrent edges of a rectangular parallelepiped whose lengths are 4, 3, 2 units respectively then find value of $$(\bar{a} + \bar{b} + \bar{c})\,.\,(\bar{a}\times\bar{b} + \bar{b}\times\bar{c} + \bar{c}\times\bar{a})$$
9. If D, E, F are three points on the sides BC, CA, AB repectively of a ΔABC, such that AD, BE, CF are concurrent, then using vector method prove that $$\left|\frac{BD}{CD}\times\frac{CE}{AE}\times\frac{AF}{BF}\right| = 1$$
10. $\bar{a}, \bar{b}$ are perpendicular vectors, find projection of the vector $$l\,\frac{\bar{a}}{|\bar{a}|} + m\,\frac{\bar{b}}{|\bar{b}|} + n\,\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$$ along the angle bisector of the vectors $\bar{a}$ and $\bar{b}$

PC1 latest correction

(3 - Marks)

1. If $\bar{a}, \bar{b}, \bar{c}$ are non co-planer non-zero vectros in the plane and $\bar{r}$ is any vector in the space then show that $[\bar{b}\ \bar{c}\ \bar{r}]\ \bar{a} + [\bar{c}\ \bar{a}\ \bar{r}]\ \bar{b} + [\bar{a}\ \bar{b}\ \bar{r}]\ \bar{c} = [\bar{a}\ \bar{b}\ \bar{c}]\ \bar{r}$

2. A parallelogram is constructed on the vector $\bar{a} = \bar{3}\ \alpha - \bar{\beta}$, $\bar{b} = \alpha + \bar{3}\ \beta$ and $|\bar{\alpha}| = |\bar{\beta}| = 2$ and angle between $\bar{\alpha}$ and $\bar{\beta}$ is $\frac{\pi^c}{3}$ then find lengths of the diagonals.

3. $\bar{a}, \bar{b}, \bar{c}$ are three vectors such that $|\bar{a}| = |\bar{b}| = |\bar{c}| = 4$ and angles between $\bar{a}$ and $\bar{b}$; $\bar{b}$ and $\bar{c}$; $\bar{c}$ and $\bar{a}$ are equal to $\frac{\pi^c}{3}$. Find volume of parallelopiped whose adjacent sides $\bar{a}, \bar{b}, \bar{c}$.

4. $\bar{a}, \bar{b}, \bar{c}$ are three vectors and vectors $\bar{a}', \bar{b}', \bar{c}'$ are three vectors such that $\bar{a} \cdot \bar{a}' = \bar{b} \cdot \bar{b}' = \bar{c} \cdot \bar{c}' = 1$ and $\bar{a} \cdot \bar{b}' = \bar{a} \cdot \bar{c}' = \bar{b} \cdot \bar{c}' = \bar{c} \cdot \bar{a}' = \bar{c} \cdot \bar{b}' = 0$

   Then prove that $[\bar{a}'\ \bar{b}'\ \bar{c}'] = \frac{1}{[\bar{a}\ \bar{b}\ \bar{c}]^3}\ [\bar{b} \times \bar{c}\quad \bar{c} \times \bar{a}\quad \bar{a} \times \bar{b}]$

5. If $\overline{OA} = \bar{a}$ and $\overline{OB} = \bar{b}$ then show that the vector along the angle bisector of $\angle AOB$ is given by $\bar{d} = \lambda \left(\frac{\bar{a}}{|\bar{a}|} + \frac{\bar{b}}{|\bar{b}|} \right)$

6. A transversal cuts the sides OL, OM and diagonal ON of the parallelogram at A, B, C repectively. Prove that $\frac{OL}{OA} + \frac{OM}{OB} = \frac{ON}{OC}$ using vector method.

7. Find all values of λ for which $(\hat{i} + \hat{j} + 3\hat{k})\,x + (3\hat{i} - 3\hat{j} + \hat{k})\,y + (-4\hat{i} + 5\hat{j})\,z = \lambda\,(x\hat{i} + y\hat{j} + z\hat{k})$ where x, y z are not all equal to 0.

8. A straight line intersects sides AB, AC and AD in point B_1, C_1, D_1.
   If $\overline{AB_1} = \lambda_1\,\overline{AB}$; $\overline{AD_1} = \lambda_2\,\overline{AD}$; $\overline{AC_1} = \lambda_3\,\overline{AC}$ then prove that $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

9. If $\cos\alpha \neq 1$; $\cos\beta \neq 1$; $\cos\gamma \neq 1$ Prove that vectors $\bar{a} = \cos\alpha\,\hat{i} + \hat{j} + \hat{k}$; $\bar{b} = \hat{i} + \cos\beta\,\hat{j} + \hat{k}$; $\bar{c} = \hat{i} + \hat{j} + \cos\gamma\,\hat{k}$ can never coplanar.

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(4 - Marks)

1. If $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{u}$, $\bar{v}$, $\bar{w}$ are vectors prove that $[\bar{a}\ \bar{b}\ \bar{c}]\ [\bar{u}\ \bar{v}\ \bar{w}] = \begin{vmatrix} \bar{a}.\bar{u} & \bar{b}.\bar{u} & \bar{c}.\bar{u} \\ \bar{a}.\bar{v} & \bar{b}.\bar{v} & \bar{c}.\bar{v} \\ \bar{a}.\bar{w} & \bar{b}.\bar{w} & \bar{c}.\bar{w} \end{vmatrix}$

2. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are vectors prove that $[\bar{a}\ \bar{b}\ \bar{c}]^2 = \begin{vmatrix} \bar{a}.\bar{a} & \bar{b}.\bar{a} & \bar{c}.\bar{a} \\ \bar{a}.\bar{b} & \bar{b}.\bar{b} & \bar{c}.\bar{b} \\ \bar{a}.\bar{c} & \bar{b}.\bar{c} & \bar{c}.\bar{c} \end{vmatrix}$

3. If $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{l}$, $\bar{m}$ are vectors prove that $[\bar{a}\ \bar{b}\ \bar{c}]\ (\bar{l} \times \bar{m}) = \begin{vmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{a}.\bar{l} & \bar{b}.\bar{l} & \bar{c}.\bar{l} \\ \bar{a}.\bar{m} & \bar{b}.\bar{m} & \bar{c}.\bar{m} \end{vmatrix}$

4. Prove that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a}\ .\ \bar{c})\ \bar{b} - (\bar{a}\ .\ \bar{b})\ \bar{c}$

5. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar unit vectors each including the angle of measure 30^0 with the other then find the volume of tetrahedron whose co-terminal edges are $\bar{a}$, $\bar{b}$, $\bar{c}$.

6. In ΔOAB, E is the mid-point of OB and D is a point on AB such that AD : DB = 2 : 1. If OD and AE intersect at P, determine ratio OP : PD using vector method.

7. $\bar{a}$ and $\bar{b}$ are two non-collinear vectors, show that the points having positions vectors.

   $l_1\bar{a} + l_2\bar{b}$; $m_1\bar{a} + m_2\bar{b}$; $n_1\bar{a} + n_2\bar{b}$ are co-linear if

   $(l_1m_2 - m_1l_2) + (m_1n_2 - n_1m_2) + (n_1l_2 - l_1n_2) = 0$

8. Let the perpendicular lines B′ B and C′C intersect at A and position vector of A w.r.t. O be $\bar{a}$. $\overline{AB}$ and $\overline{AC}$ are parallel to $\bar{b}$ and $\bar{c}$ respectively. If P is any point on the bisector

   of ∠CAB then prove that position vector of P is given by $\bar{a} + \lambda \left(\frac{\bar{b}}{|\bar{b}|} \pm \frac{\bar{c}}{|\bar{c}|} \right)$

9. $\bar{a}$, $\bar{b}$, $\bar{c}$ are position vectors of points A, B, C and P, Q, R are points BC, CA, AB respectively such that BP : PC = CQ : QA = AR : RB = 1 : 2

   Find position vector of vertices of ΔXYZ formed by lines AP, BQ and CR.

