Maharastra Board | Class 12 Physics

Previous 3-Year Questions with Detailed Solutions (2022-2024)

93 Questions

of Physics with Detailed Solution

CAREERS360

Maharashtra Board Class 12 Physics Solutions - 2022

SECTION- A

Q.1 Select and write the correct answer for the following multiple type of questions :

(i) The first law of thermodynamics is concerned with the conservation of

(a) momentum
(b) energy
(c) temperature
(d) mass

Solution:

The first law of thermodynamics, which focuses on the conservation of energy, states that the total energy in an isolated system remains constant over time. It asserts that energy can be transferred between objects or converted into different forms, but it cannot be created or destroyed.

Hence,the correct option is (b)

(ii) The average value of alternating current over a full cycle is always ________.
$[\mathrm{I}_0 =$ **Peak value of current** $]$
(a) zero
(b) $\frac{\mathrm{I}_0}{2}$
(c) $\frac{\mathrm{I}_0}{\sqrt{2}}$
(d) $2\mathrm{I}_0$

Solution:

In the case of alternating current (AC), the current changes direction periodically, completing a cycle from positive to negative. Over one complete cycle, the average value of a sinusoidal waveform, such as that of alternating current, is zero. This is because the positive and negative halves of the cycle exactly cancel each other out.

Hence,the correct option is (a)

(iii) The angle at which maximum torque is exerted by the external uniform electric field on the electric dipole is
(a) 0°

(b) 30°
(c) 45°
(d) 90°

Solution:

The torque τ exerted on an electric dipole in a uniform electric field is given by the equation:

$$\tau = pE \sin\theta$$

where:
- p is the magnitude of the dipole moment,
- E is the magnitude of the electric field,
- θ is the angle between the dipole moment and the electric field direction.

The torque is maximum when $\sin\theta$ is maximum, which occurs at $\theta = 90^{\circ}$.

Hence,the correct option is (d)

(iv) The property of light which does not change, when it travels from one medium to another is
(a) velocity
(b) wavelength
(c) frequency
(d) amplitude

Solution:

When light travels from one medium to another, its frequency remains unchanged. The velocity and wavelength of light can vary depending on the medium's refractive index, but the frequency, which is determined by the source of the light, does not depend on the medium through which the light is traveling.

Hence,the correct option is (c).

(v) The root mean square speed of the molecules of a gas is proportional to   .
[$\mathrm{T} =$ Absolute temperature of gas]
(a) $\sqrt{\mathrm{T}}$
(b) $\frac{1}{\sqrt{\mathrm{T}}}$
(c) T
(d) $\frac{1}{\mathrm{T}}$

Solution:

The root mean square speed of the molecules of a gas is given by the equation derived from the kinetic theory of gases:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

where:
- v_{rms} is the root mean square speed,
- k is the Boltzmann constant,

- T is the absolute temperature of the gas,
- m is the mass of a gas molecule.

This equation shows that v_{rms} is directly proportional to the square root of the absolute temperature T.

Hence,the correct option is (a)

(**vi) The unit Wbm^{-2} is equal to**
(a) henry
(b) watt
(c) dyne
(d) tesla

Solution:

The unit "Wb m2" is derived from "Weber per square meter," which describes the magnetic flux density. This unit is equivalent to Tesla. A Tesla (T) is the SI derived unit of magnetic flux density, representing one Weber per square meter.

Hence,the correct option is: (d)

(vii) When the bob performs a vertical circular motuon and ue string rotates in a vertical plane, the difference in the tension in the string at horizontal position and uppermost position is .
(a) mg
(b) $2\mathrm{mg}$
(c) $3\mathrm{mg}$
(d) $6\mathrm{mg}$

Solution:
At the uppermost positions differ significantly due to the influence of gravity and the need for centripetal force.

At the uppermost position of the circle, the tension T_{top} in the string must provide the necessary centripetal force to keep the bob moving in a circle, in addition to balancing the gravitational force pulling the bob downward. The equation for the tension at the top is:

$$T_{\text{top}} + mg = \frac{mv^2}{r}$$

Simplifying, it becomes:

$$T_{\text{top}} = \frac{mv^2}{r} - mg$$

At the horizontal position , the only force acting towards the center of the circle (the direction of centripetal force) is the tension in the string. Thus, the tension $T_{\text{horizontal}}$ is entirely providing the centripetal force, given by:

$$T_{\text{horizontal}} = \frac{mv^2}{r}$$

Given these expressions, the difference in the tension between the horizontal position and the top position can be calculated as:

$$\Delta T = T_{\text{horizontal}} - T_{\text{top}} = \left(\frac{mv^2}{r}\right) - \left(\frac{mv^2}{r} - mg\right) = mg$$

Hence,the correct option is (a)

(viii) A liquid rises in glass capillary tube upto a height of 2.5 cm at room temperature. If another glass capillary tube having radius half that of the earlier tube is immersed in the same liquid, the rise of liquid in it will be

(a) $1.25\ \mathrm{cm}$.

(b) $5\ \mathrm{cm}$

(c) $2.5\ \mathrm{cm}$

(d) $10\ \mathrm{cm}$

Solution:

The rise of a liquid in a capillary tube due to capillary action is given by the equation:
$h = \frac{2\gamma\cos(\theta)}{\rho g r}$
Given that the liquid rises to 2.5 cm in a capillary tube, and another tube with half the radius of the first is used, the height of the liquid rise can be determined as follows:

The height of the liquid rise is inversely proportional to the radius of the tube. Therefore, if the radius is halved, the height of the rise will double. So, for a tube with half the radius, the liquid will rise:

$$h = 2 \times 2.5\,\mathrm{cm} = 5\,\mathrm{cm}$$

Hence,the correct option is(b)

(ix) In young's double slit experiment the two coherent sources have different amplitudes. If the ratio of maximum intensity to minimum intensity is $16:1$, then the ratio of amplitudes of the two source will be

(a) $4:1$

(b) $5:3$

(c) $1:4$

(d) $1:16$

Solution:

In Young's double slit experiment, the intensities for constructive and destructive interference depend on the amplitudes of the two coherent sources.

Let A_1 and A_2 be the amplitudes of the two sources. The maximum intensity $I_{\max}$ and the minimum intensity can be expressed as follows:

$I_{\max}=(A_1+A_2)^2$

$I_{\min}=(A_1-A_2)^2$

Given that the ratio of maximum to minimum intensity is 16:1:

$\frac{I_{\max}}{I_{\min}}=\frac{(A_1+A_2)^2}{(A_1-A_2)^2}=16$

Taking the square root of both sides:

$\frac{A_1+A_2}{A_1-A_2}=4$

This equation forms a system with another equation $A_1+A_2=4(A_1-A_2)$, simplifying to:

$A_1+A_2=4A_1-4A_2$

$5A_2=3A_1$

$\frac{A_1}{A_2}=\frac{5}{3}$

Hence,the correct option is: (b)

(x) The equation of a simple harmonic progressive wave travelling on a string is $y=8\sin(0.02x-4t)\text{cm}$**. The speed of the wave is**
(a) $10\ \text{cm/s}$
(b) $20\ \text{cm/s}$
(c) $100\ \text{cm/s}$
(d) $200\ \text{cm/s}$

Solution:

The equation of a simple harmonic progressive wave given is:

$$y=8\sin(0.02x-4t)\ \text{cm}$$

Here, the wave function is in the form

$$y=A\sin(kx-\omega t)$$

A is the amplitude,
k is the wave number,
ω is the angular frequency,
t is time,
x is position.

In this equation:

$$k = 0.02\,\mathrm{cm}^{-1},$$
$$\omega = 4\,\mathrm{s}^{-1}.$$

The speed v of the wave is given by the relation:

$$v = \frac{\omega}{k}$$

Substituting the given values:

$$v = \frac{4\,\mathrm{s}^{-1}}{0.02\,\mathrm{cm}^{-1}} = 200\,\mathrm{cm/s}$$

Hence,the correct option is(d)

Q2. Answer the following questions :

(i) Define potential gradient of the potentiometer wire.

Solution:

The fall of potential per unit length along potentiometer wire is called the potential gradient. If L is length of wire AB and V is the potential difference across it then Potential gradient k = V/L The S.I. unit of potential gradient is volt/metre

(ii) State the formula for critical velocity in terms of Reynold's number for a flow of a fluid.

Solution:

The formula for critical velocity in terms of Reynolds number for fluid flow can be expressed by rearranging the definition of the Reynolds number. The Reynolds number (Re) is given by:

$$\mathrm{Re} = \frac{\rho v_c D}{\mu}$$

Where:
- ρ is the density of the fluid,
- v_c is the critical velocity,
- D is the characteristic length (e.g., diameter of a pipe),
- μ is the dynamic viscosity of the fluid.

To find the critical velocity v_c in terms of Reynolds number, rearrange the equation to solve for v_c:

$$v_c = \frac{\mathrm{Re}\mu}{\rho D}$$

Here, the critical Reynolds number value (denoted typically as Re_c) that indicates the transition from laminar to turbulent flow is often used as a threshold. This value depends on the specific conditions and geometry of the flow but is typically around 2000 for flow in pipes.

(iii) Is it always necessary to use red light to get photoelectric effect?

Solution:

The photoelectric effect does not occur when the red light strikes the metallic surface because the frequency of red light is lower than the threshold frequency of the metal. so, it is not necessary to use red light in photoelectric effect

(iv) Write the Boolean expression for Exclusive - OR (X $\mathrm{OR})$ gate.

Solution:

The Boolean expression for an Exclusive-OR (XOR) gate, which outputs true only when the inputs differ, can be written in several equivalent forms using basic logic operations (AND, OR, NOT). One common expression for the XOR operation of two inputs A and B is:

$$A \oplus B=(A \wedge \neg B) \vee(\neg A \wedge B)$$

This expression states that $A \oplus B$ (XOR) is true when either A is true and B is false, or A is false and B is true.

(v) Write the differential equation for angular S.H.M.

Solution:

The differential equation for angular simple harmonic motion (S.H.M.) is similar to that of linear S.H.M. and describes the motion of objects like pendulums. It is expressed as:

$$\frac{d^2 \theta}{d t^2}=-\omega^2 \theta$$

Here, θ is the angular displacement, ω is the angular frequency, and the equation highlights that the angular acceleration is proportional to the negative of the angular displacement, characterizing the oscillatory nature of S.H.M.

(vi) What is the mathematical formula for third postulate of Bohr's atomic model?

Solution:

The third postulate of Bohr's atomic model relates to the quantization of the angular momentum of an electron in orbit around the nucleus. According to this postulate, the angular momentum of the electron in a stable orbit is quantized and can only take on certain discrete values. The mathematical formula for this postulate is:

$$L=n \frac{h}{2 \pi}$$

Here:
- L is the angular momentum of the electron,
- n is the principal quantum number (a positive integer),

- h is Planck's constant.

This formula states that the angular momentum of an electron in an orbit is an integer multiple of $\frac{h}{2\pi}$, also known as the reduced Planck constant ($\hbar$). This quantization condition explains the stability of orbits and the emission or absorption of energy through quantized photons when electrons transition between these orbits.

(vii) Two inductor coils with inductance $10\mathrm{mH}$ and $20\mathrm{mH}$ are connected in series. What is the resultant inductance of the combination of the two coils?

Solution:

When two inductor coils are connected in series, their resultant inductance is simply the sum of their individual inductances. This is because the magnetic field created by each coil adds together, enhancing the overall inductance of the circuit.

Given the inductances of the two coils are 10 mH and 20 mH, the resultant inductance L_{total} of the combination is calculated as follows:

$$L_{\text{total}} = L_1 + L_2 = 10\text{ mH} + 20\text{ mH} = 30\text{ mH}$$

Therefore, the resultant inductance of the two coils connected in series is 30 mH.

(viii) Calculate the moment of inertia of a uniform disc of mass 10 kg and radius 60 cm about an axis perpendicular to its length and passing through its centre.

Solution:

To calculate the moment of inertia of a uniform disc of mass m and radius r about an axis perpendicular to its length and passing through its center, we use the formula:

$$I = \frac{1}{2}mr^2$$

Given:

$$m = 10\,\mathrm{kg}$$
$$r = 60\,\mathrm{cm} = 0.6\,\mathrm{m}\text{ (since we need the radius in meters for SI units)}$$

Plugging these values into the formula, we get:

$$I = \frac{1}{2} \times 10\,\mathrm{kg} \times (0.6\,\mathrm{m})^2$$

$$I = \frac{1}{2} \times 10 \times 0.36\,\mathrm{m}^2$$

$$I = 1.8\,\mathrm{kg} \cdot \mathrm{m}^2$$

Thus, the moment of inertia of the disc is $1.8\,\mathrm{kg}\cdot\mathrm{m}^2$.

SECTION- B

Q. 3. Define moment of inertia of a rotating rigid body. State it unit and dimensions.

Solution:

Moment of inertia of a rigid body about an axis of rotation is defined as the sum of product of each point mass and square of its perpenducular distance from the axis of rotation. The S.I. unit of moment of inertia is kg. m 2 . The dimension of moment of inertia is [M 1 L 2 T 0]

Q. 4. What are polar dielectrics and non polar dielectrics?

Solution:

NON POLAR DIELECTRICS:

Are made of non-polar molecules.
The center of mass of positive charges coincides with the center of mass of the negative charges, in the molecule.
In its normal state (i.e., in the absence of any external force, esp electric field), each molecule has zero dipoles and has a symmetric shape.

POLAR DIELECTRICS:

Are made of polar molecules.
The center of mass of positive charges does not coincide with the center of mass of the negative charges, in the molecule.
In its normal state (i.e., in the absence of any external force, esp electric field), each molecule has some intrinsic permanent dipole and has an asymmetric shape.

Q. 5. What is a thermodynamic process? Give any two types of it.

Solution:

A Thermodynamic process is a process in which the thermodynamic state of a system is changed. A change in a system is defined by a passage from an initial to a final state of thermodynamic equilibrium.
Types of Thermodynamic Processes

The state of a system can be changed by different processes. In Thermodynamics, types of processes include: Isobaric process in which the pressure (P) is kept constant (ΔP =0). Isochoric process in which the volume (V) is kept constant (ΔV =0).

Q. 6. Derive an expression for the radius of the n^{th} Bohr orbit of the electron in hydrogen atom.

Solution:

To derive the expression for the radius of the n-th Bohr orbit of the electron in a hydrogen atom, we start with Bohr's model of the hydrogen atom. Bohr proposed that electrons move in circular orbits around the nucleus under the influence of the Coulomb force of attraction between the positively charged nucleus and the negatively charged electron.

Key Assumptions and Equations

1. Centripetal Force and Coulomb Force:
The centripetal force required to keep the electron in a circular orbit is provided by the Coulomb force of attraction between the electron and the proton. This gives us the equation:

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

where:
- m is the mass of the electron
- v is the velocity of the electron
- r is the radius of the orbit
- k is Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0}$)
- e is the charge of the electron

2.Quantization of Angular Momentum:
Bohr proposed that the angular momentum of the electron is quantized and given by:

$$mvr = n\hbar$$

where:
- $\hbar$ is the reduced Planck's constant ($\hbar = \frac{h}{2\pi}$)
- n is a positive integer (the principal quantum number)

Derivation Steps

Solve for v from the angular momentum quantization condition:

$$v = \frac{n\hbar}{mr}$$

Substitute v into the centripetal force equation

$$\frac{m\left(\frac{n\hbar}{mr}\right)^2}{r} = \frac{ke^2}{r^2}$$

Simplify this equation:

$$\frac{mn^2\hbar^2}{m^2r^3} = \frac{ke^2}{r^2}$$

$$\frac{n^2\hbar^2}{mr^3} = \frac{ke^2}{r^2}$$

Solve for r:

$$\frac{n^2\hbar^2}{mr^3} = \frac{ke^2}{r^2}$$

Multiply both sides by r^3 and simplify:

$$n^2\hbar^2 = ke^2mr$$

$$r = \frac{n^2\hbar^2}{ke^2m}$$

Final Expression

The radius r_n of the n-th Bohr orbit is given by:

$$r_n = \frac{n^2\hbar^2}{ke^2m}$$

Substituting Constants

Using the known values of constants:
- $\hbar = \frac{h}{2\pi}$
- $k = \frac{1}{4\pi\epsilon_0}$
- e is the elementary charge
- m is the mass of the electron

The expression becomes:

$$r_n = \frac{n^2\hbar^2}{ke^2m} = \frac{n^2(\frac{h}{2\pi})^2 4\pi\epsilon_0}{e^2m} = \frac{n^2h^2\epsilon_0}{\pi e^2m}$$

Thus, the radius of the n-th Bohr orbit in a hydrogen atom is:

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

Q. 7. What are harmonics and overtones (Two points)?

Solution:

Harmonic frequencies are whole number multiples of the fundamental frequency or the lowest frequency of vibration. Consider a vibrating string. The modes of vibration are all multiples of the fundamental and are related to the string length and wave velocity. Higher frequencies are found via the relationship
fn = nf1

Wavelength = 2L/n where L is the string length.

An overtone is a name given to any resonant frequency above the fundamental frequency or fundamental tone. The list of successive overtones for an object is called the overtone series. The first overtone as well as all subsequent overtones in the series may or may not be an integer multiple of the fundamental. Sometimes the relationship is that simple, and other times it is more complex, depending o the properties and geometry of the vibrating object.

Q. 8. Distinguish between potentiometer and voltmeter.

Solution:

Potentiometer

1. Measure emf of cell very accurately

2. Does not draw any current from known emf source while measuring current

4. While measuring emf resistance of potentiometer becomes infinite.

4. sensitivity is high

5. Based on null deflection method

Voltmeter

1 Measure emf of cell approximately

2 Does not draw any current from known emf source while measuring current

3 While measuring emf resistance of potentiometer becomes very high but measurable.

4 sensitivity is low

5. Based on deflection method

Q. 9. What are mechanical equilibrium and thermal equilibrium?

Solution:

Mechanical equilibrium: Consider a gas container with piston. When some mass is placed on the piston, it will move downward due to downward gravitational force and after certain humps and jumps the piston will come to rest at a new position. When the downward gravitational force given by the piston is balanced by the upward force exerted by the gas, the system is said to be in mechanical equilibrium. A system is said to be in mechanical equilibrium if no unbalanced force acts on the thermodynamic system or on the surrounding by thermodynamic system.

Thermal equilibrium: When a hot cup of coffee is kept in the room, heat flows from coffee to the surrounding air. After sometime the coffee reaches the same temperature as the surrounding air and there will be no heat flow from coffee to air or air to coffee. It implies that the coffee and surrounding air are in thermal equilibrium with each other. Two systems are said to be in thermal equilibrium with each other if they arc at the same temperature, which Mechanical equilibrium will not change with time.