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Chapter 6 : Co-ordinate Geometry

(2 - Marks)

1. Find the values of λ for which the triangle with vertices A (6, 10, 10) ; B (1, 0, –5) ; C (6, –10, λ) is a right angled triangle at B.
2. Find direction ratios of the line which bisects the angle between the lines whose direction consines are l_1, m_1, n_1 and l_2, m_2, n_2.
3. The equation of motion of a particle in space is given by $x = 2t$; $y = -4t$; $z = 4t$, where t is measured in second and co-ordinates of particle in kilometre. Then find the distance covered from the starting point by the particle in 15 seconds.
4. The equation of motion of a particle in space is given by $x = 2t$; $y = -4t$; $z = 4t$, where t is measured in second and co-ordinates of particle in kilometre. Then find the speed of particle in km / sec.
5. If distance of the point P (4, 3, 5) from the Y- axis is λ, then find the value of $7\lambda^2$.
6. A (3, 2, 0) ; B (5, 3, 2) ; C (–9, 6, –3) are the vertices of ΔABC. If the bisector of $\angle BAC$ meets BC at D then find the ratio in which C divides BD.
7. Planes are drawn parallel to the co-ordinate planes through the point (1, 2, 3) and (3, –4, –5). Find the lengths of edges of the parallelepiped so formed.
8. Find the ratio in which the plane $ax + by + cz + d = 0$ divides the join the points (x_1, y_1, z_1) and (x_2, y_2, z_2) .
9. Find $\overline{r}$, if direction ratio of vector $\overline{r}$ are 2, –3 6 and $|\overline{r}| = 21$ and r makes obtuse angle with the x -axis.
10. Let PM be the perpendicular drawn from the point P (x, y, z) on XY plane and OP makes an angle θ with the positive direction of Z - axis, OM makes an anlge θ with positive direction of X - axis then Prove that

    $x = r \sin\theta \cos\phi$; $y = r \sin\theta \sin\phi$; $z = r \cos\theta$

(3 - Marks)

1. The points (0, 1, –2) ; (3, λ, –1) ; (μ, –3, –4) are collinear show that the point (12, 9, 2) lies on the same line.

2. If θ is the angle between the lines having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2. Then prove that $\sin\theta = \sqrt{(l_1m_2 - m_1l_2)^2 + (m_1n_2 - n_1m_2)^2 + (n_1l_2 - l_1n_2)^2}$

3. l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of perpendicular lines. Find the direction ratios of the line perpendicular to both these lines.

4. If the diretion cosines of the line in two adjacent positions are l, m, n and $l + \delta l, m + \delta m, n + \delta n$ then show that the small angel $\delta\theta$ between two position is given by $\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2$

5. Find the angle included between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$

6. If direction cosines of the line satisfy the relation $\lambda(l + m) = n$ and $mn + nl + lm = 0$ then find the value of λ for which the two lines are perpendicular.

7. If l_1, m_1, n_1 ; l_2, m_2, n_2; and l_3, m_3, n_3 are direction cosines of mutually perpendicular vectors $\overline{OA}, \overline{OB}, \overline{OC}$. respectively then prove that the line having direction cosines proportional to $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$ and $n_1 + n_2 + n_3$ make equal angles with $\overline{OA}$, $\overline{OB}$, $\overline{OC}$.

8. Let PM be the perpendicular drawn from the point P (1, 2, 3) on XY plane. OP makes an angle θ with the positive direction of z - axis. OM makes an angle ϕ with positive direction of X - axis find θ and ϕ.

9. A (2, 3, 5) ; B (–1, 3, 2) ; C (λ, 5, μ) are vertices of ΔABC and median through vertex A is equally inclined to the axes then find area of ΔABC.

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Chapter 7 : Line

(2 - Marks)

1. Find the vector equation of line passing through the point A (5, 3, 8) and parallel to vector $3\hat{i} + 4\hat{j} + 5\hat{k}$.
2. A line passes through the point with position vecter $3\hat{i} - 4\hat{j} + \hat{k}$ and is in the direction of $2\hat{i} + \hat{j} - 2\hat{k}$, find the equation of the line in vector and cartesian form.
3. Find the equation of line in symmetric form passes through the point (8, 3, 7) and (–2, 5, –3)
4. Find the equation of line in cartesian form passing through the point (2, 1, –2) and perpendicular to the vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $2\hat{i} - 2\hat{j} + 3\hat{k}$.
5. Find the vector equation of line perpendicular to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $x = 5$, $\frac{y-3}{2} = \frac{z-2}{3}$, and passing through (3, –1, 11)
6. Write symmetric form of the equation of the line $3x - 1 = 4y + 8 = 3z - 3$
7. Find the angle between the pair of line
   $\bar{r} = (4\hat{i} + 7\hat{j} - 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and
   $\bar{r} = (5\hat{i} - 4\hat{j} + 3\hat{k}) + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$
8. Find the distane between the parallel lines
   $\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and
   $\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$
9. Find the direction cosines of the line
   $$\frac{2x-1}{3} = 3y = \frac{4z+3}{2}$$
10. Find the vecter equation of a line passing through the point with position vector $2\hat{i} - \hat{j} + \hat{k}$ and parallel to the line joining the points $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.

(3 - Marks)

1. Show that the lines $\bar{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + \hat{k})$ and $\bar{r} = (\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} + 3\hat{j} + 2\hat{k})$ intersect find their point of intersection.
2. Find the value of k, if the points A (1, 2, –1), B (4, –2, 4) and C (0, 0, k) form a triangle right angled, at C.
3. Find the foot of the perpendicular drawn from the point A (1, 0, 3) to the line joining the points B (4, 7, 1) and C (3, 5, 3)
4. Find the vector equation of the line passing through the point (2, 3, –4) and perpendicular to XZ - Plane, Hence find the equation in cartesian form.
5. Find the distance of P (1, 2, –2) from the line $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z}{-1}$
6. A line makes the same angle θ with each of X and Z - axis. If the angle β which it makes with Y - axis is such that $\sin^2\beta = 3\sin^2\theta$, then find the value of $\cos^2\theta$.
7. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{-1}$

   interect then find the value of k.
8. Find the shortest distance between the line $1 + x = 2y = -12z$ and $x = y + 2 = 6z - 6$
9. Find the distance of P (2, –1, 3) from the line $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-2}{2}$
10. Find the two points on the line $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z-5}{2}$ on either side (2, –3, –5)

    which are at a distance of 3 units from it.

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Chapter 8 : Plane

(2 - Marks)

1. Find the vector equation of the plane passing through the point A (4, –2, 3) and parallel to the plane $x + 2y - 5z + 8 = 0$
2. Find the vector equation of plane which passes through the point A (1, –1, 1) and perpendicular to the vector $4\hat{i} + 2\hat{j} + 2\hat{k}$.
3. Find the acute angle between the planes $\bar{r} . (2\hat{i} + \hat{j} + \hat{k}) = -5$ and $\bar{r} . (\hat{i} - \hat{j} + 2\hat{k}) = 8$
4. Find the value of p, if the planes $x - y + pz + 7 = 0$ and $3x + y - z = 4$ are perpendicular to each other.
5. Find the vector equation of the plane passing through the points (5, 2, –1) (2,2, 3) and origin.
6. Find the equation of the plane through the point (2, –3, 1) and perpendicular to the line whose d.r's are 3, –1, 2.
7. Find the angle between the line $\frac{x-1}{5} = \frac{y}{2} = \frac{z}{14}$ and plane $12x + 4y - 3z = 25$
8. Find the equation of a plane whose distance from the origin is 5 units and normal in the direction of $\hat{n} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$
9. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are co-planar then find the value of K.
10. A plane makes intercept 1, 2, 3 on the co-ordinate axes. If the distance from origin is p then find the value of p.

3 - Marks

1. Find the equation of a plane which bisects the line joining the point A (2, 3, 4) and B (4, 5, 8) at right angles.
2. Find the equation of the plane through the intersection of the planes $\bar{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\bar{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$ whose perpendicualr distance from origin is unity.
3. Find the equation of plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$ and parallel to the plane $x + 3y + 6z = 1$
4. A variable plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ at a unit distance from the origin cuts the co-ordinate axes at A, B and C. Centroid (x, y, z) of ΔABC satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$ then find the value of k
5. If the angle between the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$ then find the value of λ
6. Find the distance of the plane passes throught (1, –2, 1) and perpendicular to planes $2x - 2y + z = 0$ and $x - y + 2z = 4$ from the point (1, 2, 2)
7. Find the vector and cartesian equation of the plane passing through the points (2, 3, 1), (4, –5, 3) and parallel to the X - axis.
8. Find the equation of the plane passing through the point (3, –2, –1) and parallel to the lines whose direction ratios are 1, –2, 4 and 3, 2, –5.
9. Find the angle between the planes $x - 2y + 2z = 7$ and $x - y - 3z = 5$
10. Find the equation of plane in vector form and cartesian form if the plane is at a distance of 3 units from the origin and has $\hat{i} + \hat{j} - 3\hat{k}$ as a normal vector.

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Chapter 9 : Linear Programming

(2 - Marks)

'SECTION B'

1. Find the point at which the maximum value $(3x + 2y)$ takes place when subject to constraints $x + y \leq 2$, $x \geq 0$, $y \geq 0$.
2. Find the feasible region graphically of the inequalities $5x + 10y \geq 100$, $x + y \leq 60$, $x \geq 0$, $y \geq 0$.
3. Find the maximum value of $z = 6x + 10y$ subject to $x \leq 6$, $y \leq 2$, $x \geq 0$, $y \geq 0$.
4. Find the corner points of the feasible region dtermned by the linear inequalities $2x + y \leq 10$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$.
5. Draw the graph of the inequalities $5x + y \geq 10$, $x + y \geq 6$, $x + 4y \geq 12$, $x \geq 0$, $y \geq 0$ on a graph paper.
6. A firm is engaged in producing two models X_1, X_2 performing only three operations assembling, painting and testing. The relevent data are as follows.

Model	Unit Sale Price	Hours required for each unit		
		Assembling	Painting	Testing
X_1	Rs. 50	1.0	0.2	0.0
X_2	Rs. 80	1.5	0.2	0.1

Total number of hours avaiable each week for assembling 600 hours, painting 100 hours and testing 30 hours. The firm wishes to determine its weekly product - mix. So as to maximize revenue. Formulate L.P.P. model for maximize the revenue.

7. Find the coordinates of the point for minimum value of $z = 7x - 8y$, subject to $x + y \leq 20$, $y \geq 5$, $x \geq 0$, $y \geq 0$.
8. Find the area of the feasible region for the constraints $x + 3y \leq 3$, $x \geq 0$, $y \geq 0$
9. Find the solution set of inequation $x - 2y \geq 0$, $2x - y + 2 \leq 0$, $x \geq 0$, $y \geq 0$.
10. Draw the geometrical shape of the common region represented by the inequalities $0 \leq x \leq 6$, $0 \leq y \leq 4$.

3 - Marks

'SECTION 'C'

1. Using graphical method solve Minimize $z = 7x - 8y$ subject to the constraints $x + y \leq 20$, $y \geq 5$, $x \geq 0$, $y \geq 0$.

2. Using graphical method solve Maximize $z = 7x + 10y$ subject to the constraints $x + y \leq 30$, $y \leq 12$, $x \geq 6$, $x \geq 0$, $y \geq 0$.

3. Using graphical method find the maximum value of $z = 10x + 25y$ subject to the $0 \leq x \leq 3$, $0 \leq y \leq 3$, $x + y \leq 5$.

4. Using graphical method find the minimum value of $z = 7x + y$ subject to $5x + y \geq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$.

5. A factory produces two products P_1 and P_2. Each of the product P_1 requires 2 hrs. for moulding, 3 hours for grinding and 4 hrs. for polishing. and each of the product P_2 requires 4 hrs for moulding, 2 hrs for grinding and 2 hrs for polishing. The factory has moulding machine available for 20 hrs, grinding machine for 24 hrs and polishing machine for 13 hrs. The profit is Rs, 5 per unit of P_1 and Rs. 3 per unit of P_2 and the factory can sell all that it produces. Formulate the problem as a linear programming problem to maximize the profit.

6. A firm can produce three types of cloths say C_1, C_2, C_3. Three kinds of wool are required for it say red wool, green wool and blue wool one unit of length C_1 needs 2 meters of red wool, 3 meters of blue wool, one unit of cloth C_2 needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool and one unit of cloth C_3 needs 5 meters of green wool and 4 meters of blue wool. The firm has only a strock of 16 meters of red wool, 20 meters of green wool and 30 meters fo blue wool. It is assumed that the income obtained from one unit of length of C_1 is Rs. 6, of cloth C_2 is Rs. 10 and of cloth C_3 is Rs. 8. Formulate the problem as a linear programming problem to maximize the income.

7. A company makes two types of leather belts A and B. A is high quality belt and B is of lower quality belt. The profits are Rs. 40 and Rs. 30 per belt. Each belt of type A requires twice as much time as a belt of type B. And if all belts were of type B, the company could make 1000 belts per day. The supply leather is sufficient for only 800 belts per day (both A and B combined) Belt A requires a fancy buckle and only 400 buckels per day are available. There are only 700 buckles available for belt B. What should be the daily production for each type of belt ? Formulate the problem as a LPP.

8. A dietician whishes to mix two types of food in such a way that the vitamin contents of the mixture contain atleast 8 units of Vitamin A and 10 units of Vitamin C. Food ‘P’ Contains 2 units per kg. of Vitamin A and 1 unit per kg of Vitamin C while food ‘Q’ contains 1 unt per kg of Vitamin A and 2 units per kg of Vitamin C. It costs Rs. 50/- per kg to purchase food ‘P’ and Rs. 70/- per kg to purchase food ‘Q’. Formulate the above linear programming problem to minimize the cost of such a mixture.

9. A rubber company is engaged in producing three types of tyres A, B and C. Each type requires processing in two plants. Plant I and Plant II. The Capacities of the two plants in number of tyres per day are as follows.

Plant	A	B	C
I	50	100	100
II	60	60	200

The monthly demand for tyre A, B and C is 2500, 3000 and 7000 respectively. If plant I costs Rs. 2500/- per day and plant II costs Rs. 3500/- per day to operate. How many days should each be run per month to minimize cost while meeting the damand ? Formulate the problem as LPP.

10. A firm is engaged in breeding goats. The goats are fed on various products grown on the farm. They need certain nutrients named as X, Y and Z. The goats are fed on two products A and B. One unit of product A contains 36 units of X, 3 units and Y and 20 units Z. while one unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units repectively. Product A costs Rs. 20/- per unit and peroduct B costs Rs. 40/- per unit. Formulate the LPP to minimize the cost.