Q. 10. An electron in an atom is revolving round the nucleus in a circular orbit of radius 5.3×10^{-11} m with a speed of 3×10^{6} m/s. Find the angular momentum of electron.

Solution:

To find the angular momentum of an electron revolving around the nucleus in a circular orbit, we use the formula for angular momentum L in terms of mass m, velocity v, and radius r:

$$L = mvr$$

Given the data:
- Radius of the orbit, $r = 5.3 \times 10^{-11}$ m
- Speed of the electron, $v = 3 \times 10^{6}$ m/s
- Mass of the electron, $m = 9.109 \times 10^{-31}$ kg (known constant)

Substitute these values into the formula:

$$L = (9.109 \times 10^{-31}\,\mathrm{kg}) \times (3 \times 10^{6}\,\mathrm{m/s}) \times (5.3 \times 10^{-11}\,\mathrm{m})$$

First, calculate the product of the constants:

$$L = 9.109 \times 10^{-31} \times 3 \times 10^{6} \times 5.3 \times 10^{-11}$$

$$L = 9.109 \times 3 \times 5.3 \times 10^{-31+6-11}$$

$$L = 9.109 \times 3 \times 5.3 \times 10^{-36}$$

Calculate the numerical part:

$$L = 9.109 \times 3 = 27.327$$

$$27.327 \times 5.3 = 144.8331$$

$$L = 144.8331 \times 10^{-36}$$

$$L = 1.448331 \times 10^{-34}\,\mathrm{kg \cdot m^2/s}$$

So, the angular momentum of the electron is:

$$L \approx 1.45 \times 10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$$

Thus, the angular momentum of the electron is $1.45 \times 10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$.

Q. 11. Plane wavefront of light of wavelength 6000 Angestrom is incident on two slits on a screen perpendicular to the direction of light rays. If the total separation of 10 bright fringes on a screen $2\ \mathrm{m}$ away is $2\ \mathrm{cm}$, find the distance between the slits.

Solution:

To find the distance between the slits, we need to use the principles of the double-slit experiment in wave optics.

Given:
- Wavelength of light, $\lambda = 6000\,\text{Å} = 6000 \times 10^{-10}\,\mathrm{m} = 6 \times 10^{-7}\,\mathrm{m}$
- Distance between the screen and the slits, $D = 2\,\mathrm{m}$
- Total separation of 10 bright fringes, $L = 2\,\mathrm{cm} = 0.02\,\mathrm{m}$

In a double-slit experiment, the fringe separation (or fringe width) β is given by:

$$\beta = \frac{\lambda D}{d}$$

where d is the distance between the slits.

Given that the total separation of 10 bright fringes is $0.02\,\mathrm{m}$, the separation between two consecutive bright fringes (fringe width) β is:

$$10\beta = 0.02\,\mathrm{m}$$

$$\beta = \frac{0.02\,\mathrm{m}}{10} = 0.002\,\mathrm{m}$$

Now, substitute β into the fringe width formula to find d:

$$\beta = \frac{\lambda D}{d}$$

$$0.002 = \frac{6 \times 10^{-7} \times 2}{d}$$

Solve for d:

$$d = \frac{6 \times 10^{-7} \times 2}{0.002}$$

$$d = \frac{1.2 \times 10^{-6}}{0.002}$$

$$d = 1.2 \times 10^{-6} \times 500$$

$$d = 6 \times 10^{-4}\,\mathrm{m}$$

Thus, the distance between the slits is:

$$d = 6 \times 10^{-4}\,\mathrm{m} = 0.6\,\mathrm{mm}$$

So, the distance between the slits is $0.6\,\mathrm{mm}$.

Q. 12. Eight droplets of water each of radius $0.2\ \mathrm{mm}$ coalesce into a single drop. Find the decrease in the surface area.

Solution:

Volume of a single small droplet:

$$V_{\text{small}} = \frac{4}{3}\pi r^3$$

Given $r = 0.2\,\mathrm{mm}$:

$$V_{\text{small}} = \frac{4}{3}\pi(0.2)^3$$

$$V_{\text{small}} = \frac{4}{3}\pi(0.008)\,\mathrm{mm}^3$$

$$V_{\text{small}} = \frac{32}{3}\pi \times 10^{-3}\,\mathrm{mm}^3$$

Total volume of eight small droplets:

$$V_{\text{total}} = 8 \times V_{\text{small}}$$

$$V_{\text{total}} = 8 \times \frac{32}{3}\pi \times 10^{-3}\,\text{mm}^3$$

$$V_{\text{total}} = \frac{256}{3}\pi \times 10^{-3}\,\text{mm}^3$$

Radius of the resulting larger drop

$$V_{\text{large}} = V_{\text{total}} = \frac{4}{3}\pi R^3$$

$$\frac{4}{3}\pi R^3 = \frac{256}{3}\pi \times 10^{-3}$$

$$R^3 = 64 \times 10^{-3}$$

$$R = (64 \times 10^{-3})^{1/3}$$

$$R = 0.4\,\text{mm}$$

Surface area of a single small droplet

$$A_{\text{small}} = 4\pi r^2$$

$$A_{\text{small}} = 4\pi(0.2)^2$$

$$A_{\text{small}} = 4\pi(0.04)$$

$$A_{\text{small}} = 0.16\pi\,\text{mm}^2$$

Total surface area of eight small droplets

$$A_{\text{total small}} = 8 \times A_{\text{small}}$$

$$A_{\text{total small}} = 8 \times 0.16\pi\,\text{mm}^2$$

$$A_{\text{total small}} = 1.28\pi\,\text{mm}^2$$

Surface area of the larger drop

$$A_{\text{large}} = 4\pi R^2$$

$$A_{\text{large}} = 4\pi(0.4)^2$$

$$A_{\text{large}} = 4\pi(0.16)$$

$$A_{\text{large}} = 0.64\pi \, \text{mm}^2$$

Decrease in surface area:

$$\Delta A = A_{\text{total small}} - A_{\text{large}}$$

$$\Delta A = 1.28\pi - 0.64\pi$$

$$\Delta A = 0.64\pi \, \text{mm}^2$$

Q. 13. A 0.1H inductor, a $25 \times 10^{-6}\ \text{F}$ capacitor and a 15Ω resistor are connected in series to a $120\ \text{V}, 50\ \text{Hz}$ AC source. Calculate the resonant frequency.

Solution:

To calculate the resonant frequency of an RLC circuit, we use the formula for the resonant frequency f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Given:
- Inductance, $L = 0.1\,\text{H}$
- Capacitance, $C = 25 \times 10^{-6}\,\text{F}$

Substitute these values into the resonant frequency formula:

$$f_0 = \frac{1}{2\pi\sqrt{0.1 \times 25 \times 10^{-6}}}$$

First, calculate the product LC:

$$LC = 0.1 \times 25 \times 10^{-6}$$

$$LC = 2.5 \times 10^{-6}$$

Now, find the square root of LC:

$$\sqrt{LC} = \sqrt{2.5 \times 10^{-6}}$$

$$\sqrt{2.5 \times 10^{-6}} = \sqrt{2.5} \times 10^{-3}$$

$$\sqrt{2.5} \approx 1.58$$

Therefore,

$$\sqrt{2.5 \times 10^{-6}} = 1.58 \times 10^{-3}$$

Now, calculate the resonant frequency:

$$f_0 = \frac{1}{2\pi \times 1.58 \times 10^{-3}}$$

$$f_0 = \frac{1}{2 \times 3.1416 \times 1.58 \times 10^{-3}}$$

$$f_0 = \frac{1}{9.948 \times 10^{-3}}$$

$$f_0 \approx 100.52\,\text{Hz}$$

Thus, the resonant frequency is approximately $100.52\,\text{Hz}$.

Q. 14. The difference between the two molar specific heats of a gas is $9000\ \text{J/kgK}$. If the ratio of the two specific heats is 1.5 , calculate the two molar specific heats.

Solution:

To calculate the two molar specific heats, C_p and C_v, given their difference and ratio, we use the following relationships:

1. $C_p - C_v = 9000\,\text{J/kg K}$
2. $\frac{C_p}{C_v} = 1.5$

Let's denote C_v as C. Then C_p can be written as $1.5C$.

From the first relationship:

$$C_p - C_v = 9000$$

Substitute C_p with $1.5C$:

$$1.5C - C = 9000$$

$$0.5C = 9000$$

$$C = \frac{9000}{0.5}$$

$$C = 18000 \, \mathrm{J/kg \ K}$$

So, $C_v = 18000 \, \mathrm{J/kg \ K}$.

Now, calculate C_p:

$$C_p = 1.5C$$

$$C_p = 1.5 \times 18000$$

$$C_p = 27000 \, \mathrm{J/kg \ K}$$

Therefore, the two molar specific heats are:

$$C_p = 27000 \, \mathrm{J/kg \ K}$$

$$C_v = 18000 \, \mathrm{J/kg \ K}$$

SECTION- C

Q. 15. With the help of a neat diagram, explain the reflection of light on a plane reflecting surface.

Solution:

Reflection of Light on a Plane Reflecting Surface

Reflection of light occurs when a light ray strikes a smooth, flat surface (like a mirror) and bounces back. This process follows certain laws known as the laws of reflection. Let's discuss these laws with the help of a diagram and an explanation.

Laws of Reflection

e Incident Ray, the Reflected Ray, and the Normal
- The incident ray,he reflected ray, and the normal to the surface at the point of incidence all lie in the same plane.

Angle of Incidence and Angle of Reflection:
- The angle of incidence (the angle between the incident ray and the normal) is equal to the angle of reflection (the angle between the reflected ray and the normal).

Explanation

1. Incident Ray:
- The ray of light that strikes the reflecting surface is called the incident ray. In the diagram, it is the ray coming from the top left towards the surface.

2.Reflected Ray:
- The ray of light that bounces off the reflecting surface is called the reflected ray. In the diagram, it is the ray moving away from the surface to the top right.

3. Normal
- The normal is an imaginary line perpendicular to the reflecting surface at the point of incidence (where the incident ray strikes the surface).

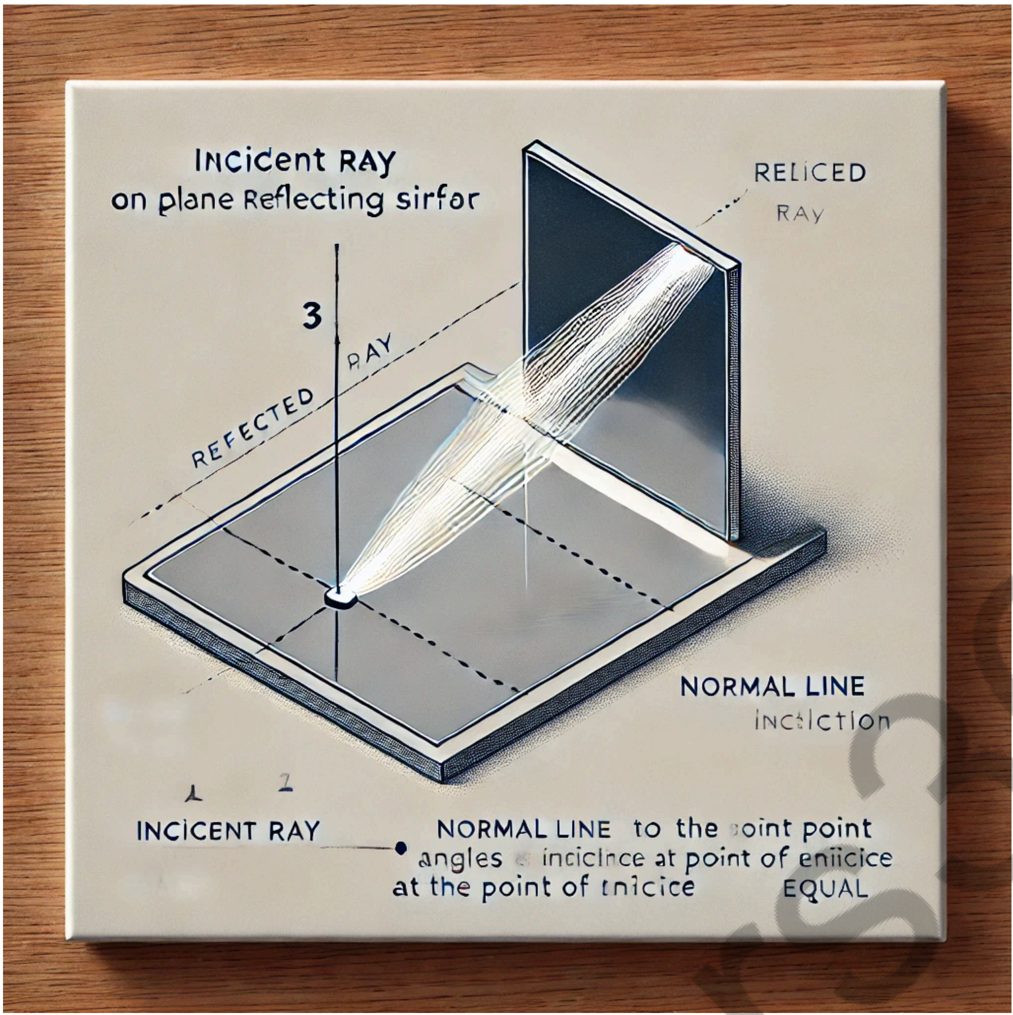
4. Angle of Incidence (i):
- The angle between the incident ray and the normal. It is denoted by i in the diagram.

5. Angle of Reflection (r):
- The angle between the reflected ray and the normal. It is denoted by r in the diagram.

According to the laws of reflection:

$$i = r$$

This means that the angle at which the light ray strikes the surface will be equal to the angle at which it reflects off the surface.



Q. 16. What is magnetization, magnetic intensity and magnetic susceptibility?

Solution:

Magnetic susceptibility is a dimensionless proportionality constant that indicates the degree of magnetization of a material in response to an applied magnetic field.

The Magnetic moment of a magnet undergoes a change when it is placed in a magnetic field. This change that is, the magnetic moment change per unit volume is the Intensity of Magnetisation.

Magnetization, also termed magnetic polarization, is a vector quantity that measures the density of permanent or induced dipole moment in a given magnetic material.

Q. 17. Prove that the frequency of beats is equal to the difference between the frequencies of the two sound notes giving rise to beats.

Solution:

To prove that the frequency of beats is equal to the difference between the frequencies of two sound notes giving rise to beats, let's consider two sound waves with slightly different frequencies.

Let the two sound waves be described by the equations:

$$y_1(t) = A\sin(2\pi f_1 t)$$

$$y_2(t) = A\sin(2\pi f_2 t)$$

Here, f_1 and f_2 are the frequencies of the two waves, and A is the amplitude (assumed to be the same for simplicity).

When these two waves are superposed, the resulting wave is the sum of $y_1(t)$ and $y_2(t)$:

$$y(t) = y_1(t) + y_2(t)$$

$$y(t) = A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t)$$

Using the trigonometric identity for the sum of sine functions:

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

We get:

$$y(t) = 2A\sin\left(\frac{2\pi f_1 t + 2\pi f_2 t}{2}\right)\cos\left(\frac{2\pi f_1 t - 2\pi f_2 t}{2}\right)$$

$$y(t) = 2A\sin\left(\pi(f_1 + f_2)t\right)\cos\left(\pi(f_1 - f_2)t\right)$$

This equation represents a wave whose amplitude is modulated by a cosine function. The term $\cos\left(\pi(f_1 - f_2)t\right)$ represents the modulation of the amplitude of the combined wave. The frequency of this cosine term is $\frac{1}{2} \times (f_1 - f_2)$.

The beat frequency f_{beat} is the frequency at which the amplitude of the resulting wave oscillates, which is the absolute value of the difference between the frequencies of the two original waves:

$$f_{\text{beat}} = |f_1 - f_2|$$

Thus, we have proved that the frequency of beats is equal to the difference between the frequencies of the two sound notes giving rise to the beats.

Q. 18. Define :
(a) Inductive reactance
(b) Capacitive reactance
(c) Impedance

Solution:

Inductive reactance is the opposition offered by the inductor in an AC circuit to the flow of AC current.

It is represented by (XL) and measured in ohms (Ω). Inductive reactance is mostly low for lower frequencies and high for higher frequencies. It is, however, negligible for steady DC current.

The inductive reactance formula is given as follows:

Inductive Reactance, XL = 2πfL

The definition of capacitive reactance states that it is the opposition offered by a capacitor to the flow of AC current in the AC circuit. A capacitor opposes the changes in the potential difference or the voltage across its plates. Capacitive reactance is said to be inversely proportional to the capacitance and the signal frequency. It is normally represented by (Xc) and measured in the SI unit of ohm (Ω).

The capacitive reactance formula is given as follows:

Capacitive reactance, Xc = 1/2fC

Impedance, represented by the symbol Z, is a measure of the opposition to electrical flow. It is measured in ohms. For DC systems, impedance and resistance are the same, defined as the voltage across an element divided by the current (R = V/I).

Q. 19. Derive an expression for the kinetic energy of a body rotating with a uniform angular speed.

Solution:

To derive the expression for the kinetic energy of a body rotating with a uniform angular speed, we need to consider the rotational motion of the body. The kinetic energy in rotational motion is analogous to that in linear motion but involves rotational variables such as angular speed and moment of inertia.

The rotational kinetic energy (K) of a rotating body is given by the formula:

$$K = \frac{1}{2} I \omega^2$$

where:
- I is the moment of inertia of the body about the axis of rotation
- ω is the angular speed of the body

Derivation

The moment of inertia I is a measure of how the mass of the body is distributed with respect to the axis of rotation. For a rigid body, it is given by:

$$I = \sum_i m_i r_i^2$$

where m_i is the mass of the i-th particle of the body, and r_i is the distance of the i-th particle from the axis of rotation.

Each particle of the rotating body has a linear velocity v_i given by:

$$v_i = \omega r_i$$

where ω is the angular speed and r_i is the radius of rotation for the i-th particle.

The kinetic energy (K_i) of each particle is:

$$K_i = \frac{1}{2} m_i v_i^2$$

Substituting $v_i = \omega r_i$:

$$K_i = \frac{1}{2} m_i (\omega r_i)^2$$

$$K_i = \frac{1}{2} m_i \omega^2 r_i^2$$

The total kinetic energy K of the rotating body is the sum of the kinetic energies of all the individual particles:

$$K = \sum_i \frac{1}{2} m_i \omega^2 r_i^2$$

Since ω is constant for all particles in a rigid body, we can factor it out of the summation:

$$K = \frac{1}{2} \omega^2 \sum_i m_i r_i^2$$

Recognizing that the sum $\sum_i m_i r_i^2$ is the definition of the moment of inertia I:

$$K = \frac{1}{2} \omega^2 I$$

Q. 20. Derive an expression for emf (e) generated in a conductor of length (l) moving in uniform magnetic field (B) with uniform velocity (v) along x-axis.