4 - Marks

'SECTION 'D'

1. Using graphical method solve maximize $Z = 12x + 3y$ subject to the constraints $x + y \leq 5$, $3x + y \leq 9$, $x \geq 0$, $y \geq 0$.
2. Using graphical method solve minimize $Z = 2x + 2y$ subject to the constraints $3x + 2y \geq 12$, $x + 3y \geq 11$, $x \geq 0$, $y \geq 0$.
3. Solve the following LPP graphically.

   Maximize $Z = x + 2y$ subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x \geq 0$, $y \geq 0$
4. Solve the following LPP graphically

   Minimize : $Z = -3x + 4y$ subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$
5. Solve the following LPP graphically method

   Minimize : $Z = 20x + 10y$ subject to $x + 2y \leq 40$, $3x + y \geq 30$, $4x + 3y \geq 60$, $x \geq 0$, $y \geq 0$
6. A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hours of work on machine A and 3 hours on machine B to produce package of nuts while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He urns a profit of Rs. 2.50 per package of nuts and Rs. 1.00 per package of bolts. How many package of each should he produce each day so as to maiximize his profit, If he operates his machine for at most 12 hours a day. Formulate this LPP and then solve it.
7. An oil Company requires 12,00, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high grade, medium grade and low grade oil respectively while refinely B produces 200, 400 and 100 barrels per day of high grade, medium grade and low grade oil respectively. If refinery A costs Rs. 400 per day and refinery B costs 300 per day to operate, how many days should each be run to minimize costs while satisfying requirements.
8. A dealer whishes to purchase a number of fans and sewing machines. He has only Rs. 5760/- to invest and had space for atmost 20 items. A fan costs him Rs. 360/- and a sweing machine Rs. 240/-. His expectation is that he can sale a fan at a profit of Rs. 22 and sweing machine at a profit of Rs. 18. Assuming that he can sale all the items that he can buy. How

should he invest his money in order to maximize his profit? Translate this problem mathematically and then solve it.

9. A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to einsure certain nutrient constituents (call then X, Y and Z), it is necessary to buy two additional products say A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 unis, 36 units and 100 units respectively. Product A costs Rs. 20/- per unit and product B costs Rs. 40 per unit formulae linear programming. Problem to minimize the total cost and solve the problem by using graphical method.

10. If a young man rides his motocylce at 25 km / hour. He had to spend Rs. 2/- per km on petrol. If he rides at a faster speed of 140 km / hour the petrol cost increases at Rs. 5/- per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.

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Chapter 10 : Continuity

(2 - Marks)

1. Prove that the function $\frac{\sin x}{|x|}$ is discontinuous at $x = 0$.

2. Let f (x) be a continuous function and g (x) be a discontiuous function, prove that f (x) + g (x) is discontinuous function.

3. Discuss the continuity of $f(x) = \frac{x^5\sqrt{x} - 32\sqrt{2}}{x^3\sqrt{x} - 8\sqrt{2}}$, $x \neq 2$

$$= \frac{44}{7}, \quad \text{at } x = 2$$

4. If $f(x) = \frac{\sqrt[n]{x^n} - 1}{\sqrt[m]{x^n} - 1}$ for $x \neq 1$ is continuous at $x = 1$, find f (1)

5. Discuss the continuity of the function

$$f(x) = x^2/a - a \quad , \quad x < a$$
$$= 0 \quad , \quad x = a$$
$$= a - x^2/a \quad x > a$$, at $x = a$

6. Find the point (s) in the interval [–1, 2] where the function f (x) = x for $x \neq 0$ and f (0) = 1 is discontinuous

7. If $f(x) = \frac{\sin[4(x-3)]}{x^2 - 9}$, $x \neq 3$ is continuous at $x = 3$ then find f (3).

8. If the function $f(x) = \frac{\cos kx - \cos 4x}{x^2}$, $x \neq 0$

$$= 6 \quad , x = 0$$

is continuous at $x = 0$, find k.

9. If the function $f(x) = \frac{\log x - 1}{x - e}$, for $x \neq e$

is continuous at $x = e$ find f (e).

10. Show that the function $f(x) = 2x - |x|$ is continuous at $x = 0$.

3 - Marks

1. If $f(x) = \left(1 + |\sin x|\right)^{\frac{a}{|\sin x|}}$, $-\pi/6 < x < 0$

   $= b$, $x = 0$

   $= e^{\tan 2x / \tan 3x}$, $0 < x < \pi/6$

   Find a and b so that the function is continuous at $x = 0$

2. Find the value of f (0) so that the function

   $$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$ is continuous for all x.

3. If the function $f(x) = \dfrac{x + x^2 + x^3 + x^4 + x^5 - 62}{x - 2}$, $x \neq 2$

   $= 3k$, $x = 2$

   is continuous at $x = 2$, find k.

4. If $f(x) = \dfrac{\sin x - \sin a}{\cos x - \cos a}$, $x \neq a$

   $= 1$, $x = a$ is continuous, at $x = a$ find a

5. If the function $f(x) = \dfrac{(a + x)^2 \sin(a + x) - a^2 \sin x}{x}$, $x \neq 0$

   is continuous at $x = 0$ find f (0)

6. If $f(x) = \dfrac{x \quad a^x - x}{\sqrt{1 + x^2} - \sqrt{1 - x^2}}$, $x \neq 0$

   $= k$, $x = 0$ is continuous at $x = 0$, find k.

7. If the function $f(x) = \dfrac{(5^x - 1)^4}{x \tan(x/5) \log(1 + x^2/5)}$, $x \neq 0$

   is continuous at $x = 0$, find f (0).

8. Discuss the continuity of $f(x) = \dfrac{2^{2x-2} - 2^x + 1}{\tan^2(x-1)}$, $x \neq 1$

$= 2 \log 2$, $x = 1$ at $x = 1$

9. If the function $f(x) = \dfrac{x + 1 - \sqrt{x + 13}}{x - 3}$, $x \neq 3$

$= k$ $\quad x = 3$

is continuous at $x = 3$, find k.

10. If $f(x) = \dfrac{(a^x - 1)^3}{\sin(x \log a) \, log\,(1 + x^2\, log\, a^2)}$, $x \neq 0$

is continuous at $x = 0$, find f (0).

4 - Marks

1. Find the value of A so that the function

$f(x) = \dfrac{2^{x+2} - 16}{4^x - 2^4}$, $x \neq 2$

$= A$, $x = 2$ is continuous at $x = 2$

2. If $f(x) = \dfrac{2^{x-} + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$, $x \neq 2$ is continuous

at x = 2, find f (2)

3. If $f(x) = \dfrac{1 - \cos(x^2 - x - 6)}{(x-3)^2}$, $x \neq 3$ is continuous at $x = 3$ find f (3)

4. If the function

$f(x) = \dfrac{1}{x^8}\left[1 - \cos\left(\dfrac{x^2}{2}\right) - \cos\left(\dfrac{x^2}{4}\right) + \cos\left(\dfrac{x^2}{2}\right)\cos\left(\dfrac{x^2}{4}\right)\right]$, $x \neq 0$

$= \dfrac{k^2}{4}$, $x = 0$ is continuous at $x = 0$ find k.

5. Discuss the continuity of the function

$$f(x) = \frac{(1 - \tan(x/2))(1 - \sin x)}{(1 + \tan(x/2))(\pi - 2x)^3}, \quad x \neq \pi/2$$

$$= \frac{1}{16}, \quad x = \pi/2 \quad \text{at } x = \pi/2$$

6. Let $f(x) = \dfrac{\log(1 + x - x^2) + \log(1 - x + x^2)}{\sec x - \cos x}$, $x \neq 0$

then find the value of f (0) so that f (x) is continuous at $x = 0$

7. If the function

$$f(x) = \frac{x \sin a - a \sin x}{x - a}, \quad x \neq a$$

is continuous at $x = a$, find f (a)

8. If the function

$$f(x) = x + a^2 \sqrt{2} \sin x, \quad 0 \leq x < \pi/4$$

$$= x \cot x + b, \quad \pi/4 \leq x < \pi/2$$

$$= b\sin 2x - a\cos 2x \quad \pi/2 \leq x \leq \pi$$

is continuous on $[0, \pi]$, find a & b

9. Discuss the continuity of the function.

$$f(x) = \frac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}, \quad x \neq \pi/4$$

$$= 3/\sqrt{2}, \quad x = \pi/4 \quad \text{at } x = \pi/4$$

10. Find K it the fuction

$$f(x) = \frac{\log(1 + 2x) - 2\log(1 + x)}{x^2}, \quad x \neq 0$$

$$= kx^2 + 5x + 3k, \quad x = 0 \text{ is continuous at } x = 0$$

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Chapter 11 : Differentation

(2 - Marks)

1. If $y = x\left[\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right) + \sin x\right] + \frac{1}{2\sqrt{x}}$ find $\frac{dy}{dx}$

2. If $y = \tan^{-1} \frac{(ax - b)}{(bx + a)}$ then find dy/dx.

3. If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ then find dy/dx.

4. If $y = \cos^{-1}\left(\sqrt{\frac{1+x}{2}}\right)$ find dy/dx.

5. If $f(x) = |x - 1| + |x - 3|$ then find $f^1(2)$

6. Find the deivative of $f(x) = \left[\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right) + x^x\right]$ *w.r.t.x* at $x=1$

7. If $x^y y^x = 16$ then find $\frac{dy}{dx}$ at (2, 2)

8. If $f(x) = \frac{x}{1 + |x|}$, for $x \in R$ then find $f'(0)$

9. If $f'(x) = \sqrt{3x^2 - 1}$ and $y = f(x^2)$ then find $\frac{dy}{dx}$

10. If $y = \tan^{-1}\left[\frac{\log(e/x^3)}{\log(ex^3)}\right] + \tan^{-1}\left[\frac{\log(e^4x^3)}{\log(e/x^{12})}\right]$ show that $\frac{d^2y}{dx^2} = 0$

3 - Marks

1. Let $f(x) = e^x$, $g(x) = \sin^{-1}x$ and $h(x) = f(g(x))$ then find $\dfrac{h'(x)}{h(x)}$

2. Find the deivative of f (tan x) w.r.t. to g (sec x) at $x = \pi/4$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$

3. If $y = \tan^{-1}\left(\dfrac{1}{1+x+x^2}\right) + \tan^{-1}\left(\dfrac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\dfrac{1}{x^2+5x+7}\right)$ + ... + n terms then find $y'(0)$

4. If $f(x) = \cos x \ \cos 2x \ \cos 4x \ \cos(8x) \,.\, \cos 16x$ then find $f'(\pi/4)$

5. If f be twice differentiable function such that $f^{11}(x) = -f(x)$ and $f'(x) = g(x)$, $h(x) = [f(x)]^2 + [g(x)]^2$ if $h(5) = 10$ then find $h(10)$.

6. If $y = \sin^2\left[\cot^{-1}\left(\dfrac{1}{\sqrt{\dfrac{1+x}{1-x}}}\right)\right]$ then find $\dfrac{dy}{dx}$

7. If $y = x \cos y$, show that $\dfrac{dy}{dx} = \dfrac{\cos^2 y}{\cos y + y \sin x}$

8. If $y = \dfrac{x^{\sin x}}{1+x+x^2}$ find $\dfrac{dy}{dx}$

9. If $y = e^{f(x)}$ where $f(x) = \sqrt{\dfrac{x-1}{x+1}}$ then show that $\dfrac{dy}{dx} = \dfrac{y \log y}{x^2 - 1}$

10. If $y = \tan x + {}^2/_3 \tan^3 x + {}^1/_5 \tan^5 x$ then find ${}^{dy}/_{dx}$ in terms of sec x.

4 - Marks

1. If $\log y = \log(\sin x) - x^2$ then show that $\frac{d^2y}{d^2x} + 4x\frac{dy}{dx} + (4x^2 + 3)y = 0$

2. If $2y = \sqrt{x+1} + \sqrt{x-1}$ show that $4(x^2 - 1)\, {}^{d^2y}/_{dx^2} + 4x\, {}^{dy}/_{dx} - y = 0$

3. If $y = |\cos x| + |\sin x|$ then find $\frac{dy}{dx}$ at $x = 2\pi/3$

4. Find $\frac{dy}{dx}$ if $y = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$ at $x = 1$

5. If $\sqrt{x+y} - \sqrt{y-x} = 8$ then prove that $\frac{d^2y}{d^2x} = {}^2/_{c^2}$.

6. If $(a - b\tan y)(a + b\tan x) = a^2 + b^2$ show that ${}^{dy}/_{dx}$ is constant and state its value.

7. If $y = \sin^{-1}[15x - 500x^3] + \cos^{-1}[1372x^3 - 21x]$ find ${}^{dy}/_{dx}$.

8. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\ldots\infty}}}}$

   show that $\frac{dy}{dx} = \frac{y^2 - x}{2y^3 - 2xy - 1}$

9. If $x^2 + y^2 = t + t^{-1}$ and $x^4 + y^4 = t^2 + t^{-2}$ then show that $\frac{dy}{dx} = {}^{-y}/_{x}$.

10. If $y = \frac{a + b\tan x}{b - a\tan x}$ show that $\frac{dy}{dx} = 1 + y^2$.

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Chapter 12 : Application of derivative

(2 - Marks)

1. An inverted conical container, R = 4 cm, H = 20 cm is full of water. Due to a small leak at the vertex, the volume of the water in the container reduce at $12 cm^3/sec$. Find the rate at which water level falls when water level is 5 cm.

2. A wire length 2 units is cuts in to two parts which are bent to form a square of side x units and a circle of radius r units. If the sum of the area of the square and the circle so formed is minimum then find the relation between x and r.

3. Find c if $f(x) = \dfrac{x^2 - 4x}{x + 2}$, $0 \leq x \leq 4$ and Rolle's theorem is applicable.

4. If $f(x) = x^n$, $0 < a < b < c$ and $n > 1$ is an odd number. Then show that $(b - c)a^n + (c - a)b^n + (a - b)c^n$ is negative.

5. Find the approximate value of $\sqrt{37}$.

6. Find the approximate value of $\log_e 99$ if $\log_e 10 = 2.3023$.

7. If $1° = 0.0175$ radians . $\sin 60° = 0.8660$ then find the approximate value of $\cos(60° 40')$.

8. If e = 2.7183. Find the approximate value of $e^{1.005}$.

9. Find the approximate value of $f(x) = x^3 + 5x^2 - 7x + 10$ at $x = 1.1$

10. If $f(x) = x(2 - x)$, $x \in [0, 1]$ verifies LMVT. Then find c.

11. On the interval [0, 1] the function $x^{25}(1 - x)^{75}$ what point takes its maximum value.

12. Find the anlge between the curves $y = a^x$ and $y = b^x$.

13. The distance covered by a particle in t second is give be $x = 3 + 8t - 4t^2$. Find the velocity after 1 second.

3 - Marks

1. Verifty Rolle's theorem for function $f(x) = x^2 - 8x + 12$ on [2, 6]

2. Verifty Rolle's theorem for function $f(x) = x(x-4)^2$ on [0, 4]

3. Verifty Rolle's theorem for function $f(x) = \dfrac{\sin x}{e^x}$ on $[0, \pi]$

4. Verifty Rolle's theorem for function $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

5. Discuss the applicability of Roll's theorem for function $f(x) = |x|$ in [–1, 1]

6. Discuss the applicability of Roll's theorem for the function

$$f(x) = \begin{cases} x^2 + 1 & 0 \le x < 1 \\ 3 - x & 1 \le x \le 2 \end{cases}$$

7. Verify LMVT for the function $f(x) = \sin x - \sin 2x - x$ on $[0, \pi]$

8. Verify LMVT for the function $f(x) = (x-3)(x-6)(x-9)$ on [3, 5]

9. Verify LMVT for the function $f(x) = \log x$ on [1, 2]

10. Using LMVT, find the point on the curve $y = \sqrt{x-2}$ defined on the interval [2, 3] where the tangent is parallel to the chord joining the end points of the curve.