Solution:

The electromotive force (EMF) induced in a conductor moving in a magnetic field can be derived using Faraday's law of electromagnetic induction. Faraday's law states that the induced EMF (8) is equal to the negative rate of change of magnetic flux () through the loop formed by the conductor.

The formula for Faraday's law is given by:

ε= do dt

For a straight conductor of length I moving at a velocity v perpendicular to a uniform magnetic field B, the magnetic flux (4) is given by:

Φ = B. A

where A is the cross-sectional area perpendicular to the magnetic field.

Since the conductor is moving along the x-axis, let's assume the cross- sectional area is A and the magnetic field is along the z-axis. Then:

A=1w

where w is the width of the conductor.

Substitute this into the expression for magnetic flux:

Φ = B · (ι· ω)

Now, differentiate with respect to time:

do dt = B. d(low) dt

Since the width (w) is constant, the derivative of Ix w with respect to time is just the velocity (v) of the conductor:

dt = Bυ

Finally, substitute this into Faraday's law:

do ε = - = -Boυ

So, the expression for the EMF (€) generated in a conductor of length I moving with velocity v in a uniform magnetic field B is ε = - B. v. The negative sign indicates the direction of the induced current according to Lenz's law.

Q. 21. Derive an expression for terminal velocity of a spherical object falling under gravity through a viscous medium.

Solution:

The maximum constant velocity acquired by object which is freely falling inside the viscous medium is known as terminal velocity . It's SI unit is m/s [LT^-1]

There are 3 phenomenal force acting on the freely falling object.

Weight = m x g (act downward)
Upthrust Force acting upward
Viscous Force acting upward
As the object falls freely it's velocity increases but this increased velocity also increase the viscous force.

Now this increased value of Viscous force along with Upthrust force balance is the downward weight of object

Fv + Fu = mg

Here net force upward is equal to the net force downward and the object acquired a dynamic equilibrium , therefore

Fnet = 0

m x a = 0

Since mass of object cannot be zero so expression become zero means a object acquired a constant velocity to travel rest of the distance is termed as Terminal velocity

Q. 22. Determine the shortest wavelengths of Balmer and Paschen series. Given the limit for Lyman series is $912 \mathrm{~A}$.

Solution:

To determine the shortest wavelengths of the Balmer and Paschen series, we need to understand the transitions of electrons in a hydrogen atom between different energy levels. The shortest wavelength in any series corresponds to the transition from the first level outside the series limit to the series limit.

Series Limits

Lyman Series:
- Transitions to $n_1 = 1$
- Shortest wavelength (series limit) corresponds to transition from $n \rightarrow \infty$ to $n = 1$.
- Given: Limit for Lyman series = 912 Å

Balmer Series:
- Transitions to $n_1 = 2$
- Shortest wavelength corresponds to transition from $n \rightarrow \infty$ to $n = 2$.

Paschen Series:
- Transitions to $n_1 = 3$
- Shortest wavelength corresponds to transition from $n \rightarrow \infty$ to $n = 3$.

Formula for Wavelength

The wavelength λ of light emitted or absorbed when an electron transitions between two energy levels in a hydrogen atom is given by the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where:
- R is the Rydberg constant ($R \approx 1.097 \times 10^7 \mathrm{~m}^{-1}$)
- n_1 and n_2 are the principal quantum numbers of the energy levels

For the shortest wavelength in each series, $n_2 = \infty$.

Shortest Wavelength of Balmer Series

For the Balmer series ($n_1 = 2$):

$$\frac{1}{\lambda_{\text{Balmer}}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = R\left(\frac{1}{4} - 0\right) = \frac{R}{4}$$

$$\lambda_{\text{Balmer}} = \frac{1}{R/4} = \frac{4}{R}$$

Shortest Wavelength of Paschen Series

For the Paschen series ($n_1 = 3$):

$$\frac{1}{\lambda_{\text{Paschen}}} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = R\left(\frac{1}{9} - 0\right) = \frac{R}{9}$$

$$\lambda_{\text{Paschen}} = \frac{1}{R/9} = \frac{9}{R}$$

Calculations Using Lyman Series Limit

Given the limit for the Lyman series ($n_1 = 1$) is 912 Å:

$$\frac{1}{\lambda_{\text{Lyman}}} = R\,(1 - 0) = R$$

So,

$$R = \frac{1}{\lambda_{\text{Lyman}}} = \frac{1}{912 \times 10^{-10}\,\text{m}}$$

Using this value of R:

Shortest Wavelength of Balmer Series

$$\lambda_{\text{Balmer}} = \frac{4}{R} = 4 \times 912\,\text{Å} = 3648\,\text{Å}$$

Shortest Wavelength of Paschen Series

$$\lambda_{\text{Paschen}} = \frac{9}{R} = 9 \times 912\,\text{Å} = 8208\,\text{Å}$$

Q. 23. Calculate the value of magnetic field at a distance of $3\ \text{cm}$ from a very long, straight wire carrying a current of $6\ \text{A}$.

Solution:

To calculate the magnetic field at a distance from a very long, straight wire carrying a current, we use Ampere's Law, specifically the formula derived from it for the magnetic field around a straight current-carrying conductor.

The formula for the magnetic field B at a distance r from a long, straight wire carrying a current I is given by:

$$B = \frac{\mu_0 I}{2\pi r}$$

where:
- μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$)
- I is the current through the wire
- r is the distance from the wire

Given:
- $I = 6\,\mathrm{A}$
- $r = 3\,\mathrm{cm} = 0.03\,\mathrm{m}$

Substitute these values into the formula:

$$B = \frac{4\pi \times 10^{-7} \times 6}{2\pi \times 0.03}$$

Simplify the expression:

$$B = \frac{4 \times 10^{-7} \times 6}{2 \times 0.03}$$

$$B = \frac{24 \times 10^{-7}}{0.06}$$

$$B = \frac{24 \times 10^{-7}}{6 \times 10^{-2}}$$

$$B = \frac{24}{6} \times 10^{-7+2}$$

$$B = 4 \times 10^{-5}\,\mathrm{T}$$

Thus, the magnetic field at a distance of 3 cm from the wire is:

$$B = 4 \times 10^{-5}\,\mathrm{T} = 40\,\mu\mathrm{T}$$

So, the magnetic field at a distance of 3 cm from the wire is $40\,\mu\mathrm{T}$.

Q. 24. A parallel plate capacitor filled with air has an area of $6\,\mathrm{cm}^2$ and plate separation of $3\,\mathrm{mm}$. Calculate its capacitance.

Solution:

To calculate the capacitance of a parallel plate capacitor, we use the formula:

$$C = \frac{\epsilon_0 A}{d}$$

where:
- C is the capacitance
- ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$)

- A is the area of the plates
- d is the separation between the plates

Given:
- Area, $A = 6\,\mathrm{cm}^2 = 6 \times 10^{-4}\,\mathrm{m}^2$
- Plate separation, $d = 3\,\mathrm{mm} = 3 \times 10^{-3}\,\mathrm{m}$

Substitute these values into the formula:

$$C = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-4}}{3 \times 10^{-3}}$$

Simplify the expression:

$$C = \frac{8.854 \times 10^{-12} \times 6}{3 \times 10^{3}}$$

$$C = \frac{8.854 \times 6}{3} \times 10^{-12}$$

$$C = 17.708 \times 10^{-12}$$

$$C = 17.708\,\mathrm{pF}$$

Thus, the capacitance of the parallel plate capacitor is approximately $17.708\,\mathrm{pF}$.

Q. 25. An emf of $91\mathrm{mV}$ is induced in the windings of a coil, when the current in a nearby coil is increasing at the rate of $1.3\ \mathrm{A/s}$, what is the mutual inductance (M) of the two coils in mH ?

Solution:

To find the mutual inductance (M) between two coils, we can use the formula relating the induced electromotive force (emf) in one coil to the rate of change of current in the other coil:

$$\mathcal{E} = M \frac{dI}{dt}$$

where:
- $\mathcal{E}$ is the induced emf
- M is the mutual inductance
- $\frac{dI}{dt}$ is the rate of change of current

Given:
- Induced emf, $\mathcal{E} = 91\,\mathrm{mV} = 91 \times 10^{-3}\,\mathrm{V}$
- Rate of change of current, $\frac{dI}{dt} = 1.3\,\mathrm{A/s}$

Substitute these values into the formula:

$$91 \times 10^{-3} = M \times 1.3$$

Solve for M:

$$M = \frac{91 \times 10^{-3}}{1.3}$$

$$M = \frac{91}{1.3} \times 10^{-3}$$

$$M \approx 70 \times 10^{-3}$$

$$M = 70\,\mathrm{mH}$$

Thus, the mutual inductance of the two coils is $70\,\mathrm{mH}$.

Q. 26. Two cells of emf $4\ \mathrm{V}$ and $2\ \mathrm{V}$ having respective internal resistance of 1Ω and 2Ω are connected in parallel, so as to send current in the same direction through an external resistance of 5Ω. Find the current through the external resistance.

Solution:

Calculate the Equivalent EMF (E_{eq}) and Equivalent Internal Resistance (r_{eq})

The equivalent EMF (E_{eq}) of cells connected in parallel is given by:

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Substitute the given values:

$$E_{\text{eq}} = \frac{(4\,\mathrm{V})(2\,\Omega) + (2\,\mathrm{V})(1\,\Omega)}{1\,\Omega + 2\,\Omega}$$

$$E_{\text{eq}} = \frac{8\,\mathrm{V} + 2\,\mathrm{V}}{3\,\Omega}$$

$$E_{\text{eq}} = \frac{10\,\mathrm{V}}{3\,\Omega}$$

$$E_{\text{eq}} \approx 3.33\,\mathrm{V}$$

The equivalent internal resistance (r_{eq}) of cells connected in parallel is given by:

$$\frac{1}{r_{\text{eq}}} = \frac{1}{r_1} + \frac{1}{r_2}$$

Substitute the given values:

$$\frac{1}{r_{\text{eq}}} = \frac{1}{1\,\Omega} + \frac{1}{2\,\Omega}$$

$$\frac{1}{r_{\text{eq}}} = 1 + 0.5$$

$$\frac{1}{r_{\text{eq}}} = 1.5$$

$$r_{\text{eq}} = \frac{1}{1.5}\,\Omega$$

$$r_{\text{eq}} = \frac{2}{3}\,\Omega \approx 0.67\,\Omega$$

Calculate the Total Resistance (R_{total})

The total resistance in the circuit is the sum of the external resistance and the equivalent internal resistance:

$$R_{\text{total}} = R + r_{\text{eq}}$$

$$R_{\text{total}} = 5\,\Omega + 0.67\,\Omega$$

$$R_{\text{total}} = 5.67\,\Omega$$

Using Ohm's law:

$$I = \frac{E_{\text{eq}}}{R_{\text{total}}}$$

$$I = \frac{3.33\,\text{V}}{5.67\,\Omega}$$

$$I \approx 0.587\,\text{A}$$

Thus, the current through the external resistance is approximately $0.587\,\text{A}$.

SECTION- D

Q. 27. Derive an expression for a pressure exerted by a gas on tic hacis of kinetic theory of gases.

Solution:

Let there be n moles of an ideal gas enclosed in a cubical box of volume $V = L^3$ and constant temperature T.

The molecule is moving with the velocity $\mathbf{v}$ and collides with the shaded wall of the cube. The wall is parallel to the yz plane. The change in momentum of the x-component is:

$$\Delta p_x = \text{final momentum} - \text{initial momentum}$$

$$\Delta p_x = (-mv_x) - (mv_x) = -2mv_x$$

The momentum transferred to the wall is $+2mv_x$ in magnitude. The rebounded molecule then goes to the opposite wall and collides with it.

As L is the length of the cubical box, the time for the molecule to travel back and forth to the shaded wall is:

$$\Delta t = \frac{2L}{v_x}$$

The average force exerted on the shaded wall by molecule 1 is given as:

$$\text{Average force} = \frac{\text{Average rate of change of momentum}}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

The total average force on the wall from all molecules is:

$$\text{Total average force} = \frac{m(v_{x1}^2 + v_{x2}^2 + \ldots)}{L}$$

The average pressure P is given by:

$$P = \frac{\text{Average force}}{\text{Area of the shaded wall}} = \frac{\frac{m(v_{x1}^2+v_{x2}^2+\ldots)}{L}}{L^2} = \frac{m(v_{x1}^2 + v_{x2}^2 + \ldots)}{L^3}$$

Since the molecule has no preferred direction to move, the velocities are evenly distributed in all directions. Therefore, the average pressure is:

$$P = \frac{1}{3V} m \left\langle v^2 \right\rangle$$

Thus, we obtain the required expression for the pressure exerted by a gas:

$$P = \frac{1}{3}\left(\frac{N}{V}\right) m v^2$$

Q. 28. What is a rectifier? With the help of a neat circuit diagram. explain the working of a half wave rectifier.

Solution:

A device which converts A.C. to D.C. is called rectifier. In this case output exists only for half cycle Hence it is called half wave rectifier. Construction: The circuit diagram of a half wave rectifier using a junctiondiode is as shown in fig. The alternating voltage source is connected to the primary coil of a transformer. The secondary coil is connected to the diode in series with a resistance RL called the load resistance.

T=Transformer

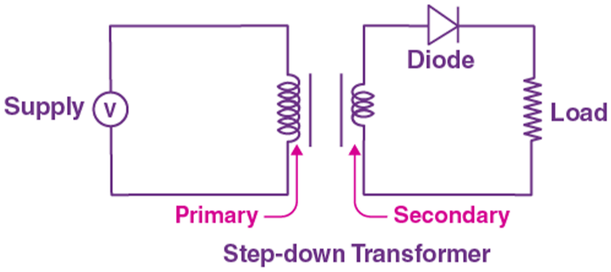
D=Diode

V0=output voltage

v1=input votage
RL=load resistance

Working: In first cycle of input voltage, the anode of the diode is positive potential w.r.t. cathode. Hence the diode is in forward-biased. Hence it conduct current. The current flows through load resistance giving voltage drop iRL.

This voltage drop is called output voltage. During next half cycle the anode of diode is in negative potential w.r.t. Hence it is in reversed- biased. Hence it does not conduct the current. Hence current does not flow through load resistance giving no P.D. across it. Hence output voltage is unidirectional. It is called as D.C.



Q. 29. Draw a neat, labelled diagram of a suspended coll type movil! coil galvanometer.
The initial pressure and volume of a gas enclosed in a cylinder are $2 \times 10^5 \mathrm{~N/m^2}$ and $6 \times 10^{-3} \mathrm{~m^3}$ respectively. If the work done in compressing the gas at constant pressure is $150 \mathrm{~J}$, find the final volume of the gas.

Solution:

Given data,

rho = 2 10 ^ 5 N / (m ^ 2)

Vi = 6 10 ^ - 3

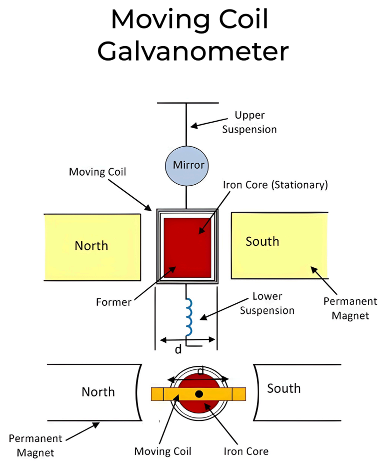
W = - 150J

Vf = ?

W = P(Vf - Vi)

= 6 10 ^ - 3 + (- 150/(2 10 ^ 5))
= 6 10 ^ - 3 + (- 75 10 ^ - 5)
= 6 10 ^ - 3 - 0.75 10 ^ - 3 V_{f}
= 5.25 10 ^ - 3 m ^ 3

Final volume of the gas is 5.25 × 10 3 m^3.



Q. 30. Define second's pendulum. Derive a formula for the length of second's pendulum.
A particle performing linear S.H.M. has maximum velocity 25 $\mathrm{cm/s}$ and maximum acceleration $100\ \mathrm{cm/s^2}$. Find period of oscillations.

Solution:

For second's pendulum, 2 = 2πλ Ls g Where Ls is the length of seconds pendulum having period T = 2s L = S g 2 p

This is a formula for the length of second's pendulum.
[08:34, 7/4/2024] Shivam: Given data,

Vmax = 25 cm/s

amax = 100 cm/s^2

T = ?

Vmax = aw

amax = aw2

2 max V max 3

100 25 ω

= 4 rad/s

2π W T =

2π == π 2

3.142 2 T =

= 1.571 s

The period of oscillations is 1.57

Q. 31. Explain de Broglie wavelength. Obtain an expression for de Broglie wavelength of wave associated with material particles. The photoelectric work function for a metal is $4.2 \mathrm{eV}$. Find the threshold wavelength.

Solution:

De-Broglie equated the energy equation of Plank (wave nature) and Einstein (particle nature) such that,

E = hv (Plank energy relation)

$E = mc^2$ (Einsteins mass-energy relation)

Where,

E = energy associated with the particle

h = planks constant

v = frequency associated with the particle

m = moss of the particle

c = speed of light

After equating equations (1) and (2) we

get:

$hv = mc^2$

$h = mc^2$

If the particle is moving with velocity 'v' Use then equation (3) becomes,

The wavelength of the particle if energy is in electron volt is, λ = 12, 400 Å E

Where E should be in eV.

After substituting the value of E i.e. 4.2 eV in equation (4) we get:

λ= 12, 400 Å /4.2
= 2952.38 Å

Hence, the threshold wavelength of the particle is 2952.38 Å.

Maharashtra Board Class 12 Physics Solutions - 2023

SECTION- A

Q.1 Select and write the correct answer for the following multiple type of questions :

(i) If ' n ' is the number of molecules per unit volume and ' d ' is the diameter of the molecules, the mean free path ' λ ' of molecules is

(a) $\sqrt{\frac{2}{\pi n d}}$

(b) $\frac{1}{2\pi n d^2}$

(c) $\frac{1}{\sqrt{2}\pi n d^2}$

(d) $\frac{1}{\sqrt{2\pi n d}}$

Solution:

The mean free path λ of molecules, given the number density n and the diameter d of the molecules, can be described by the formula:

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

This equation accounts for the effective collision cross-sectional area πd^2 and the density of molecules n, incorporating a factor of $\sqrt{2}$ to adjust for the relative motion between pairs of molecules. The correct expression reflects the interaction between these variables to determine the average distance a molecule travels before colliding with another.