11. Using LMVT, find a point on the curve $y = x^2 + x$, where the tangent is parallel to the chord joining (0,0) and (1, 2).

12. A stone is thrown vertically upward from the top of a tower 64 m high accordingly the law of motion given by $s = 48t - 16t^2$ what is the greatest height attained by the stone above the ground.

13. If the surface area of a sphere of radius r in increasing uniformaly at the rate 8 cm^2/s then show that the rate of change of volume is propertional to radius r.

4 - Marks

1. Each side of an eqilateral triangle is increasing at the rate of $\sqrt{3}$ cm/sec. Find the rate of which its area is increasing when its side 2 meters.

2. A circular blot of ink increases in area in such a way that the radius 'r' cm at a time 't' sec. is given by $r = 2t^2 - t^3/4$ what is the rate of increase of the area when t = 4.

3. Water is being poured at the rate $36m^2$/min in to cylindrical vessel whose base is a circle of radius 3 meters. Find the rate at which the level of water is rising.

4. The height of cone is 30 cm and it is constant, the radius of the base is increasing at the rate of 2.5 cm/sec. Find the rate of increase of volume of the cone when the radius is 10cm.

5. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the point on the curve where the y - coordinate is changing twice as fast as the x - co-ordinate.

6. An edge of a veriable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing when the edge is 5 cm long ?

7. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.

8. Find the slope of the tangent and the normal when $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$ at $= \pi/2$

9. Find a point on the curve $xy = -4$ where the tangents are inclined at an angle 45^0 with x - axis.

10. Show that the tangents to the curve $y = 2x^2 - 3$ at a point $x = 2$ and $x = -2$ are parallel.

11. FInd the point on the curve $y = 6x - x^2$ where the tangents has slope -4. Also find the equation of the tanent at that point.

12. Find the second degree polynomial $f(x)$ satisfying $f(0) = 0$, $f(1) = 1$, $f^1(x) > 0$ for all $x \in (0, 1)$

13. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ attains its maximum and minimum at p and q respectively such that $p^2 = q$ then find the value of a.

14. If $x + y = 8$ then show that maximum value of x^2y is $\frac{2048}{27}$

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Chapter 13 : Indefinite Integeals

SECTION B
(2 - Marks)

1. Evaluate $\int \sqrt{\dfrac{e^x - 1}{e^x + 1}}\, dx$

2. Evaluate $\int \dfrac{\sec^2 x - 7}{\sin^7 x}\, dx$

3. Evaluate $\int \sin(101\, x) \sin^{99} x\, dx$

4. Evaluate $\int \dfrac{\sqrt{\cot x}}{\sin x \cdot \cos x}\, dx$

5. Evaluate $\int \dfrac{\log(e^x + 1)}{e^x}\, dx$

6. Evaluate $\int \dfrac{x^4 + 1}{x^6 + 1}\, dx$

7. Evaluate $\int \sqrt{\dfrac{\cos x - \cos^3 x}{1 - \cos^3 x}}\, dx$

8. Evaluate $\int \dfrac{1 + x}{1 + \sqrt[3]{x}}\, dx$

9. Evaluate $\int x \cdot (x^x)^x (2 \log x + 1)\, dx$

10. Show that $\int \dfrac{dx}{\sin^4 x}$ is a polynomial of degree three in cot x.

SECTION C
3 - Marks

1. Evaluate $\int \tan^{-1}\left(\sqrt{\dfrac{1-x}{1+x}}\right) dx$

2. Evaluate $\int \dfrac{dx}{\cos^3 x \sqrt{\sin 2x}}$

3. Evaluate $\int \dfrac{dx}{x^2 (x^4 + 1)^{3/4}}$

4. Evaluate $\int \dfrac{1}{1 + x + x^2 + x^3} dx$

5. Evaluate $\int \dfrac{1}{x} \sqrt{\dfrac{1-x}{1+x}} dx$

6. Evaluate $\int \dfrac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2\theta - 4 \sin \theta} d\theta$

7. Evaluate $\int x \sin x \,.\, \sec^3 x \, dx$

8. Evaluate $\int \dfrac{\sin x}{\sqrt{1 + \sin x}} dx$

9. Evaluate $\int \dfrac{\cos 4x - 1}{\cot x - \tan x} dx$

10. Show that $\int (\sqrt{\tan x} + \sqrt{\cot x}) \, dx = \sqrt{2} \tan^{-1}\left(\dfrac{\tan x - 1}{\sqrt{2 \tan x}}\right) + c$

SECTION D

4 - Marks

1. Evaluate $\int \tan^{-1}(1 + \sqrt{x})\, dx$

2. Evaluate $\int \dfrac{1}{\sin^3 x + \cos^3 x}\, dx$

3. Evaluate $\int \log\left|\sqrt{1 - x} + \sqrt{1 + x}\right| dx$

4. Evaluate $\int \dfrac{1}{\cos^2 x + \cot^2 x}\, dx$

5. Evaluate $\int \dfrac{1}{\sec x + \operatorname{cosec} x}\, dx$

6. Evaluate $\int \dfrac{\sin x}{\sin 4x}\, dx$

7. If $\int [\log(\log x) + (\log x)^{-2}]\, dx = f(x) + c$ then find $f(x)$.

   Also find $f(x)$ when graph of $y = f(x)$ passes through the point (e, e)

8. Evaluate $\int \sin^{-1}\left(\sqrt{\dfrac{x}{a + x}}\right) dx$

9. Evaluate $\int \dfrac{(x + 1)\sqrt{x + 2}}{\sqrt{x - 2}}\, dx$

10. Evaluate $\int \dfrac{dx}{\sqrt{2x + 3} + \sqrt{x + 2}}\, dx$

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Chapter 14 : Definite Integrals and It's Applications

SECTION B

2 - Marks

1. Evaluate $\int_{0}^{\pi/2} \frac{3\sin\theta + 4\cos\theta}{\sin\theta + \cos\theta}$

2. Evaluate $\int_{1/3}^{1} \frac{(x - x^3)^{\frac{1}{3}}}{x^4}\,dx$

3. Show that $\int_{0}^{\pi/2} \cos^{99}x\;dx = 0$

4. Find the area of the region enclosed by the lines y = x, x = e and the curve y = $\frac{1}{x}$ and posiitve x - axis.

5. Evaluate $\int_{1}^{e^{37}} \frac{\pi\sin(\pi\log x)}{x}\,dx$

6. Evaluate $\int_{-3}^{3} f(x - [x])\,dx$ where f is signum function and [.] denotes greates integer function.

7. Evaluate $\int_{\alpha}^{\beta} x\,|x|\,dx$ where $\alpha < 0 < \beta$.

8. Show that $\int_{0}^{\pi/2} (2\log\sin x - \log\sin 2x)\,dx = \frac{-\pi}{2}\log 2$

9. Evaluate $\int_{\frac{1}{e}}^{e} |\log x|\;dx$

10. Show that $\int_{0}^{\pi} x\,f(\sin x)\,dx = \pi\int_{0}^{\pi/2} f(\cos x)\,dx$.

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SECTION C

3 - Marks

1. Evaluate $\int_0^{\pi/2} \frac{1}{1 + e^{\sin x}} \, dx$

2. Evaluate $\int_0^{1} [5x] \, dx$. Where [.] denotes greatest integer function.

3. Find the area bounded between the parablas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the strainght line $y = 2$.