Hence, the answer is option (c)

(ii) The first law of thermodynamics is consistent with the law of conservation of

(a) momentum

(b) energy

(c) mass

(d) velocity

Solution:

The first law of thermodynamics, which is a fundamental principle in physics, is consistent with the law of conservation of energy. This law states that energy cannot be created or destroyed in an isolated system; it can only be transformed from one form to another or transferred between objects.

Hence, the answer is option (b)

(iii) $\mathrm{Y}=\overline{\mathrm{A}+\mathrm{B}}$ **is the Boolean expression for**
(a) OR - gate
(b) AND - gate
(c) NOR - gate
(d) NAND) - gate

Solution:

The Boolean expression given as $Y=\overline{A+B}$ uses the notation for the NOR operation. In Boolean algebra, the plus symbol $+$ signifies an OR operation, and the overline indicates a NOT operation. Therefore, the expression $\overline{A+B}$ translates to NOT (A OR B), which is the definition of a NOR gate.

Hence, the answer is option (c)

(iv) The property of light which remains unchanged when it travels from one medium to another is .
(a) velocity
(b) wavelength
(c) amplitude
(d) frequency

Solution:

When light travels from one medium to another, its frequency remains unchanged. While the velocity and wavelength of light can vary depending on the medium's refractive index, the frequency, determined by the source of the light, is independent of the medium through which the light is traveling.

Hence, the answer is option (d)

(v) If a circular coil of 100 turns with a cross-sectional area of $1 \mathrm{~m}^2$ **is kept with its plane perpendicular to the magnetic field of I** T**, the magnetic flux linked with the coil will be**
(a) $1 \mathrm{~Wb}$
(b) $50 \mathrm{~Wb}$
(c) $100 \mathrm{~Wb}$
(d) $200 \mathrm{~Wb}$

Solution:

The magnetic flux Φ linked with a coil is given by the formula:

$$\Phi=N B A$$

where:
- N is the number of turns in the coil,
- B is the magnetic field strength,
- A is the cross-sectional area of the coil.

Given:
- $N = 100$ turns,
- $B = 1$ T (Tesla),
- $A = 1$ m^2,

Substituting these values into the formula, we get:

$$\Phi = 100 \times 1\,\mathrm{T} \times 1\,\mathrm{m}^2 = 100\,\mathrm{Wb}\,(\mathrm{Weber})$$

Hence, the answer is option (c)

(vi) If ' θ ' represents the angle of contact made by a liquid which completely wets the surface of the container then
(a) $0 = 0$
(b) $0 < \theta < \frac{\pi}{2}$
(c) $\theta = \frac{\pi}{2}$
(d) $\frac{\pi}{2} < 0 < \pi$

Solution:

When a liquid completely wets the surface of the container, the angle of contact (or contact angle) θ is very small, approaching zero. This occurs because the adhesive forces between the liquid and the container surface are much stronger than the cohesive forces within the liquid.

Therefore, for a liquid that completely wets the surface of the container.

Hence, the answer is option (a)

(vii) The LED emits visible light when its .
(a) junction is reversc biased
(b) depletion region widens
(c) holes and electrons recombine
(d) junction becomes hot

Solution:

An LED (Light Emitting Diode) emits visible light when electrons and holes recombine. This recombination of charge carriers occurs when the LED is forward biased, causing electrons to drop from a higher energy level to a lower energy level, releasing energy in the form of photons (light)

Hence, the answer is option (c)

(**viii) Soft iron is used to make the core of transformer because of its**
(a) low coercivity and low retentivity
(b) low coercivity and ligh retentivity

(c) high coereivity and ligh retentivity
(d) high coereivity and low retentivity

Solution:

Soft iron is used to make the core of transformers because of its low coercivity and low retentivity. Low coercivity means that the material can be easily magnetized and demagnetized, which is essential for the efficient operation of transformers that require frequent changes in the direction of the magnetic field. Low retentivity ensures that the material does not retain significant magnetization after the external magnetizing field is removed, reducing energy losses.

Hence, the answer is option (a)

(ix) If the maximum kinetic energy of emitted electrons in photoelectric effect is $2 \mathrm{eV}$, the stopping potential will be
(a) $0.5 \mathrm{~V}$
(b) $1.0 \mathrm{~V}$
(c) $1.5 \mathrm{~V}$
(d) $2.0 \mathrm{~V}$

Solution:

The stopping potential V_s in the photoelectric effect is related to the maximum kinetic energy $K_{\max}$ of the emitted electrons by the equation:

$$K_{\max} = e V_s$$

where e is the elementary charge (charge of an electron).

Given that the maximum kinetic energy of the emitted electrons is 2 eV, we can directly use this value as the stopping potential because 1 eV of energy corresponds to a stopping potential of 1 V.

Hence, the answer is option **(d)**

(x) The radius of eighth orbit of electron in H-atom will be more than that of fourth orbit by a factor of
(a) 2
(b) 4
(c) 8
(d) 16

Solution:

In the Bohr model of the hydrogen atom, the radius of the n-th orbit is given by:

$$r_n = n^2 r_1$$

where r_n is the radius of the n-th orbit, and r_1 is the radius of the first orbit.

To find the factor by which the radius of the eighth orbit ($n=8$) is more than that of the fourth orbit ($n=4$), we can use the formula for the radii:

$$r_8 = 8^2 r_1 = 64 r_1$$

$$r_4 = 4^2 r_1 = 16r_1$$

The factor by which r_8 is greater than r_4 is:

$$\frac{r_8}{r_4} = \frac{64r_1}{16r_1} = 4$$

Hence, the answer is option (b)

Q2. Answer the following questions :

(i) What is the valuc of resistance for an ideal voltmeter?

Solution:

An ideal voltmeter is designed to measure the voltage across two points in a circuit without affecting the circuit itself. To achieve this, an ideal voltmeter must have infinite resistance. This ensures that it draws no current from the circuit, thereby not altering the voltage being measured.

Therefore, the value of resistance for an ideal voltmeter is:

Infinite (or very high).

(ii) What is the value of force on a closed circuit in a magnetic field?

Solution:

The net force on a closed circuit in a uniform magnetic field is zero. This is because the magnetic forces acting on different segments of the circuit cancel each other out due to symmetry and the fact that the magnetic force on a current-carrying conductor is perpendicular to both the direction of the current and the magnetic field.

Therefore, the value of the force on a closed circuit in a magnetic field is:Zero.

(iii) What is the average value of alternating current over a complete cycle?

Solution:

The average value of an alternating current (AC) over a complete cycle is zero. This is because the positive half-cycle of the current cancels out the negative half-cycle when averaged over a full period.

Therefore, the average value of alternating current over a complete cycle is:

Zero.

(iv) An clectron is accelerated through a potential difference of 100 volt. Calculate de-Broglic wavelength in nm.

Solution:

Calculate the kinetic energy E_k of the electron:

$$E_k = eV$$

where e is the elementary charge (approximately 1.602×10^{-19} coulombs) and V is the potential difference (100 volts).

Since the kinetic energy is also given by:

$$E_k = \frac{1}{2}mv^2$$

we can solve for the velocity v of the electron.

Use the de Broglie wavelength formula:

$$\lambda = \frac{h}{p}$$

where h is Planck's constant (approximately 6.626×10^{-34} Js) and p is the momentum of the electron ($p = mv$).

First, calculate the kinetic energy:

$$E_k = eV = (1.602 \times 10^{-19}\,\mathrm{C})(100\,\mathrm{V}) = 1.602 \times 10^{-17}\,\mathrm{J}$$

Next, we can relate this to the momentum p using the fact that:

$$E_k = \frac{p^2}{2m}$$

$$p = \sqrt{2mE_k}$$

The mass of the electron m is approximately $9.109 \times 10^{-31}\,\mathrm{kg}$.

Now, calculate p:

$$p = \sqrt{2mE_k} = \sqrt{2(9.109 \times 10^{-31}\,\mathrm{kg})(1.602 \times 10^{-17}\,\mathrm{J})}$$

$$p \approx \sqrt{2.917 \times 10^{-47}\,\mathrm{kg}^2\,\mathrm{m}^2\,\mathrm{s}^{-2}}$$

$$p \approx 5.4 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}}$$

Finally, calculate the de Broglie wavelength λ:

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\,\mathrm{Js}}{5.4 \times 10^{-24}\,\mathrm{kg\,m\,s^{-1}}}$$

$$\lambda \approx 1.227 \times 10^{-10}\,\mathrm{m}$$

$$\lambda \approx 0.1227\,\mathrm{nm}$$

So, the de Broglie wavelength of the electron is approximately $0.1227\,\mathrm{nm}$.

(v) If friction is made zero for a road, can a vehicle move safely on this road?

Solution:

If friction were made zero on a road, a vehicle would find it very difficult, if not impossible, to move safely. Here's why:

1. **Starting Motion:** To start moving, a vehicle relies on the friction between its tires and the road. Without friction, the tires would simply spin without gaining any traction, preventing the vehicle from moving forward effectively.
2. **Steering:** Steering a vehicle also depends on friction. When a driver turns the steering wheel, friction allows the tires to grip the road and change direction. Without friction, the vehicle would continue moving in the same direction regardless of steering input.
3. **Stopping:** Braking systems are designed to stop a vehicle by applying a force that generates friction between the brake components and the wheels. Without friction, the brakes would be unable to slow down the wheels, leading to a vehicle that cannot stop.

In summary, zero friction on a road would compromise the ability to start, steer, and stop a vehicle, making safe driving impossible.

(vi) State the formula giving relation between electric field intensity and potential gradient.

Solution:

The formula relating electric field intensity $(\vec{E})$ and potential gradient is given by:
$\vec{E} = -\nabla V$
Represents the gradient of the electric potential V. The negative sign indicates that the electric field points in the direction of decreasing potential. This relation tells us that the electric field at a point is equal to the

negative of the rate of change of the electric potential at that point with respect to position.

(vii) Calculate the velocity of a particle performing S.H.M. after 1 second, if its displacement is given by $x=5\sin\left(\frac{\pi t}{3}\right)\mathrm{m}$.

Solution:

To calculate the velocity of a particle performing simple harmonic motion (SHM), we differentiate the displacement equation with respect to time. The displacement x is given by:

$x=5\sin\left(\frac{\pi t}{3}\right)\mathrm{~m}$
The velocity v is the first derivative of displacement with respect to time $v=\frac{dx}{dt}$
$\text{Differentiating } x=5\sin\left(\frac{\pi t}{3}\right) \text{ using the chain rule:} v=5\cdot\cos\left(\frac{\pi t}{3}\right)\cdot\frac{d}{dt}\left(\frac{\pi t}{3}\right)$
$\text{Since } \frac{d}{dt}\left(\frac{\pi t}{3}\right)=\frac{\pi}{3}, \text{ we have:} v=5\cdot\cos\left(\frac{\pi t}{3}\right)\cdot\frac{\pi}{3} v=\frac{5\pi}{3}\cos\left(\frac{\pi t}{3}\right)$
$\text{Now, plug in } t=1 \text{ second to find the velocity at that time:}$
$v=\frac{5\pi}{3}\cos\left(\frac{\pi}{3}\right)\text{Recall that } \cos\left(\frac{\pi}{3}\right)=\frac{1}{2}, v=\frac{5\pi}{3}\cdot\frac{1}{2} v=\frac{5\pi}{6}\mathrm{~m/s}$
$\text{Therefore, the velocity of the particle after 1 second is } \frac{5\pi}{6}\mathrm{~m/s}.$

(viii) Write the mathematical formula for Bohr magncton for an electron revolving in n^{th} orbit.

Solution:

The Bohr magneton is a physical constant that represents the magnetic dipole moment of an electron due to its orbital motion around the nucleus in an atom. The formula for the Bohr magneton (μ_B) is given by:
$\mu_B=\frac{e\hbar}{2m_e}$ \
$\text{Here:} -e \text{ is the elementary charge } (1.602\times10^{-19} \text{ Coulombs}),$
$-\hbar \text{ (h-bar) is the reduced Planck's constant } \left(\frac{h}{2\pi}\right), \text{ where } h$
$\text{ is Planck's constant approximately } 6.626\times10^{-34} \text{ Joule-second},$
$-m_e \text{ is the mass of an electron } (9.109\times10^{-31}\mathrm{~kg}).$$
This formula provides the magnitude of the magnetic dipole moment for an electron in its ground state orbit within the Bohr model of the atom.

SECTION- B

Q. 3. Define coefficient of viscosity. State its formula and S.I. units.

Solution:

The coefficient of viscosity, also known as dynamic viscosity, quantifies a fluid's internal resistance to flow. It is a measure of how much a fluid resists deformation at a given rate.

The formula for dynamic viscosity (η) is derived from Newton's law of viscosity, which can be stated as:

$$\tau = \eta \frac{dv}{dy}$$

Here, τ is the shear stress, η is the dynamic viscosity, $\frac{dv}{dy}$ is the velocity gradient perpendicular to the direction of shear.

The SI unit of viscosity is the Pascal-second $(Pa \cdot s)$, which is equivalent to $(N \cdot s)/m^2$ or kg/(m·s).

Q4. Obtain an exprcssion for mugnetic induction of a toroid of ' N ' turns about an axis passing through its centre and perpendicular

Solution:

The expression for the magnetic field at the center of a toroid (a doughnut-shaped coil) can be derived using Ampere's Law. When a current I lows through a toroid having N turns and a mean radius R, the magnetic field B is directed along the circular path of the toroid's cross-section and is uniform along this path.

Ampere's Law states that the line integral of the magnetic field B around a closed loop is equal to the permeability of free space μ_0 times the enclosed current I_{enc} :

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

For a toroid, the path of integration is along a circle at the center of the toroid. The enclosed current I_{enc} is the total current through all the turns, which is $N \times I$.

Given the symmetry and uniformity of the magnetic field along this circular path, the line integral simplifies to the magnetic field magnitude B times the circumference $2\pi R$ of the circle:

$$B \times 2\pi R = \mu_0 N I$$

Solving for B, we find the expression for the magnetic field inside a toroid:

$$B = \frac{\mu_0 N I}{2\pi R}$$

This formula shows that the magnetic field inside a toroid is directly proportional to the number of turns N, the current I, and inversely proportional to the mean radius R of the toroid.

Q. 5. State and prove principle of conservation of angular momentum.

Solution:

he principle of conservation of angular momentum states that the total angular momentum of a closed system remains constant if no external torques are acting on it.

To prove this principle, consider a system of particles. The angular momentum $\vec{L}$ of a single particle with respect to a point is given by:

$$\vec{L} = \vec{r} \times \vec{p}$$

where $\vec{r}$ is the position vector of the particle relative to the point, and $\vec{p}$ \text{ is the linear momentum of the particle.}

For a system of particles, the total angular momentum is the vector sum of the angular momenta of all the particles:

$$\vec{L}_{\text{total}} = \sum_i \vec{L}_i = \sum_i \vec{r}_i \times \vec{p}_i$$

Taking the time derivative of $\vec{L}_{\text{total}},$ we get:

$$\frac{d\vec{L}_{\text{total}}}{dt} = \sum_i \left(\frac{d\vec{r}_i}{dt} \times \vec{p}_i + \vec{r}_i \times \frac{d\vec{p}_i}{dt} \right)$$

Since $\frac{d\vec{r}_i}{dt}$ is the velocity $\vec{v}_i$ of the particle, and $\vec{v}_i \times \vec{p}_i$ is zero (because $\vec{v}_i$ and $\vec{p}_i$ are parallel), the equation simplifies to:

$$\frac{d\vec{L}_{\text{total}}}{dt} = \sum_i \vec{r}_i \times \frac{d\vec{p}_i}{dt}$$

The term $\frac{d\vec{p}_i}{dt}$ is the net force $\vec{F}_i$ on the particle. Thus,

$$\frac{d\vec{L}_{\text{total}}}{dt} = \sum_i \vec{r}_i \times \vec{F}_i = \vec{\tau}_{\text{total}}$$

where $\vec{\tau}_{\text{total}}$ is the total torque on the system. If the system is closed and no external torques act on it, $\vec{\tau}_{\text{total}} = 0,$ and therefore:

$$\frac{d\vec{L}_{\text{total}}}{dt} = 0$$

This implies that $\vec{L}_{\text{total}}$ is conserved.

Q.6 . Obtain an expression for equivalent capacitance of two capacitors C_1 and C_2 connected in series.

Solution:

To find the equivalent capacitance of two capacitors C_1 and C_2 connected in series, we start by considering the fact that the charge Q on each capacitor must be the same because charge cannot accumulate at the connection point between capacitors in a series circuit.}

The voltage across each capacitor can be expressed in terms of its capacitance and the charge on it:

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}$$

The total voltage V across the series combination of the two capacitors is the sum of the voltages across each:

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

The equivalent capacitance C_{eq} for the series combination is defined by the equation:

$$V = \frac{Q}{C_{eq}}$$

Equating the two expressions for V :

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Simplifying by canceling Q (assuming $Q \neq 0$):

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Hence, the equivalent capacitance of two capacitors connected in series is given by:}

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

Q. 7. Explain, why the equivalent inductance of two coils connected in parallel is less than the inductance of either of the coils.

Solution:

When two coils are connected in parallel, the equivalent inductance of the configuration is less than the inductance of either of the individual coils. This can be understood by considering how the total inductance changes based on how the magnetic fields of the individual coils interact and how the total impedance of the circuit is affected.

The formula for the equivalent inductance L_{eq} of two inductors L_1 and L_2 connected in parallel is:

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

From this formula, we see that the equivalent inductance is given by the reciprocal of the sum of the reciprocals of the individual inductances. The formula guarantees that L_{eq} will be less than either L_1 or L_2.

This occurs because, when inductors are connected in parallel, the total flux linkage is the same across all parallel branches, and the voltage across each inductor is the same. However, the parallel connection offers multiple paths for the current, effectively reducing the overall opposition to changes in current flow compared to each inductor individually. This reduction in opposition means that the circuit's ability to store magnetic energy is lessened, resulting in a lower equivalent inductance.

Q. 8. How will you convert a moving coil galvanometer into an ammeter?

Solution:

To convert a moving coil galvanometer into an ammeter, a very low resistance called a shunt resistance must be connected in parallel to the galvanometer. This shunt resistance allows most of the current in the circuit to bypass the galvanometer, enabling it to measure higher currents without being damaged by excessive current flow through its coil.

The value of the shunt resistance R_s can be calculated using the formula:
$R_s = \frac{R_g I_g}{I - I_g}$
where:
R_g is the resistance of the galvanometer,
I_g is the maximum current that the galvanometer can safely measure,
I is the maximum current that the ammeter is expected to measure.
This arrangement ensures that the galvanometer coil carries only a small fraction of the total current, proportional to its resistance relative to the shunt. As a result, the galvanometer now functions as an ammeter capable of measuring up to the total current I.