4. Show that $\int_0^{\pi/4} \log (1 + \tan^2\theta + 2 \tan \theta) \, d\theta = \frac{\pi}{4} \log 2$

5. Evaluate $\int_{-1}^{3} |x - 2| + [x]) \, dx$, where [.] denotes greatest integer function.

6. If $f(x) = \int_1^{x} \sqrt{2 - t^2} \, dt$ then find the roots of the equation $x^2 - f'(x) = 0$

7. Find the area of the region bounded by the curves $y = \begin{cases} x^2 & , x < 0 \\ x & , x > 0 \end{cases}$ and the line $y = 4$

8. Evaluate $\int_0^{\pi/2} (\sin 2x \,.\, \tan^{-1} (\sin x)) \, dx$.

9. If $f(x)$ is a continuous function such taht $f(2 - x) + f(x) = 0$ for all x, then find

$$\int_0^{2} \frac{1}{1 + 2^{f(x)}} \, dx$$

10. Evaluate $\int_0^{1} \sin \left(2 \tan^{-1} \left(\sqrt{\frac{1 + x}{1 - x}}\right)\right) dx$

****

SECTION D

4 - Marks

1. Evaluate $\int_0^1 \tan^{-1}(1 - x + x^2)\, dx$.

2. Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} | x \cos \pi x |\, dx$.

3. Evaluate $\int_0^{\infty} \frac{x \log x}{(1 + x^2)^2}\, dx$.

4. Evaluate $\int_0^1 \frac{2 - x^2}{(1 + x)\sqrt{1 - x^2}}\, dx$.

5. $\int_{\frac{1}{e}}^{e} \frac{| \log x |}{x^2}\, dx$.

6. Find the area bounded by the curves $x = y^2$ and $x = 3 - 2y^2$.

7. Evaluate $\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(| x | + \frac{\pi}{3}\right)}\, dx$.

8. Find the area enclosed within the curves $| x | + | y | = 1$

9. If $I = \int_2^4 \frac{\sqrt{x^2 - 4}}{x^4}\, dx$ then show that $I = \sqrt{3}$

10. Let $f(x)$ = maximum of $\{ x + | x | , x - [x] \}$ where $[.]$ denotes greatest integer function

    then find $\int_{-2}^{2} f(x)\, dx$.

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Chapter 15 : Differential Equation

(2 - Marks)

1. Find the differential equation of all circles passing through the origin and having their centres on the X - axis.

2. Verify $y = \frac{1}{4} e^{-2x} + cx + d$, is the solution of the differential equation $\frac{d^2y}{dx^2} = e^{-2x}$

3. Find the equation of the curve whose slope $\frac{dy}{dx} = \frac{2y}{x}$, x, y > 0 which passes through the point (1, 1)

4. Find the equation of curve passing through (1, π/4) and having slope $\frac{\sin 2y}{x + \tan y}$ at (x, y)

5. Find the integrating factor (I.F.) of differential equation $\frac{dy}{dx} = e^{x-y}(1 - e^{y})$

6. If sin x is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$ then find the value of P.

7. Find the integrating factor of the differential equation $(xy - 1)\frac{dy}{dx} + y^2 = 0$

8. State order and degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} \left(\frac{dy}{dx}\right)^{\frac{1}{3}} = 5$

9. By eliminating arbitary constant of equation $y = c^2 + \frac{c}{x}$ find differential equation.

10. If $y = \sin^{-1} x$ then show that $(1 - x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx}$

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3 - Marks

1. Find the differential equation associated to the primitive $y = ae^{4x} - be^{-3x} + c$

2. Find A if $x = 4t^3$, $y = 4t^2 - t^4$ constitute a solution of the differential equation

$$36 \frac{d^2y}{dx^2} \left[y - (2x)^{2/3} \right] = A + \left(\frac{x}{4} \right)^{2/3}$$

3. Find the particular solution of the differenital equation given as

$$e^{\frac{dy}{dx}} = x + 1 \text{ at } y(0) = 3$$

4. Find $y(\pi/2)$ if $(2 + \sin x) \frac{dy}{dx} + (y + 1) \cos x = 0$ and $y(0) = 1$

5. Find the equation of the curve passing through (2, –2) and having slope $\frac{y+1}{x^2 - x}$

6. If curve $y = f(x)$ passes through the point (1, –1) and satisfies the differenital

   quation $y(1 + xy)\, dx = x\, dy$, then find $f\left(-\frac{1}{2}\right)$

7. Find the general solution of the differenital equation

$$(e^{x^2} + e^{y^2})\, y \frac{dy}{dx} + e^{x^2}(xy^2 - x) = 0$$

8. The differential equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ whose general solution is given by

   $f(x, y)\, e^{e^x} =$ constant, then find $f(0, 0)$

9. Find the general solution of the differential equation

   $\cos x\, dy = y(\sin x - y)\, dx, \quad 0 < x < \pi/2$

10. If $f'(x) = f(x)$ and $f(-1) = 1$ then find $f(5)$.

11. Form and differential equation satisfying $\sqrt{1 - x^4} + \sqrt{1 - y^4} = a(x^2 - y^2)$ and find the degree.

12. If the population of a country doubles in 50 years. Then find the number of years in which population will be triple under the assumption that the rate of increase of population proportional to the number of inhabitants.

13. A ray of light coming from origin after reflecting at the point $P \equiv (x, y)$ of any curve become parallel to x - axis find the equation of curve.

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4 - Marks

1. The rate at which radioactive substance decay is known to be proportional to the number of such nuclei that are present at that time in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergoan disintegration in a period of 100 years. Find what percentage of the original radioactive nuclei will remain after 1000 years.

2. In a college hostel accomodating 1000 students, one of them came in carrying a flue virus, then the hostel was isoleted. If the rate of which the virus spreads is assumed to be proportional to the product of the number N of infected students and the number of non infected students and if the number of infected students is 50 after 4 days. Show that more than 95% of the students will be infected after 10 days.

3. Water flows from the base of rectangular tank of depth 16 meters. The rate of flowing the water is proportional to the square root of depth at any time 't'. After 2 hours depth of water is 4 meter, find the depth of water after 4 hours.

4. Water at 100^0C cools in 10 minutes to 88^0C, in the room temperature of 25^0C. Find the temperature of water after 20 minutes.

5. A tank of 100 m^3 capacity is full with pure water. Begining at t = 0 brine containing 1 kg/m^3 of salt runs in at the rate 1 m3/min. The mixture is kepts uniform by stirring. It runs out at the same rate when will there by 50 kg. of dissolved salt in the tank ?

6. Show that the singular solution of the differential equation $y = mx + m - m^3$ where $m = \frac{dy}{dx}$ passes through the point (–1, 0)

7. Let the population of rabiits surviving at a time 't' be governed by the differential equation

   $$\frac{dp\,(t)}{dt} = -\frac{1}{2} P\,(t) - 200.$$ If p (0) = 100 then find p (t).

8. A hemispherical tank (radius = 1m) is initially full of water and has an out let of 12 cm^2 at the bottom. When the out let is opened the flow of water is according to the law $v\,(t) = \lambda \sqrt{h(t)}$ where v (t) is the velocity of flow in cm/sec. h (t) is the water level in cms. and λ is constant. Find the time taken to empty of tank is sec.