Q. 9. A 100Ω resistor is connected to a $220 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. Calculate :
(a) r.m.s. value of current and
(b) net power consumed over the full cycle

Solution:

Part (a): RMS Value of Current

The RMS value of the current I_{rms} flowing through the resistor can be calculated using Ohm's law, which states that $I = \frac{V}{R}$, where V is the RMS voltage and R is the resistance. For an AC circuit with a peak voltage V_{peak}, the RMS voltage V_{rms} is $\frac{V_{\text{peak}}}{\sqrt{2}}$. However, the 220 V given is already the RMS value. Therefore:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220 \text{ V}}{100 \ \Omega} = 2.2 \text{ A}$$

Part (b): Net Power Consumed

The net power consumed P by the resistor is given by:

$$P = V_{\text{rms}} \times I_{\text{rms}}$$

Using the values calculated:

$$P = 220 \, \text{V} \times 2.2 \, \text{A} = 484 \, \text{W}$$

Thus, the power consumed over the full cycle is 484 watts.

Q. 10. A bar magnet of mass $120 \mathrm{~g}$ in the form of a rectangular parallelopiped, has dimensions $1 = 40 \mathrm{~mm}, \mathrm{~b} = 10 \mathrm{~mm}$ and $h = 80 \mathrm{~mm}$, with its dimension ' h ' vertical, the magnet performs angular oscillations in the plane of the magnctic field with period

π seconds. If the magnetic moment is $3.4\mathrm{Am}^2$, determine the influencing magnetic field.

Solution:

The problem presents a bar magnet of mass 120 g, shaped as a rectangular parallelepiped, with dimensions:

$l = 40$ mm, $b = 10$ mm, and $h = 80$ mm, performing angular oscillations in a magnetic field. However, the question seems to be incomplete as it does not specify what needs to be calculated or further described regarding the period of these oscillations.

Typically, one might be asked to calculate the period of oscillation if the magnetic moment μ and the magnetic field } B were known. For a magnetic dipole μ in a uniform magnetic field B, the torque τ can be calculated by $\tau = \mu B \sin(\theta)$, and the small-angle approximation leads to the harmonic motion equation, which resembles that of a physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{\mu B}}$$

Where I is the moment of inertia of the bar magnet about its pivot point. The moment of inertia for a rectangular prism rotated about an axis through its center perpendicular to its length l is:

$$I = \frac{1}{12}m(b^2 + h^2)$$

Substituting the values and converting units where necessary (remembering m = 120 g, $b = 10$ mm, h = 80 mm, you can compute I and thus find T if μ and B were known.

However, since these values are not provided, we cannot complete the calculation here. If you wish to compute these given more information, please provide μ and B.

Q. 11. Distinguish between free vibrations and forced vibrations (Two points).

Solution:

The difference between the free and forced vibration is tabulated below,

Free vibration	Forced vibration
When the body gets disturbed during its equilibrium position it produced the free vibrations	When external periodic force act on a body it produced the forced vibration
The force is required to initiate the free vibration	Continuous periodic force is required to initiate the forced vibration.
It is a self-sustained vibration	It is an externally sustained vibration
The free vibrations always depend on the body hence it is also called the natural frequency	The forced vibration frequency is equal to the external periodic force.

The free vibrations are stopped fastly	The forced vibrations are stopped based on the external periodic force
Example: simple pendulum oscillations	Example: music instruments that have soundboards.

Q. 12. Compare the rate of loss of heat from n metal sphere at $827^{\circ}\mathrm{C}$ with rate of loss of heat from the same at $427^{\circ}\mathrm{C}$, if the temperature of surrounding is $27^{\circ}\mathrm{C}$.

Solution:

To compare the rate of heat loss from a metal sphere at two different temperatures, we use the Stefan-Boltzmann law. Let's denote the temperatures as follows:
- $T_1 = 100^{\circ}\mathrm{C}$ for the first case.
- $T_2 = 427^{\circ}\mathrm{C}$ for the second case.
- The temperature of the surroundings (T_s) is $27^{\circ}\mathrm{C}$.

First, we need to convert all temperatures to Kelvin:

$$T_1 = 100^{\circ}\mathrm{C} + 273 = 373\,\mathrm{K}$$

$$T_2 = 427^{\circ}\mathrm{C} + 273 = 700\,\mathrm{K}$$

$$T_s = 27^{\circ}\mathrm{C} + 273 = 300\,\mathrm{K}$$

According to the Stefan-Boltzmann law, the rate of heat loss (P) from the sphere is given by:

$$P = \sigma A(T^4 - T_s^4)$$

where:
- σ is the Stefan-Boltzmann constant ($\sigma \approx 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$)
- A is the surface area of the sphere

Let P_1 be the rate of heat loss at T_1 and P_2 be the rate of heat loss at T_2:

$$P_1 = \sigma A(373^4 - 300^4)$$

$$P_2 = \sigma A(700^4 - 300^4)$$

To find the ratio of the rates of heat loss $\frac{P_2}{P_1}$:

$$\frac{P_2}{P_1} = \frac{\sigma A(700^4 - 300^4)}{\sigma A(373^4 - 300^4)} = \frac{700^4 - 300^4}{373^4 - 300^4}$$

Now we calculate the fourth powers of the temperatures:

$$700^4 = 2.401 \times 10^{11}$$

$$373^4 = 1.938 \times 10^{10}$$

$$300^4 = 8.1 \times 10^9$$

So,

$$700^4 - 300^4 = 2.401 \times 10^{11} - 8.1 \times 10^9 = 2.3209 \times 10^{11}$$

$$373^4 - 300^4 = 1.938 \times 10^{10} - 8.1 \times 10^9 = 1.128 \times 10^{10}$$

Therefore,

$$\frac{P_2}{P_1} = \frac{2.3209 \times 10^{11}}{1.128 \times 10^{10}} \approx 20.58$$

The rate of loss of heat from the metal sphere at $427^\circ\mathrm{C}$ is approximately 20.58 times the rate of loss of heat at $100^\circ\mathrm{C}$.

Q. 13. An ideal mono-atomic gas is adiabatically compressed so that its final temperature is twice its initial temperature. Calculate the ratio of final pressure to its initial pressure.

Solution:

To solve this problem, we need to use the principles of adiabatic processes for an ideal gas. For an adiabatic process, the following relations hold:

1. $PV^\gamma = \text{constant}$
2. $TV^{\gamma-1} = \text{constant}$

where γ (gamma) is the adiabatic index or heat capacity ratio, which for a monoatomic ideal gas is $\frac{5}{3}$.

Given:
- The final temperature T_f is twice the initial temperature T_i. So, $T_f = 2T_i$.

We need to find the ratio of the final pressure P_f to the initial pressure P_i.

Using the adiabatic relation $TV^{\gamma-1} = \text{constant}$:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

Since $T_f = 2T_i$, we can write:

$$T_i V_i^{\gamma-1} = 2T_i V_f^{\gamma-1}$$

Dividing both sides by T_i:

$$V_i^{\gamma-1} = 2V_f^{\gamma-1}$$

Taking the $(\gamma - 1)$th root of both sides:

$$V_i = 2^{\frac{1}{\gamma-1}} V_f$$

For a monoatomic gas, $\gamma = \frac{5}{3}$, so $\gamma - 1 = \frac{2}{3}$:

$$V_i = 2^{\frac{1}{\frac{2}{3}}} V_f = 2^{\frac{3}{2}} V_f = 2^{1.5} V_f = 2\sqrt{2} V_f$$

Now we use the adiabatic relation $PV^{\gamma} = \text{constant}$:

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

Substitute $V_i = 2\sqrt{2} V_f$:

$$P_i (2\sqrt{2} V_f)^{\gamma} = P_f V_f^{\gamma}$$

$$P_i (2\sqrt{2})^{\gamma} V_f^{\gamma} = P_f V_f^{\gamma}$$

$$P_i (2\sqrt{2})^{\frac{5}{3}} = P_f$$

$$P_i (2^{1.5} \cdot 2^{0.5})^{\frac{5}{3}} = P_f$$

$$P_i (2^2)^{\frac{5}{3}} = P_f$$

$$P_i 2^{\frac{10}{3}} = P_f$$

$$P_f = P_i \cdot 2^{\frac{10}{3}}$$

Since $2^{\frac{10}{3}}$ can be simplified as:

$$2^{\frac{10}{3}} = (2^3)^{\frac{10}{9}} = 8^{\frac{10}{9}} = 2^{3.333} \approx 10.079$$

So,

$$P_f \approx 10.079 P_i$$

Therefore, the ratio of the final pressure to the initial pressure is approximately 10.079.

Q. 14. Disintegration rate of a radio-active sample is 10^{10} per hour at 20 hours from the start. It reduces to 5×10^9 per hour after 30 hours. Calculate the decay constant.

Solution:

To calculate the decay constant of a radioactive sample, we can use the fact that the disintegration rate (or activity) decreases exponentially over time. The general formula for the activity A at time t is given by:

$$A(t) = A_0 e^{-\lambda t}$$

where:
- $A(t)$ is the activity at time t.
- A_0 is the initial activity.
- λ is the decay constant.
- t is the time.

We are given the following information:
- The activity at $t = 20$ hours is $A_1 = 10^{10}$ disintegrations per hour.
- The activity at $t = 30$ hours is $A_2 = 5 \times 10^9$ disintegrations per hour.

We can set up the following equations based on the given information:

1. $A_1 = A_0 e^{-\lambda \cdot 20}$
2. $A_2 = A_0 e^{-\lambda \cdot 30}$

Divide the second equation by the first equation to eliminate A_0:

$$\frac{A_2}{A_1} = \frac{A_0 e^{-\lambda \cdot 30}}{A_0 e^{-\lambda \cdot 20}}$$

$$\frac{5 \times 10^9}{10^{10}} = e^{-\lambda \cdot 30} / e^{-\lambda \cdot 20}$$

$$\frac{1}{2} = e^{-\lambda \cdot 30 + \lambda \cdot 20}$$

$$\frac{1}{2} = e^{-\lambda \cdot 10}$$

Take the natural logarithm of both sides:

$$\ln\left(\frac{1}{2}\right) = -\lambda \cdot 10$$

$$\ln\left(2^{-1}\right) = -\lambda \cdot 10$$

$$-\ln(2) = -\lambda \cdot 10$$

$$\lambda = \frac{\ln(2)}{10}$$

We know that $\ln(2) \approx 0.693$, so:

$$\lambda \approx \frac{0.693}{10}$$

$$\lambda \approx 0.0693 \, \text{per hour}$$

Therefore, the decay constant λ is approximately $0.0693 \, \text{per hour}$.

SECTION- C

Q. 15. Derive laws of reflection of light using Huygens' principle.

Solution:

Huygens' principle is a method of analysis applied to problems of wave propagation both in the far-field limit and in near-field diffraction. The principle states that every point on a wavefront is itself the source of spherical wavelets, and the wavefront at any later time is the envelope of these wavelets. Using Huygens' principle, we can derive the laws of reflection of light.

Laws of Reflection

1. The angle of incidence is equal to the angle of reflection.
2. The incident ray, the reflected ray, and the normal to the reflecting surface at the point of incidence all lie in the same plane.

Derivation using Huygens' Principle

Consider a plane wavefront AB incident on a reflecting surface XY. Let AB be inclined at an angle i to the normal ON to the reflecting surface at point O, where O is the point of incidence.

1. Wavefronts and Point Sources:
   - According to Huygens' principle, every point on the incident wavefront AB can be considered as a source of secondary wavelets that spread out in all directions with the same speed as the wave.
   - Let c be the speed of light in the medium.

2. Incident Wavefront:
   - When the incident wavefront AB reaches the reflecting surface, point A reaches the surface at point O at time $t = 0$.
   - After a time t, point B will reach point C on the surface, where $AC = ct$.

3. Reflected Wavefront:
   - According to Huygens' principle, point O will act as a source of secondary wavelets.
   - After time t, these wavelets will have traveled a distance ct in the medium.

4. Constructing the Reflected Wavefront:
   - To construct the new wavefront after reflection, draw an arc of radius ct from point O.
   - This arc intersects the surface at point C. From point C, draw a perpendicular line CD of length ct, representing the distance the secondary wavelet would have traveled in time t.

5. Formation of the Reflected Wavefront:
   - The line CD is tangent to the arc and represents the new position of the wavefront after reflection.
   - Therefore, CD is the reflected wavefront.

6. Angles of Incidence and Reflection:
   - The angle of incidence i is the angle between the incident wavefront AB and the normal ON.
   - The angle of reflection r is the angle between the reflected wavefront CD and the normal ON.

7. Using Geometry:
   - Since $AC = ct$ and $CD = ct$, triangle AOC is congruent to triangle COD.
   - Therefore, angle i is equal to angle r.

Thus, we have derived the law of reflection:

$$\text{Angle of incidence} = \text{Angle of reflection}$$

$$i = r$$

Q. 16. State postulates of Bohr's atomic model.

Solution:

Niels Bohr's atomic model, proposed in 1913, was a significant advancement in understanding atomic structure and the behavior of electrons. Bohr's model was based on the following key postulates:

Quantized Orbits:
- Electrons revolve around the nucleus in fixed orbits or energy levels without radiating energy. These specific orbits are called quantized orbits or stationary states.
- Each orbit corresponds to a definite energy level, and these energy levels are quantized, meaning that an electron can only exist in certain discrete energy states.

Angular Momentum Quantization:
- The angular momentum of an electron in a given orbit is quantized and is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant.
- Mathematically, the angular momentum L of an electron in a quantized orbit is given by:

$$L = n\frac{h}{2\pi}$$

where n is a positive integer (n = 1, 2, 3, ...), known as the principal quantum number.

Energy Emission and Absorption:
- An electron can move between orbits by absorbing or emitting a photon of energy. The energy of the photon corresponds to the difference in energy between the initial and final orbits.
- When an electron jumps from a higher energy orbit (E_2) to a lower energy orbit (E_1), it emits a photon with energy equal to the difference in energy levels:

$$E_2 - E_1 = h\nu$$

where ν is the frequency of the emitted photon.
- Conversely, an electron absorbs a photon of energy $h\nu$ to move from a lower energy orbit (E_1) to a higher energy orbit (E_2).

Stable Orbits:
- The electrons in an atom are in stable orbits that are determined by the balance between the electrostatic force of attraction between the positively charged nucleus and the negatively charged electron and the centrifugal force due to the electron’s motion.
- As long as an electron remains in a stable orbit, it does not emit radiation.

These postulates allowed Bohr to explain the stability of atoms and the line spectra of hydrogen and hydrogen-like atoms. The model was particularly successful in explaining the Rydberg formula for the spectral emission lines of atomic hydrogen, providing a theoretical basis for the observed spectral lines.

However, it's important to note that while Bohr's model was a significant improvement over previous atomic models, it was later superseded by the more comprehensive and accurate quantum mechanical model of the atom, which incorporates the principles of wave-particle duality and the probabilistic nature of electron positions.

Q. 17. Define and state the unit and dimensions of:
(a) Magnetization
(b) Explain magnetic susceptibility

Solution:

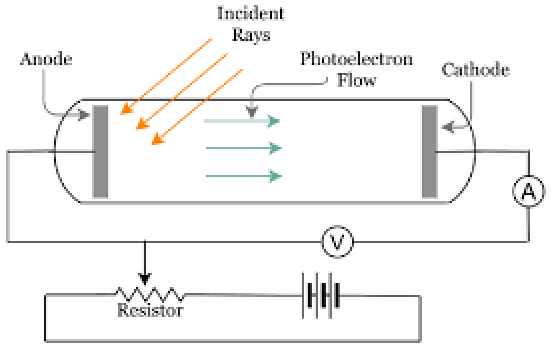
(a) Magnetization:
- Definition: Magnetization $(\mathbf{M})$ is the measure of the density of magnetic dipole moments within a magnetic material. It indicates the extent to which a material can be magnetized in the presence of an external magnetic field.
- Unit: The unit of magnetization in the SI system is amperes per meter $(\mathrm{A/m})$.
- Dimensions: The dimensions of magnetization are $\left[\mathrm{L}^{-1}\ \mathrm{T}\right]$, where L represents length and T represents time.
(b) Magnetic Susceptibility:
- Explanation: Magnetic susceptibility (χ) is a dimensionless proportionality constant that indicates how much a material will become magnetized in an applied magnetic field.
- Unit: Since it's dimensionless, magnetic susceptibility has no units.
- Dimensions: Magnetic susceptibility is dimensionless and does not have any specific dimensions.

Q. 18. With neat labelled circuit diagram, describe an experiment to study the characteristics of photoelectric effect.

Solution:

The photoelectric effect experiment demonstrates the emission of electrons from a material when it is exposed to light. To study the characteristics of the photoelectric effect, we typically use an experimental setup consisting of a light source, a phototube (or a photoemissive cell), an adjustable voltage supply, and measuring instruments such as a voltmeter and an ammeter. Here is a step-by-step description along with a neat labeled circuit diagram:

Circuit Diagram



Experimental Setup

1. Light Source:
   - A monochromatic light source, such as a mercury vapor lamp with filters, to provide light of different frequencies (wavelengths).

2. Phototube:
   - A phototube or photoemissive cell consists of a vacuum tube with a photosensitive material (cathode) and an anode inside. The cathode emits electrons when illuminated by the light source.

3. Adjustable Voltage Supply:
   - A variable DC power supply to adjust the potential difference (voltage) between the cathode and the anode.

4. Voltmeter and Ammeter:
   - A voltmeter connected across the phototube to measure the applied voltage.
   - An ammeter connected in series with the circuit to measure the photoelectric current (current due to emitted electrons).

5. Stopping Potential Setup:
   - A reversing switch or polarity control to apply a retarding potential (negative voltage) to stop the emitted electrons from reaching the anode, used to measure the stopping potential.

Steps of the Experiment

1. Setup Preparation:
   - Assemble the circuit as shown in the diagram.
   - Ensure the light source is focused on the phototube's cathode.

2. Initial Observation:
   - Turn on the light source and observe the current generated due to the photoelectric effect.
   - Measure the photoelectric current with the ammeter.

3. Varying Light Intensity:
   - Change the intensity of the light while keeping the frequency constant.
   - Observe the change in photoelectric current with varying light intensity.
   - Note that the current increases with increasing light intensity, demonstrating that the number of emitted electrons depends on the light's intensity.

4. Varying Light Frequency:
   - Use filters to change the light frequency (wavelength).
   - Observe the photoelectric current and note the threshold frequency below which no electrons are

emitted.
- This demonstrates the existence of a threshold frequency, below which the photoelectric effect does not occur, regardless of the light intensity.

5. Stopping Potential Measurement:
- Gradually apply a negative voltage (retarding potential) to the anode using the adjustable voltage supply.
- Measure the stopping potential (the voltage at which the photoelectric current drops to zero).
- Repeat this for different light frequencies and plot the stopping potential against frequency.