9. A mothball evaporate at a rate proportional to the instantaneus surface area. Its radius to half in value in one day. Find the requaired for the ball to disapper compleletly.

10. An inverted conical tank of 2 m radius and 4n height is initially full of water has an out let at bottom. The outlet is opened at some instant. The rate of flow through the outlet at any time t is $6\,h^{3/2}$, where h is height of water level above the out let at time t. Then fine the time it takes to empty the tank.

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Chapter 16 : Probability Distribution- VSA Qns

(2 - Marks)

1. Two cards are drawn from a pack of 52 cards.
   Prepare the probability distribution of the random variable defined as, number of black cards.
2. State with reasons whether the following represent the p.m.f of a random variable.

x :	0	1	2	3
p(x) :	0.1	0.2	-0.1	0.7

y	0	1	2
P(y)	0.1	0.2	0.5

3. Verify whether the following function is p.m.f. of continuous r.v.X.

   $f(x) = x/2 \quad -2 < x < 2$

   $= 0$, otherwise
4. Verify whether the following function can be regarded as the p . m. f for the given values of X

   $P(X = x) = \frac{x-2}{5}$ X = 1,2,3,4,5

   $= 0$, otherwise
5. Verify whether the following function can be regarded as the p.m.f. for the given values of X

   $P(X = x) = 1/5$ for x = 0,1,2,3,4

   $= 0$ otherwise
6. Determine k such that the following function is a p.m.f.

   $P(X = x) = kx$, x = 1,2,3,4,5

   $= 0$, otherwise
7. Determine k such that the following function is a p.m.f.

   $P(X = x) = k(.2^x/x!)$, x = 0,1,2,3

   $= 0$ otherwise
8. In a Pizza Hut , the following distribution is found for a daily demand of Pizzas. Find the expected daily demand.

No. of Pizzas :	5	6	7	8	9	10
Probability :	0.07	0.2	0.3	0.3	0.07	0.06

3 - Marks

1. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces.

2. Find the probability distribution of number of doublets in three throws of a pair of dice.

3. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some constant.

$$P(X = x) = \begin{cases} 0.1 & \text{if } x = 0, \\ kx & \text{if } x = 1 \text{ or } 2, \\ k(5 - x) & \text{if } x = 3 \text{ or } 4, \\ 0, & \text{otherwise.} \end{cases}$$

Then a) Find the value of k ,

b) What is the probability that you study atleast two hours ?

4. Find the probability distribution of

i) Number of heads in two tosses of a coin

ii) Number of tails in the simultaneous tosses of three coins

iii) Number of heads in four tosses of a coin.

5. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

6. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice. Find the probability distribution of number of tails.

7. A random variable X has the following probability distribution:

X :	0	1	2	3	4	5	6	7
P(X = x) :	0	k	2k	2k	3k	k^2	$2k^2$,	$7k^2 + k$

Determine ; i) k, ii) P(X < 3), iii) P (0 < X < 3).

8. The r.v X has the following probability distribution function as

$$P(x) = \begin{cases} k & \text{if } x = 0 \\ 2k, & \text{if } x = 1, \\ 3k, & \text{if } x = 2, \\ 0, & \text{otherwise.} \end{cases}$$

Then find i) k ii) P (X < 2) iii) P $(X \leq 2)$.

9. Given below is the probability distribution of a discrete r.v. X:

X :	1	2	3	4	5	6
P (X = x) :	k	0	2k	5k	k	3k

Find k and hence find $P(2 \leq X \leq 3)$

10. Two fair dice are rolled. X denotes the sum of numbers appearing on the uppermost faces of the dice. Find i) $P(X < 4)$ ii) $P(3 < X < 7)$.

11. It is known that a box of 8 batteries contains 3 defective pieces and a person randomly selects 2 batteries from this box. Find the probability distribution of the number of defective batteries.

12. The probability distribution of a r.v X is as follows:

X :	-1.5	-0.5	0.5	1.5	2.5
P (X = x) :	0.05	0.2	0.15	0.25	0.35

Construct c.d.f. F(x) of X.

13. In the following table c.d.f of r.v X is given

X :	–2	–1	0	1	2
F (X_i) :	0.2	0.5	0.65	0.9	1

Find p.m.f. of X. Also find $P[X \leq 0]$.

14. An urn contains 4 white and 6 red balls . 4 balls are drawn at random from the urn . Find the probability distribution of the number of white balls.

15. 4 bad oranges are mixed accidently with 16 good oranges. Find the probability distribution of the number of bad oranges in a draw of 2 oranges.

16. In a roadside joint, veg and non-veg samosas are served along with other things. Profit per samosa is 3 rupees for a veg samosa and 5 rupees for a non-veg samosa .The probability distribution for the demand of veg and non-veg samosa are as follows:

i)

Demand (veg) :	10	15	20	25	30
Probability :	0.3	0.2	0.3	0.15	0.05

ii)

Demand (non-veg) :	5	7	9	11
Probability :	0.4	0.3	0.15	0.15.

Which type of samosa brings in more expected profit?

17. Show that the function f(x) defined by , f (x) = 1/7 for $1 \leq x < 8$,

= 0 , otherwise,

is a probability density function for a random variable. Hence find $P(3 < X < 10)$.

18. Find the c.d.f F (x) associated with the following pdf. f (x)

$f(x) = 12x^2 (1 - x),\ 0 < x < 1,$

$= 0$, otherwise. Also, find $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$ by using c.d.f. and sketch the graph of F(x).

19. A die is tossed twice. Getting a number greater than 4 is considered a "success". Find the mean and variance of the probability distribution of the number of successes.

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Chapter 17 : Binomail Distribution -VSA

(2 - Marks)

1. Write the Binomial distribution if mean for the distribution is 3 and the standard deviation is 3/2.
2. If X ~ B (6, p) and 2 P (X = 3) = P (X = 2) then find the value of p.
3. If a die is thrown twice, then find the probability of occurence of 4 atleast once.
4. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn one-by-one with replacement then find the variance of the number of yellow balls.
5. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting point 0, 1, 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent. Find the probability of India getting at least 7 points .
6. 100 identical coins, each with probability p, of showing up heads are tossed once. If $0 < p < l$ and the possibility of heads showing on 50 coins is equal to that of heads showing on 51 coins, then find the value of p .

3 Marks

1. If the mean and the variance of the binomial varite X are 2 and 1 respectively, then find the probability that X takes a value greater than one.
2. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02, 03,....99 with replacement. An event E occurs if and only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times.
3. A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability at the end of eleven steps he is one step away from the starting point.
4. Find the minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96.
5. A lot of 100 pens contains 10 defective pens. 5 pens are selected at random from the lot and sent to the retail store. What is the probability that the store will receive at least one defective pen?
6. In a production process, producing bulbs, the probability of getting a defective bulb remains constant and it is 0.3. If we select a sample of 10 bulbs, what is the probability of getting 3 defective bulbs?

7. In a bag containing 100 eggs, 10 eggs are rotten. Find the probability that out of a sample of 5 eggs none are rotten, if the sampling is with replacement.
8. A fair coin is tossed six times. What is the probability of obtaining four or more heads?
9. Each of two persons A and B toss 3 fair coins. Find the probability that both get the same number of heads.
10. The probability of India winning a test match against England is 2/3. Assuming independence from match to match, find the probability that in a 7 match series India's third win occurs at the 5^{th} match..
11. Let x denotes the number of times heads occur in n tosses of a fair coin, if P(X = 4), P (X = 5) and P(X = 6) are in A.P., find n.

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