Observations and Conclusion

- Current vs. Intensity: The photoelectric current is directly proportional to the light intensity.
- Threshold Frequency: There is a minimum frequency (threshold frequency) below which no photoelectrons are emitted.
- Stopping Potential: The stopping potential is directly proportional to the frequency of the incident light, confirming the linear relationship predicted by Einstein's photoelectric equation:

$$eV_0 = h\nu - \phi$$

where V_0 is the stopping potential, h is Planck's constant, ν is the frequency of the incident light, and ϕ is the work function of the material.

The experiment demonstrates the key features of the photoelectric effect, including the dependence of photoelectric current on light intensity, the existence of a threshold frequency, and the linear relationship between stopping potential and light frequency. These observations support the quantum theory of light and the particle nature of photons.

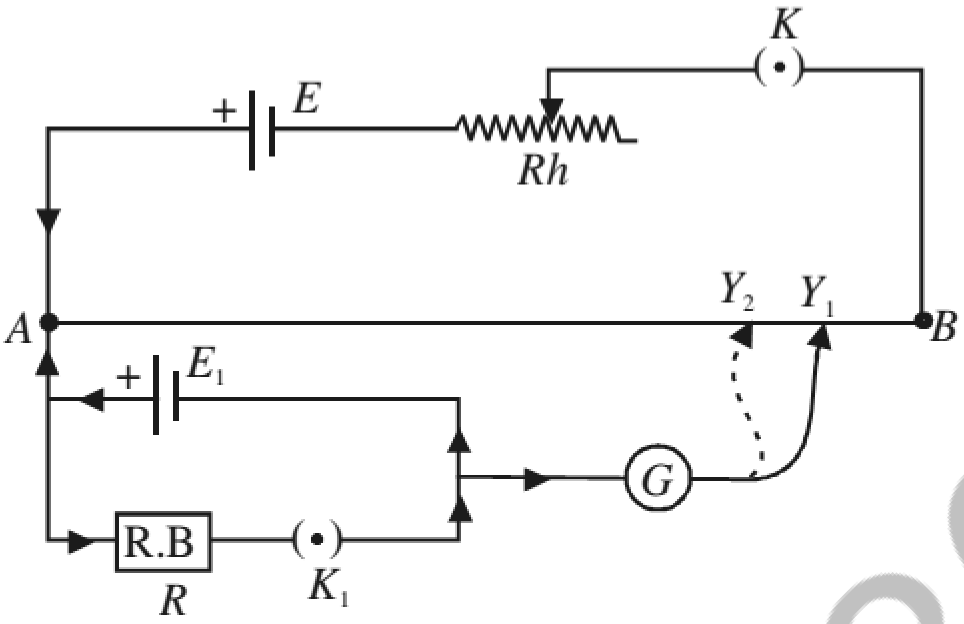
Q. 19. Explain the use of potentiometer to determine internal resistance of a cell.

Solution:

A potentiometer is a device used to measure the internal resistance of a cell accurately. The principle behind the potentiometer is that it measures the potential difference without drawing any current from the circuit, thus avoiding errors that would arise due to the current draw. Here's a step-by-step explanation of the experimental setup and procedure to determine the internal resistance of a cell using a potentiometer.

Apparatus Required
- Potentiometer
- A standard cell (reference cell)
- The cell whose internal resistance is to be measured
- A galvanometer
- A high resistance rheostat (variable resistor)
- A jockey (sliding contact)
- A key (switch)
- A known resistor (load resistor)
- Connecting wires

Circuit Diagram
The circuit diagram for determining the internal resistance of a cell using a potentiometer is as follows:



Procedure

1. Standardization of the Potentiometer:
   - Connect the standard cell (known EMF E) in the primary circuit of the potentiometer.
   - Adjust the rheostat to ensure a steady current through the potentiometer wire.
   - Move the jockey along the potentiometer wire to find the null point (no deflection in the galvanometer). Let's say the length corresponding to the null point is L_1.

2. Measurement of EMF of the Cell:
   - Replace the standard cell with the cell whose internal resistance is to be measured. Ensure the cell is connected such that its positive terminal is connected to the same end of the potentiometer as the standard cell.
   - Adjust the rheostat to maintain a steady current.
   - Find the null point on the potentiometer wire using the jockey. Let the length corresponding to this null point be L_2.
   - The EMF of the cell E is proportional to this length, i.e., $E \propto L_2$.

3. Measurement of Terminal Voltage:
   - Connect a known resistor R (load resistor) across the cell.
   - Measure the potential difference (terminal voltage V) across the cell using the potentiometer.
   - Find the new null point on the potentiometer wire using the jockey. Let this length be L_3.
   - The terminal voltage V is proportional to this length, i.e., $V \propto L_3$.

4. Calculation of Internal Resistance:
   - The EMF E and the terminal voltage V can be related using the lengths L_2 and L_3:

$$E = kL_2 \quad \text{and} \quad V = kL_3$$

   where k is a constant of proportionality.
   - The current I flowing through the circuit when the resistor R is connected can be found using Ohm's

law:

$$I = \frac{V}{R}$$

- The internal resistance r of the cell can be calculated using the formula:

$$r = \frac{E - V}{I} = \frac{E - V}{V/R} = R\left(\frac{E}{V} - 1\right)$$

- Substitute E and V with kL_2 and kL_3 respectively:

$$r = R\left(\frac{L_2}{L_3} - 1\right)$$

Conclusion
Using the potentiometer, we can determine the internal resistance of a cell accurately without drawing significant current from the cell. This method ensures that the measurement of EMF and terminal voltage is precise, leading to an accurate calculation of the internal resistance.

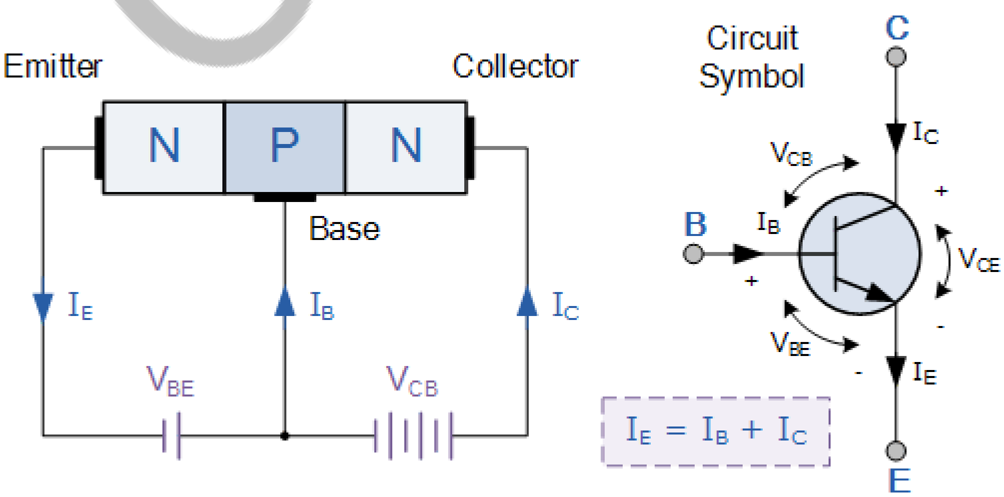
Q. 20. Explain the working of n-p-n transistor in common base configuration.

Solution:

The n-p-n transistor in a common base configuration is a type of bipolar junction transistor (BJT) where the base terminal is common to both the input and output circuits. This configuration is often used for applications requiring high-frequency response and voltage amplification.

Circuit Diagram

The basic circuit diagram of an n-p-n transistor in common base configuration is shown below:



Here:
- E : Emitter
- B : Base
- C : Collector
- V_{BE} : Voltage between base and emitter
- V_{CB} : Voltage between collector and base
- I_E : Emitter current
- I_B : Base current
- I_C : Collector current
- V_C : Collector voltage
- R_C : Collector resistor
- R_B : Base resistor

Working Principle

In the common base configuration, the input is applied between the emitter and the base, and the output is taken between the collector and the base. The base is common to both the input and the output circuits, hence the name "common base."

Current Relationships

The current relationships in an n-p-n transistor are given by:

$$I_E = I_C + I_B$$

Since I_B is very small compared to I_C, we can approximate:

$$I_E \approx I_C$$

Voltage Relationships

For the transistor to operate in the active region:
- The emitter-base junction must be forward biased.
- The collector-base junction must be reverse biased.

This means:

$$V_{BE} > 0$$

(typically around 0.7V for silicon transistors)

$$V_{CB} > 0$$

Operation

1. Emitter-Base Junction (Forward Biased):
   - When a positive voltage is applied to the emitter relative to the base, the emitter-base junction is forward biased.
   - This reduces the potential barrier, allowing electrons (majority carriers in n-p-n transistor) to flow from the emitter to the base.

2. Base-Collector Junction (Reverse Biased):
   - The base-collector junction is reverse biased by applying a positive voltage to the collector relative to the base.

- This widens the depletion region, preventing majority carriers from crossing the junction.

3. Injection and Collection:
   - Electrons injected from the emitter into the base region are minority carriers in the p-type base.
   - Due to the thin base region and the reverse bias of the collector-base junction, most of these electrons are swept into the collector region.
   - Only a small fraction of electrons recombine with holes in the base, constituting the base current I_B.

4. Output Characteristics:
   - The collector current I_C is primarily due to the electrons injected from the emitter and is almost equal to the emitter current I_E.
   - The relationship between the currents is given by the current gain α (common base current gain), which is slightly less than 1:

$$\alpha = \frac{I_C}{I_E}$$

Common Base Configuration Characteristics

- Input Characteristics: The input current (I_E) is plotted against the input voltage (V_{BE}) for different values of output voltage (V_{CB}). The input characteristics resemble that of a forward-biased diode.
- Output Characteristics: The output current (I_C) is plotted against the output voltage (V_{CB}) for different values of input current (I_E). The output characteristics show that I_C is almost independent of V_{CB} for a given I_E, indicating high output impedance.

Applications

- High-Frequency Amplifiers: The common base configuration is used in high-frequency applications due to its low input impedance and high output impedance.
- Voltage Amplifiers: It provides voltage amplification with low input voltage and high output voltage.

Conclusion

The n-p-n transistor in common base configuration demonstrates unique characteristics where the base is common to both input and output circuits. Its operation involves the forward biasing of the emitter-base junction and reverse biasing of the collector-base junction, leading to efficient current flow and voltage amplification, making it suitable for specific high-frequency and amplification applications.

Q. 21. State the differential equation of linear S.H.M. Hence, obtain expression for :
(a) acceleration
(b) velocity

Solution:

Differential Equation of Linear Simple Harmonic Motion (SHM)

Simple harmonic motion (SHM) is a type of periodic motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement. The differential equation governing SHM can be derived from Hooke's Law.

The restoring force F in SHM is given by:

$$F = -kx$$

where:
- k is the force constant (spring constant).
- x is the displacement from the equilibrium position.

According to Newton's second law:

$$F = ma$$

where:
- m is the mass of the object.
- a is the acceleration.

Combining these two equations:

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

The acceleration a can also be written as the second derivative of displacement x with respect to time t:

$$a = \frac{d^2x}{dt^2}$$

Therefore, the differential equation of SHM is:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency of the motion.

Expressions for Acceleration and Velocity

(a) Acceleration in SHM:

From the differential equation:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The acceleration $a(t)$ as a function of time is:

$$a(t) = \frac{d^2x}{dt^2} = -\omega^2 x(t)$$

If the displacement $x(t)$ is given by:

$$x(t) = A\cos(\omega t + \phi)$$

where:
- A is the amplitude of the motion.
- ω is the angular frequency.
- ϕ is the phase constant.

Then, the acceleration $a(t)$ is:

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

(b) Velocity in SHM:

The velocity $v(t)$ is the first derivative of displacement $x(t)$ with respect to time:

$$v(t) = \frac{dx}{dt}$$

Given $x(t) = A\cos(\omega t + \phi)$, we can find the velocity by differentiating $x(t)$:

$$v(t) = \frac{d}{dt}[A\cos(\omega t + \phi)]$$

$$v(t) = -A\omega\sin(\omega t + \phi)$$

Thus, the velocity $v(t)$ as a function of time is:

$$v(t) = -A\omega\sin(\omega t + \phi)$$

Q. 22. Two tuning forks of frequencies $3201\ \mathrm{Hz}$ and $340\ \mathrm{Hz}$ are sounded together to produce sound wnve. The velocily of sound in air is $326.4\ \mathrm{m/s}$. Caleulate the difference in wavelengths of these waves.

Solution:

To calculate the difference in wavelengths of the two sound waves produced by tuning forks with frequencies of 320 Hz and 340 Hz, we can use the formula for the wavelength of a wave:

$$\lambda = \frac{v}{f}$$

where:
- λ is the wavelength.
- v is the velocity of sound in air.
- f is the frequency of the sound wave.

Given:
- Velocity of sound in air, $v = 326.4\,\mathrm{m/s}$
- Frequencies of the tuning forks, $f_1 = 320\,\mathrm{Hz}$ and $f_2 = 340\,\mathrm{Hz}$

First, calculate the wavelength of the sound wave produced by the tuning fork with frequency 320 Hz:

$$\lambda_1 = \frac{v}{f_1} = \frac{326.4\,\mathrm{m/s}}{320\,\mathrm{Hz}}$$

$$\lambda_1 = 1.02\,\mathrm{m}$$

Next, calculate the wavelength of the sound wave produced by the tuning fork with frequency 340 Hz:

$$\lambda_2 = \frac{v}{f_2} = \frac{326.4\,\mathrm{m/s}}{340\,\mathrm{Hz}}$$

$$\lambda_2 = 0.96\,\mathrm{m}$$

Now, calculate the difference in wavelengths:

$$\Delta\lambda = \lambda_1 - \lambda_2$$

$$\Delta\lambda = 1.02\,\mathrm{m} - 0.96\,\mathrm{m}$$

$$\Delta\lambda = 0.06\,\mathrm{m}$$

Therefore, the difference in wavelengths of the sound waves produced by the two tuning forks is $0.06\,\mathrm{m}$.

Q. 23. In a biprism experiment, the fringes are observed in the focal plane of the eyc-piece at a distance of $1.2\ \mathrm{m}$ from the slit. The distance between the central bright band and the 20th bright band is $0.4\ \mathrm{cm}$. When a convex lens is placed between the biprism and the eye-piece, $90\ \mathrm{cm}$ from the cyc-piece, the distance between the two virtual magnified images is found to be $0.9\ \mathrm{cm}$. Determine the wavelength of light used.

Solution:

Calculate the Fringe Width (β)

The fringe width β is the distance between two consecutive bright bands (fringes). Since the distance between the central bright band and the 20th bright band is 0.4 cm, the fringe width β can be calculated as:

$$\beta = \frac{\text{Distance between central bright band and 20th bright band}}{\text{Number of fringes}}$$

$$\beta = \frac{0.4\,\text{cm}}{20} = \frac{0.4\,\text{cm}}{20} = 0.02\,\text{cm} = 0.0002\,\text{m}$$

When a convex lens is placed in the setup, it magnifies the separation between the virtual images of the slits. The distance between the two virtual magnified images is given as 0.9 cm (or 0.009 m).

Given that the lens is placed 90 cm from the eyepiece, and the total distance between the slit and the eyepiece is 1.2 m, the distance between the lens and the slits is:

$$\text{Distance between lens and slits} = 1.2\,\text{m} - 0.9\,\text{m} = 0.3\,\text{m}$$

The magnification M due to the lens can be calculated using the distances:

$$M = \frac{\text{Distance from lens to eyepiece}}{\text{Distance from lens to slits}} = \frac{0.9\,\text{m}}{0.3\,\text{m}} = 3$$

The actual slit separation d can then be calculated as:

$$d = \frac{\text{Distance between virtual magnified images}}{M} = \frac{0.009\,\text{m}}{3} = 0.003\,\text{m}$$

Using the formula for fringe width in a double-slit experiment:

$$\beta = \frac{\lambda L}{d}$$

where:
- β is the fringe width
- L is the distance between the slits and the eyepiece
- d is the slit separation

Rearranging to solve for the wavelength λ:

$$\lambda = \frac{\beta d}{L}$$

Substituting the known values:

$$\lambda = \frac{0.0002\,\text{m} \times 0.003\,\text{m}}{1.2\,\text{m}}$$

$$\lambda = \frac{0.0000006\,\text{m}^2}{1.2\,\text{m}}$$

$$\lambda = 0.0000005\,\text{m} = 5 \times 10^{-7}\,\text{m} = 500\,\text{nm}$$

Therefore, the wavelength of the light used in the biprism experiment is $500\,\text{nm}$.

Q. 24. Calculate the current flowing through two long parallel wires carrying equal currents and separated by a distance of $1.35\ \text{cm}$ experiencing a force per unit length of $4.76 \times 10^{-2}\ \text{N/m}$.

Solution:

To calculate the current flowing through two long parallel wires experiencing a given force per unit length, we can use the formula for the magnetic force between two parallel current-carrying wires. The formula is given by:

$$F=\frac{\mu_0 I_1 I_2}{2 \pi d}$$

where:
- F is the force per unit length between the wires.
- μ_0 is the permeability of free space ($4 \pi \times 10^{-7} \mathrm{~N} \cdot \mathrm{~A}^{-2}$).
- I_1 and I_2 are the currents flowing through the wires.
- d is the separation distance between the wires.

Given:
- The separation distance, $d=1.35 \mathrm{~cm}=0.0135 \mathrm{~m}$.
- The force per unit length, $F=4.26 \times 10^{-2} \mathrm{~N} / \mathrm{m}$.
- Since the currents are equal, $I_1=I_2=I$.

Substituting the given values into the formula:

$$4.26 \times 10^{-2}=\frac{\mu_0 I^2}{2 \pi d}$$

$$4.26 \times 10^{-2}=\frac{4 \pi \times 10^{-7} \cdot I^2}{2 \pi \cdot 0.0135}$$

Simplifying the equation:

$$4.26 \times 10^{-2}=\frac{2 \times 10^{-7} \cdot I^2}{0.0135}$$

$$4.26 \times 10^{-2} \times 0.0135=2 \times 10^{-7} \cdot I^2$$

$$5.751 \times 10^{-4}=2 \times 10^{-7} \cdot I^2$$

$$I^2=\frac{5.751 \times 10^{-4}}{2 \times 10^{-7}}$$

$$I^2=2.8755 \times 10^3$$

$$I=\sqrt{2.8755 \times 10^3}$$

$$I \approx 53.6 \mathrm{~A}$$

Therefore, the current flowing through each of the parallel wires is approximately $53.6 \mathrm{~A}$.

Q. 25. An alternating voltage given by $\mathrm{c}=140 \sin (314.2 \mathrm{t})$ is connected across a pure resistor of 50Ω.
Calculate :
(i) the frequency of the source
(ii) the r.m.s. current through the resistor

Solution:

Given an alternating voltage $V(t) = 140\sin(314.2t)$ connected across a pure resistor of $R = 500\,\Omega$:

(i) Frequency of the Source

The general form of an alternating voltage is given by:

$$V(t) = V_0\sin(\omega t)$$

where:
- V_0 is the peak voltage.
- ω is the angular frequency in radians per second.

Comparing this with the given equation $V(t) = 140\sin(314.2t)$, we can see that:

$$\omega = 314.2\,\mathrm{rad/s}$$

The relationship between angular frequency ω and frequency f in Hz is:

$$\omega = 2\pi f$$

Solving for f:

$$f = \frac{\omega}{2\pi} = \frac{314.2}{2\pi} \approx \frac{314.2}{6.2832} \approx 50\,\mathrm{Hz}$$

(ii) RMS Current through the Resistor

First, we calculate the RMS voltage V_{rms} of the alternating voltage. The RMS value of a sinusoidal voltage is given by:

$$V_{\mathrm{rms}} = \frac{V_0}{\sqrt{2}}$$

For the given peak voltage $V_0 = 140\,\mathrm{V}$:

$$V_{\mathrm{rms}} = \frac{140}{\sqrt{2}} \approx 140 \times 0.7071 \approx 98.99\,\mathrm{V}$$

Next, we use Ohm's law to find the RMS current I_{rms} through the resistor:

$$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{R}$$

Substituting the given resistance $R = 500\,\Omega$:

$$I_{\mathrm{rms}} = \frac{98.99}{500} \approx 0.198\,\mathrm{A}$$

Therefore, the frequency of the source is $50\,\mathrm{Hz}$, and the RMS current through the resistor is $0.198\,\mathrm{A}$.

Q. 26. An electric dipole consists of two opposite charges each of magnitude $1\mu\mathrm{C}$, separated by $2\ \mathrm{cm}$. The dipole is placed in an external electric field of $10^5\ \mathrm{N/C}$.
Calculate the :

(i) maximum torque experienced by the dipole and
(ii) work done by the external field to turn the dipole through 180°.

Solution:

Given an electric dipole with charges of magnitude $q = 1\,\mu\text{C}$ (microcoulombs) separated by a distance $2\,\text{cm}$ (0.02 meters), placed in an external electric field $E = 10^5\,\text{N/C}$:

(i) Maximum Torque Experienced by the Dipole

The torque τ experienced by an electric dipole in an electric field is given by:

$$\tau = pE\sin\theta$$

where:
- p is the dipole moment.
- E is the electric field.
- θ is the angle between the dipole moment and the electric field.

The dipole moment p is given by:

$$p = q \cdot d$$

where:
- $q = 1\,\mu\text{C} = 1 \times 10^{-6}\,\text{C}$
- $d = 0.02\,\text{m}$

Thus,

$$p = 1 \times 10^{-6}\,\text{C} \times 0.02\,\text{m} = 2 \times 10^{-8}\,\text{C} \cdot \text{m}$$

The maximum torque occurs when $\sin\theta = 1$ (i.e., $\theta = 90^\circ$):

$$\tau_{\text{max}} = pE$$

Substitute the values:

$$\tau_{\text{max}} = 2 \times 10^{-8}\,\text{C} \cdot \text{m} \times 10^5\,\text{N/C}$$

$$\tau_{\text{max}} = 2 \times 10^{-3}\,\text{N} \cdot \text{m}$$

(ii) Work Done by the External Field to Turn the Dipole Through 180°

The work done W by the external field in rotating the dipole through an angle θ is given by:

$$W = pE(\cos\theta_i - \cos\theta_f)$$

For a rotation of 180° ($\theta_i = 0^\circ$ and $\theta_f = 180^\circ$):

$$W = pE(\cos 0^\circ - \cos 180^\circ)$$

$$\cos 0^\circ = 1$$

$$\cos 180^\circ = -1$$

So,

$$W = pE(1 - (-1))$$

$$W = pE(1 + 1)$$

$$W = 2pE$$

Substitute the values:

$$W = 2 \times 2 \times 10^{-8}\,\mathrm{C \cdot m} \times 10^{5}\,\mathrm{N/C}$$

$$W = 4 \times 10^{-3}\,\mathrm{J}$$

SECTION- D

Q. 27. On the basis of kinetic theory of gases obtain an expression for pressure exerted by gas molecules enclosed in a container on its walls.

Solution:

$P = \frac{1}{3}\frac{m}{V}N\langle v^2\rangle$

Where:
- P is the pressure exerted by the gas
- m is the mass of a gas molecule
- V is the volume of the container
- N is the number of gas molecules
- $\langle v^2\rangle$ is the mean square velocity of the gas molecules

This expression is derived from the kinetic theory of gases, which states that the pressure exerted by a gas is due to the collisions of gas molecules with the walls of the container.

Q. 28. Derive an expression for energy stored in the magnetic field in terms of induced current.
A wire $S\ \mathrm{m}$ long is supported horizontally at a height of $15\ \mathrm{m}$. along east-west direction. When it is about to hit the ground, calculate the average e.m.f. induced in it. $\left(\mathrm{g} = 10\ \mathrm{m/s^2}\right)$.

Solution:

$\text{Energy stored in the magnetic field}(U) = \frac{1}{2} L I^2$

Where:
- U is the energy stored
- L is the inductance
- I is the current

For the second part of the question, we calculate the average e.m.f. induced in the wire when it falls:

Given:
- Length of the wire, $l = 5\,\mathrm{m}$
- Height, $h = 15\,\mathrm{m}$
- Acceleration due to gravity, $g = 10\,\mathrm{m/s^2}$

The velocity v of the wire just before hitting the ground can be found using the equation of motion:

$$v = \sqrt{2gh} = \sqrt{2 \times 10\,\mathrm{m/s^2} \times 15\,\mathrm{m}} = \sqrt{300}\,\mathrm{m/s} = 10\sqrt{3}\,\mathrm{m/s}$$

The average e.m.f. $\mathcal{E}$ induced in the wire can be calculated using the formula:

$$\mathcal{E} = Blv$$

Assuming a magnetic field B perpendicular to the length and motion of the wire, the average e.m.f. induced is:

$$\mathcal{E} = B \times 5\,\mathrm{m} \times 10\sqrt{3}\,\mathrm{m/s}$$

So, the average e.m.f. induced in the wire is:

$$\mathcal{E} = 50\sqrt{3}\,B\,\mathrm{V}$$

Where B is the magnetic field strength.

Q. 29. Derive an expression for the work done during an isothermal process.
$104\ \mathrm{J}$ of work is done on certain volume of a gas. If the gas releases $125\ \mathrm{kJ}$ of heat, calculate the change in internal energy of the gas.

Solution:

$\text{Work done during an isothermal process}(W) = nRT \ln\left(\frac{V_f}{V_i}\right)$

Where:
- W is the work done
- n is the number of moles of the gas
- R is the universal gas constant
- T is the temperature (constant in an isothermal process)
- V_f is the final volume
- V_i is the initial volume

For the second part of the question, we use the first law of thermodynamics:

$$\Delta U = Q - W$$

Given:
- Work done on the gas, $W = 104\,\mathrm{J}$
- Heat released by the gas, $Q = -125\,\mathrm{kJ} = -125 \times 10^3\,\mathrm{J}$

The change in internal energy ΔU is:

$$\Delta U = -125 \times 10^3\,\mathrm{J} - 104\,\mathrm{J} = -125104\,\mathrm{J}$$

So, the change in internal energy of the gas is:

$$\Delta U = -125104\,\mathrm{J}$$

Q. 30. Obtain the relation between surface energy and surface tension.
Calculate the work done in blowing a soap bubble to a radius of $1\ \mathrm{cm}$. The surface tension of soap Solution is $2.5 \times 10^{-2}\ \mathrm{N/m}$.

Solution:

$$\text{Surface energy} = \text{Surface tension} \times \text{Surface area}$$

For a liquid with surface tension γ and surface area A, the surface energy U is given by:

$$U = \gamma A$$

For the second part of the question, we calculate the work done in blowing a soap bubble to a radius of 1 cm.

Given:
- Radius of the bubble, $r = 1\,\mathrm{cm} = 0.01\,\mathrm{m}$
- Surface tension of the soap Solution, $\gamma = 2.5 \times 10^{-2}\,\mathrm{N/m}$

A soap bubble has two surfaces (inner and outer), so the total surface area A is:

$$A = 2 \times 4\pi r^2 = 8\pi r^2$$

The work done W in blowing the bubble is equal to the change in surface energy:

$$W = \gamma \times A = 2.5 \times 10^{-2}\,\mathrm{N/m} \times 8\pi (0.01\,\mathrm{m})^2$$

$$W = 2.5 \times 10^{-2} \times 8\pi \times 10^{-4}\,\mathrm{J}$$

$$W = 2.5 \times 10^{-2} \times 8 \times 3.14159 \times 10^{-4}\,\mathrm{J}$$

$$W = 6.28 \times 10^{-6}\,\mathrm{J}$$

So, the work done in blowing the soap bubble is:

$$W \approx 6.28 \times 10^{-6}\,\mathrm{J}$$

Q. 31. Derive expressions for linear velucity at lowest position, midway position and the top-most position for a particle revolving in a vertical circle, if it has to just complete circular motion without string slackening at top.

Solution:

For a particle to complete a vertical circle, it must have sufficient velocity at the lowest, midway, and top-most positions. The following derivations provide the expressions for these velocities.

At the top-most position:

$v_t = \sqrt{gR}$

where v_t is the linear velocity at the top-most position, g is the acceleration due to gravity, and R is the radius of the vertical circle.

At the lowest position:

Using the conservation of mechanical energy,

$\frac{1}{2}mv_b^2 - \frac{1}{2}mv_t^2 = mg(2R)$

$\frac{1}{2}mv_b^2 - \frac{1}{2}m(gR) = 2mgR$

$\frac{1}{2}mv_b^2 - \frac{1}{2}mgR = 2mgR$

$\frac{1}{2}mv_b^2 = 2mgR + \frac{1}{2}mgR$

$v_b = \sqrt{5gR}$

where v_b is the linear velocity at the lowest position.

\textbf{At the midway position (at R height from the bottom):}

Using the conservation of mechanical energy,

$\frac{1}{2}mv_m^2 - \frac{1}{2}mv_t^2 = mg(R)$

$\frac{1}{2}mv_m^2 - \frac{1}{2}m(gR) = mgR$

$\frac{1}{2}mv_m^2 = mgR + \frac{1}{2}mgR$

$v_m = \sqrt{3gR}$

where v_m is the linear velocity at the midway position.

Maharashtra Board Class 12 Physics Solutions - 2024

SECTION- A

Q.1 Select and write the correct answer for the following multiple types of questions :

(i) The moment of inertia (MI) of a disc of radius R and mass M about its central axis is _____.
(a) $\frac{\mathrm{MR}^2}{4}$
(b) $\frac{\mathrm{MR}^2}{2}$
(c) MR^2
(d) $\frac{3\mathrm{MR}^2}{2}$

Solution:

The moment of inertia of a solid disc of radius R and mass M about its central axis can be calculated using the formula:

$I=\frac{1}{2} M R^2$

This formula is derived assuming the mass distribution is uniform across the disc.

Therefore, the correct answer to the question about the moment of inertia of a disc of radius R and mass M about its central axis is:

$\frac{M R^2}{2}$

Hence, the answer is option (b)

(ii) The dimensional formula of surface tension is:
(a) $\left[\mathrm{L}^{-1} \mathrm{M}^1 \mathrm{~T}^{-2}\right]$
(b) $\left[\mathrm{L}^2 \mathrm{M}^1 \mathrm{~T}^{-2}\right]$
(c) $\left[\mathrm{L}^1 \mathrm{M}^1 \mathrm{~T}^{-1}\right]$
(d) $\left[\mathrm{L}^0 \mathrm{M}^1 \mathrm{~T}^{-2}\right]$

Solution:

The dimensional formula for surface tension can be determined by understanding its definition and units. Surface tension is typically defined as force per unit length. The dimensional formula for force is $\left[M L T^{-2}\right]$

where M represents mass, L represents length, and T represents time. Since surface tension is force per unit length, you divide the dimensional formula for force by length (L).

$$\text{The dimensional formula of surface tension is } [ML^0T^{-2}]$$

Hence, the answer is option (d).

(iii) Phase difference between a node and an adjacent antinode in a stationary wave is
(a) $\frac{\pi}{4}\text{rad}$
(b) $\frac{\pi}{2}\text{rad}$
(c) $\frac{3\pi}{4}\text{rad}$
(d) πrad

Solution:

The phase difference between a node and an adjacent antinode in a stationary wave is π/2 radians. This is because a stationary wave is formed by the interference of two travelling waves of the same frequency, moving in opposite directions. The points of zero amplitude (nodes) and maximum amplitude (antinodes) are π/2 radians out of phase.

Phase difference between a node and an adjacent antinode in a stationary wave is

$$\frac{\pi}{2}\text{ rad.}$$

A quarter of a wavelength separates a node and an antinode in a stationary wave, which corresponds to a phase difference of π/2 radians, as a full wave cycle is 2π radians.

Hence, the answer is option (b)

(iv) The work done in bringing a unit positive charge from infinity to a given point against the direction of electric field is known as
(a) electric flux
(b) magnetic potential
(c) electric potential
(d) gravitational potential

Solution:

The work done in bringing a unit positive charge from infinity to a given point against the direction of an electric field is defined as the **electric potential** at that point. This work is done to overcome the electric forces and is stored as potential energy.

Electric potential, also known as voltage, quantifies the potential energy per unit charge at a point in an electric field. It is a scalar quantity and is critical for understanding the energy changes associated with the movement of charges in electric fields.

Hence, the answer is option (c)

(v) To convert a moving coil galvanometer into an ammeter we need to connect a
(a) small resistance in parallel with it
(b) large resistance in series with it
(c) small resistance in series with it
(d) large resistance in parallel with it

Solution:

To convert a moving coil galvanometer into an ammeter, which is designed to measure higher currents, you need to connect a **small resistance in parallel** with it. This small resistance is known as a **shunt** resistance and allows most of the current to bypass the sensitive galvanometer coil, thus protecting it from high currents that could damage it.

Hence, the answer is option (a)

(vi) If the frequency of incident light falling on a photosensitive material is doubled, then kinetic energy of the emitted photoelectron will be ____
(a) the same as its initial value
(b) two times its initial value
(c) more than two times its initial value
(d) less than two times its initial value

Solution:

If the frequency of incident light falling on a photosensitive material is doubled, then the kinetic energy of the emitted photoelectron will be more than two times its initial value.

This simplifies to:

$$\text{K.E.} = 2hf - hf_1$$

This can be rearranged as:

$$\text{K.E.} = hf + (hf - hf_1)$$

If we consider the initial kinetic energy to be $hf - hf_1$, the new kinetic energy becomes $2hf - hf_1$, which is more than just double the initial kinetic energy since we are adding more than hf_1 to the original kinetic energy.

Hence, the answer is option (c).

(vii) In a cyclic process, if $\Delta U =$ internal energy, $W =$ work done, $\mathrm{Q} =$ Heat supplied then
(a) $\Delta U = Q$
(b) $\mathrm{Q} = \mathrm{O}$
(c) $\mathrm{W} = \mathrm{O}$
(d) $\mathrm{W} = \mathrm{Q}$

Solution:

In a cyclic process, the internal energy of the system returns to its initial value, which means the change in internal energy (ΔU) is zero. According to the first law of thermodynamics, the change in internal energy of a system is equal to the heat added to the system minus the work done by the system, expressed as:

ΔU=Q−W

Given ΔU=0 for a cyclic process, this equation simplifies to:

0=Q−W

Which means:

Q=WQ

Therefore, the heat supplied to the system is equal to the work done by the system in a cyclic process.

Hence, the answer is option (d).

(viii) The current in a coil changes from $50 \mathrm{~A}$ to $10 \mathrm{~A}$ in 0.1 second. The self-inductance of the coil is $20 \mathrm{H}$. The induced e.m.f. in the coil is
(a) $800 \mathrm{~V}$
(b) $6000 \mathrm{~V}$
(c) $7000 \mathrm{~V}$
(d) $8000 \mathrm{~V}$

Solution:

To find the induced electromotive force (e.m.f.) in a coil due to a change in current, we use the formula for the e.m.f. induced in an inductor, which is given by:

$\epsilon=-L \frac{\Delta I}{\Delta t}$

ϵ is the induced e.m.f.
L is the self-inductance of the coil.
ΔI is the change in current.
Δt is the time over which the current change occurs.

$L=20 \mathrm{~H}$
$\Delta I=10 \mathrm{~A}-50 \mathrm{~A}=-40 \mathrm{~A}$
$\Delta t=0.1 \mathrm{~s}$

$\epsilon=-20 \mathrm{~H} \cdot \frac{-40 \mathrm{~A}}{0.1 \mathrm{~s}}=8000 \mathrm{~V}$

The induced e.m.f. in the coil is $8000 \mathrm{~V}$.

Hence, the answer is option (d).

(ix) The velocity of bob of a second's pendulum when it is $6 \mathrm{~cm}$ from its mean position and amplitude of $10 \mathrm{~cm}$, is
(a) $8 \pi \mathrm{cm} / \mathrm{s}$
(b) $6 \pi \mathrm{cm} / \mathrm{s}$
(c) $4 \pi \mathrm{cm} / \mathrm{s}$
(d) $2 \pi \mathrm{cm} / \mathrm{s}$

Solution:

Given:

$T=2 \mathrm{~s} \quad$ (Period of a second's pendulum)

$A=10 \mathrm{~cm} \quad$ (Amplitude)

$x=6 \mathrm{~cm} \quad$ (Displacement from mean position)

Angular frequency, ω :

$\omega=\frac{2 \pi}{T}=\frac{2 \pi}{2}=\pi \mathrm{rad} / \mathrm{s}$

Using the energy conservation formula for velocity:

$v=\omega \sqrt{A^2-x^2}=\pi \sqrt{10^2-6^2}=\pi \sqrt{100-36}=\pi \sqrt{64}=8 \pi \mathrm{~cm} / \mathrm{s}$

Therefore, the velocity of the bob when it is 6 cm from its mean position is $8 \pi \mathrm{~cm} / \mathrm{s}$.

Hence, the answer is option (d)

(x) In biprism experiment, the distance of $20^{\text {th }}$ bright band from the central bright band is $1.2 \mathrm{~cm}$. Without changing the experimental set-up, the distance of $30^{\text {th }}$ bright band from the central bright band will be -
(a) $0.6 \mathrm{~cm}$
(b) $0.8 \mathrm{~cm}$
(c) $1.2 \mathrm{~cm}$
(d) $1.8 \mathrm{~cm}$

Solution:

In a biprism experiment, the distance of bright bands from the central maximum is given by the relation:

$y_n=n \cdot \frac{\lambda D}{d}$

where y_n is the distance of the n-th bright band, λ is the wavelength,
D is the distance to the screen, and d is the slit separation.

Given, the distance of the 20th bright band $(n=20)$ is $1.2 \mathrm{~cm}$:

$y_{20}=20 \cdot \frac{\lambda D}{d}=1.2 \mathrm{~cm}$

To find the distance of the 30th bright band $(n=30)$:

$y_{30} = 30 \cdot \frac{\lambda D}{d}$

The ratio of distances is proportional to the ratio of their order numbers:

$\frac{y_{30}}{y_{20}} = \frac{30}{20} = 1.5$

$y_{30} = 1.5 \times 1.2\,\text{cm} = 1.8\,\text{cm}$

Therefore, the distance of the 30th bright band from the central bright band is $1.8\,\text{cm}$.

Hence, the answer is option (d)

Q2. Answer the following questions

(i) Define centripetal force.

Solution:

Centripetal force is the force that is necessary to keep an object moving in a circular path and is directed towards the center around which the object is moving. This force is always perpendicular to the motion of the object and prevents the object from flying off away from the center due to its inertia, which would otherwise cause it to move in a straight line. The magnitude of the centripetal force required to maintain circular motion depends on the object's mass, the speed at which it is traveling, and the radius of the circle

(ii) Why a detergent powder is mixed with water to wash clothes?

Solution:

Detergent powder is mixed with water to wash clothes primarily because detergents are designed to remove dirt, oils, and organic compounds from fabrics effectively. Here are the key reasons why detergents are used in water for cleaning clothes:

1. **Surfactant Action**: Detergents contain surfactants, which reduce the surface tension of water. This allows the water to spread and penetrate more effectively into the fabrics of the clothes. Surfactants also help to lift soil and grease from fabric surfaces, which can then be rinsed away with water.

2. **Emulsification**: Detergents help in emulsifying the grease or oil particles, meaning they break down the oils into smaller droplets and suspend them in water. This prevents the grease from re-depositing onto the clothes, allowing them to be washed away.

3. **Chemical Action**: Detergents chemically react with soil and stains, breaking them down into smaller, more soluble substances that can be removed during the washing process.

4. **Temperature Efficacy**: Detergents are formulated to work effectively across a range of temperatures, providing flexibility in the washing conditions, from cold washes to hot washes.

5. **Water Hardness Management**: Detergents often contain ingredients that soften water (by binding to calcium and magnesium ions), which increases the cleaning effectiveness since hard water can hinder the cleaning power of detergents.

In essence, mixing detergent with water creates a Solution that can effectively interact with fabric to remove dirt, stains, and odors, resulting in clean and fresh-smelling clothes.

(iii) What is the resistance of an ideal voltmeter?

Solution:

The resistance of an ideal voltmeter is infinitely high. An ideal voltmeter is designed this way because it must not draw any current from the circuit it is measuring. If the voltmeter had a finite resistance, it would draw some current, thus altering the circuit conditions and potentially leading to inaccurate voltage readings.

By having infinite resistance, an ideal voltmeter ensures that it has a negligible impact on the circuit, maintaining the original circuit conditions and providing an accurate measurement of the voltage between the points it is connected to.

(iv) Write the formula for the torque acting on the rotating current-carrying coil in terms of magnetic dipole moment, in vector form.

Solution:

The formula for the torque $\vec{\tau}$ acting on a rotating current-carrying coil in terms of its magnetic dipole moment }

$\vec{\mu}$ and an external magnetic field $\vec{B}$ is given by:
$\vec{\tau} = \vec{\mu} \times \vec{B}$
where:
$\vec{\tau}$ is the torque vector,
$\vec{\mu}$ is the magnetic dipole moment vector of the coil,
$\vec{B}$ is the external magnetic field vector,
$\times$ denotes the vector cross product.

The direction of $\vec{\tau}$ is perpendicular to the plane formed by $\vec{\mu}$ and $\vec{B}$, as determined by the right-hand rule. The magnitude of the torque can be found using the equation:

$\tau = \mu B \sin(\theta)$

where θ is the angle between $\vec{\mu}$ and $\vec{B}$. This torque tends to rotate the coil such that $\vec{\mu}$ aligns with $\vec{B}$, and its effect is maximal when θ is 90 degrees.

(v) What is the binding energy of a hydrogen atom?

Solution:

The binding energy of a hydrogen atom refers to the energy required to remove the electron from the atom, thereby ionizing it. This energy is essentially the energy needed to break the attraction between the positively charged proton in the nucleus and the negatively charged electron.

For a hydrogen atom, this binding energy is the same as the ionization energy of its ground state. The value can be calculated using the Bohr model of the hydrogen atom or obtained directly from known physical constants.

(vi) What is surroundings in thermodynamics?

Solution:

In thermodynamics, "surroundings" refers to everything outside the system that can interact with it. A system in thermodynamics is any part of the universe that is under consideration, usually separated by real or imaginary boundaries from the rest of the universe. The surroundings are thus the rest of the universe that lies outside of these boundaries.

The interactions between the system and its surroundings can occur in the form of heat transfer, work done by or on the system, or matter exchange. The concept of surroundings helps to define and isolate what is being studied, allowing precise calculations and observations about how a system exchanges energy or matter with everything else around it.

(vii) In a photoelectric experiment, the stopping potential is 1.5V. What is the maximum kinetic energy of a photoelectron?

Solution:

In the context of a photoelectric experiment, the stopping potential, denoted as V_s, is the potential needed to stop all photoelectrons from reaching the anode, thus giving a measure of the maximum kinetic energy of the electrons emitted from the metal surface.
The relationship between the maximum kinetic energy $K_{\max}$ and the stopping potential V_s is given by:

$$K_{max} = eV_s$$

where:
$K_{\max}$ is the maximum kinetic energy of the photoelectrons,
e is the elementary charge,
V_s is the stopping potential.

Kmax = evs

$$K_{\max} = e \times V_s$$
$$K_{\max} = 1.602 \times 10^{-19} \text{ Coulombs} \times 1.5 \text{ V}$$
$$K_{\max} = 2.403 \times 10^{-19} \text{ Joules}$$
$$K_{\max} \approx \frac{2.403 \times 10^{-19} \text{ Joules}}{1.602 \times 10^{-19} \text{ Joules/eV}} \approx 1.5 \text{ eV}$$

(viii) Two capacitors of capacities $5 \mu \mathrm{F}$ and $10 \mu \mathrm{F}$ respectively are connected in series. Calculate the resultant capacity of the combination.

Solution:

To calculate the resultant capacity of two capacitors connected in series, you use the formula for the equivalent capacitanc Ceq of capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$\frac{1}{C_{\text{eq}}} = \frac{1}{5\,\mu\text{F}} + \frac{1}{10\,\mu\text{F}}$$
$$\frac{1}{C_{\text{eq}}} = 0.2\,\text{F}^{-1} + 0.1\,\text{F}^{-1}$$
$$\frac{1}{C_{\text{eq}}} = 0.3\,\text{F}^{-1}$$
$$C_{\text{eq}} = \frac{1}{0.3} \approx 3.33\,\mu\text{F}$$

Thus, the equivalent capacitance of the combination when the two capacitors are connected in series is approximately 3.33 μF.

SECTION- B

Q. 3. Explain the change in internal energy of a thermodynamic system (the gas) by heating it.

Solution:

When you heat a gas within a thermodynamic system, its internal energy typically changes. Here's how this process works:

Internal Energy of a Gas

Internal energy, denoted as U, is the total energy stored within the microscopic constituents of a system — in this case, the gas molecules. It includes kinetic energy due to the random motions of the molecules and potential energy arising from the interactions between these molecules.

How Heating Affects Internal Energy

1. **Kinetic Energy Increase**: Heating the gas typically increases the kinetic energy of its molecules. As the temperature of the gas rises, the average speed and thus the kinetic energy of the molecules increases. According to the kinetic theory of gases, the average kinetic energy of the particles in a gas is directly proportional to the temperature of the gas.

2. **Equation of State**: For an ideal gas, the internal energy change can be described using the equation of state

Q. 4. Explain the construction of a spherical wavefront by using Huygens' principle.

Solution:

Huygens' Principle provides a convenient method to visualize and calculate the propagation of waves, including the formation of spherical wavefronts. Here's how the construction of a spherical wavefront can be explained using Huygens' Principle:

Imagine a point source of light or sound. When it emits energy, it sends out waves in all directions uniformly.

Huygens' Principle states that every point on a wavefront acts as a source of tiny secondary wavelets that spread out in all directions at the speed of the wave. The wavefront at any subsequent time is the envelope (or surface) that is tangent to these secondary wavelets.

- **Initial Wavefront**: Consider a point source emitting waves. Initially, you might visualize a small spherical wavefront centered on the source.
- **Secondary Wavelets**: After a small time interval Δt\Delta tΔt, every point on this spherical wavefront will generate its own spherical wavelet with a radius equal to cΔtc \Delta tcΔt, where ccc is the speed of the wave in the medium.
- **New Wavefront**: The new position of the wavefront after time Δt\Delta tΔt is a sphere formed by the envelope of all these secondary wavelets. The radius of this new wavefront will be larger, reflecting the distance cΔtc \Delta tcΔt added to the radius of the original wavefront.

- As time progresses, this process continues with each new wavefront acting as the source of further secondary wavelets.
- The center of each secondary wavelet remains fixed relative to the point on the original wavefront where it originated. The radius of each secondary wavelet increases linearly with time.

Example: Sound or Light from a Point Source

- For sound or light emanating from a point source in an isotropic medium (i.e., properties are uniform in all directions), the wavefronts are always spherical.
- The distance from the source to any point on the wavefront is the same, and the wavefront expands uniformly in all directions.

Q 5. Define magnetization. State its SI unit and dimensions.

Solution:

Magnetization is defined as the vector quantity that represents the magnetic moment per unit volume of a material. It indicates the extent to which a material is magnetized and is a measure of the density of magnetic dipole moments in a magnetic material. Magnetization can be induced by placing the material in a magnetic field, leading to the alignment of its microscopic magnetic dipoles (atomic or molecular magnetic moments).

SI Unit and Dimensions of Magnetization

- **SI Unit**: The SI unit of magnetization is amperes per meter (A/m).

- **Dimensions**: The dimensional formula for magnetization can be derived as follows:

  Magnetization (M) is the magnetic moment per unit volume (V)

  The magnetic moment has dimensions of current times area $[I \times L^2]$, \text{ and volume has dimensions of } [L^3].$

Q. 6. Obtain the differential equation of linear simple harmonic motion.

Solution:

The differential equation for linear simple harmonic motion (SHM) can be derived by considering the forces acting on a system that obeys Hooke's Law, which states that the force exerted by a spring is proportional to the displacement from the equilibrium position and acts in the opposite direction.

Step-by-Step Derivation:

1. **Force and Hooke's Law**: For a mass mmm attached to a spring with spring constant kkk, the restoring force FFF exerted by the spring when displaced xxx meters from its equilibrium position is:

   F=−kx
2. **Newton's Second Law**: According to Newton's second law of motion, the force acting on an object is equal to the mass of the object multiplied by its acceleration aaa, or:

   F=ma

Q. 7. A galvanometer has a resistance of 30Ω and its full scale deflection current is 20 microampere ($\mu \mathrm{A}$). What resistance should be added to it to have a range $0-10$ volt?

Solution:

Given:

$$
\begin{aligned}
R_g &= 30\,\Omega \quad \text{(Resistance of the galvanometer)} \\
I_{\text{fs}} &= 20\mu\text{A} = 20 \times 10^{-6}\ \text{A} \quad \text{(Full scale deflection current)} \\
V_{\text{range}} &= 10\ \text{V} \quad \text{(Desired range for the voltmeter)}
\end{aligned}
$$

The shunt resistance R_s should be chosen such that the current through the galvanometer at full scale deflection matches the current through R_s when 10 V is applied:
$I_{\text{fs}} = \frac{V_{\text{range}}}{R_g + R_s}$

Solving for R_s:
$R_s = \frac{V_{\text{range}}}{I_{\text{fs}}} - R_g = \frac{10}{20 \times 10^{-6}} - 30$

Calculating the value of R_s:
$R_s = \frac{10}{20 \times 10^{-6}} - 30 = 500000 - 30 = 499970\,\Omega$

Therefore, the required resistance to be added (shunt resistance) is $499970\,\Omega$.

Q. 8. Explain Biot-Savart law.

Solution:

The Biot-Savart Law is a fundamental principle in electromagnetism that describes how currents produce magnetic fields. This law is crucial for calculating the magnetic field generated by any configuration of moving charges, typically currents in wires.

Key Features:

1. **Direction**: The direction of $\vec{dB}$ is perpendicular to the plane formed by $\vec{dl}$ and $\vec{r}$, determined by the right-hand rule. If you curl the fingers of your right hand around the direction from $\vec{dl}$ to $\vec{r}$, your thumb points in the direction of $\vec{dB}$.
2. **Magnitude**: The magnitude of the magnetic field decreases with the cube of the distance from the current element (r^3) and depends directly on the current I and the length of $\vec{dl}$.
3. **Integration for Finite Lengths**: To find the total magnetic field due to a finite length of current-carrying wire, you need to integrate dB⃗\vec{dB}dB over the entire length of the wire:

Applications:

The Biot-Savart Law is used in various applications including:

- Calculating the magnetic field around wire loops and solenoids,
- Understanding the magnetic effects of different current configurations in devices like motors and generators,
- Designing magnetic systems in physics experiments and industrial applications.

Q. 9. What is a Light Emitting Diode? Draw its circuit symbol.

Solution:

A Light Emitting Diode (LED) is a semiconductor device that emits light when an electric current flows through it. LEDs are used in a wide range of applications, including indicator lights, displays, and lighting. The light production in an LED results from a process called electroluminescence, where electrons recombine with electron holes within the device, releasing energy in the form of photons. The color of the light (corresponding to the energy of the photons) is determined by the energy band gap of the semiconductor.

LEDs are known for their efficiency and longevity compared to traditional incandescent bulbs, primarily because they do not have a filament that can burn out or fail.

Circuit Symbol of an LED:

The circuit symbol for an LED is similar to that of a standard diode, but it includes two arrows pointing away from the diode, indicating that light is being emitted. Below, I'll create an image showing the circuit symbol of an LED.

Q. 10. An aircraft of wing span of $60\ \mathrm{m}$ flies horizontally in earth's magnetic field of $6 \times 10^{-5}\ \mathrm{T}$ at a speed of $500\ \mathrm{m/s}$. Calculate the e.m.f. induced between the tips of wings of aircraft.

Solution:

The electromotive force (emf) induced between the tips of the aircraft's wings is calculated using the formula:
$\mathcal{E} = B \times v \times l$
where B is the magnetic field strength $(6 \times 10^{-5}\,\mathrm{T})$, v is the speed of the aircraft $(500\,\mathrm{m/s})$, and l is the wingspan of the aircraft $(60\,\mathrm{m})$.

Plugging these values into the formula, we get:
$\mathcal{E} = 6 \times 10^{-5} \times 500 \times 60 = 1.8\,\mathrm{volts}$

Q. 11. Derive an expression for the maximum speed of a vehicle moving along a horizontal circular track.

Solution:

To derive the expression for the maximum speed of a vehicle moving along a horizontal circular track, we need to consider the forces acting on the vehicle. The primary forces involved are the centripetal force required to keep the vehicle moving in a circular path and the frictional force between the tyres and the track.

1. \textbf{Centripetal Force}: The centripetal force (F_c) required to keep the vehicle moving in a circular path is given by:

$$F_c = \frac{mv^2}{r}$$

where m is the mass of the vehicle, v is the velocity of the vehicle, and r is the radius of the circular path.

2. \textbf{Frictional Force}: The frictional force (F_f) that provides the centripetal force is given by:

$$F_f = \mu N$$

where μ is the coefficient of friction between the tyres and the track, and N is the normal force. For a vehicle on a horizontal track, the normal force is equal to the weight of the vehicle, $N = mg$.

To achieve maximum speed without skidding, the centripetal force must be equal to the maximum frictional force:

$$\frac{mv^2}{r} = \mu mg$$

Solving for v (the maximum speed), we get:

$$v^2 = \mu rg$$

$$v = \sqrt{\mu rg}$$

Thus, the maximum speed v_{max} of a vehicle moving along a horizontal circular track is:

$$v_{\text{max}} = \sqrt{\mu rg}$$

Q. 12. A horizontal force of $0.5\ \mathrm{N}$ is required to move a metal plate or area $10^{-2}\ \mathrm{m}^2$ with a velocity of $3 \times 10^{-2}\ \mathrm{m/s}$, when it rests on $0.5 \times 10^{-3}\ \mathrm{m}$ thick layer of glycerin. Find the coefficient of viscosity of glycerin.

Solution:

To find the coefficient of viscosity of glycerin, we can use the formula for the force required to move a plate through a viscous fluid:
$F = \eta \frac{A \Delta v}{d}$

Rearranging the formula to solve for η:
$\eta = \frac{F \cdot d}{A \cdot \Delta v}$

Plugging in the values:
$\eta = \frac{0.5\ \mathrm{N} \times 0.5 \times 10^{-3}\ \mathrm{m}}{10^{-2}\ \mathrm{m}^2 \times 3 \times 10^{-2}\ \mathrm{m/s}}$

Now, we'll calculate this step-by-step:
$\eta = \frac{0.5 \times 0.5 \times 10^{-3}}{10^{-2} \times 3 \times 10^{-2}}$

$\eta = \frac{0.25 \times 10^{-3}}{3 \times 10^{-4}} = \frac{0.25}{3} \approx 0.0833$

Therefore, the coefficient of viscosity of glycerin is approximately $0.0833\,\mathrm{Ns/m^2}$.

Q. 13. Two tuning forks having frequencies $320\ \mathrm{Hz}$ and $340\ \mathrm{Hz}$ are sounded together to produce sound waves. The velocity of sound in air is $340\ \mathrm{m/s}$. Find the difference in wavelength of these waves.

Solution:

To find the difference in the wavelengths of the sound waves produced by the two tuning forks with frequencies 320 Hz and 340 Hz, we can use the formula for the wavelength of a wave:

$$\lambda = \frac{v}{f}$$

where:
- λ is the wavelength,
- v is the velocity of sound in air (340 m/s)
- f is the frequency of the sound wave.

Let's calculate the wavelength for each frequency:

1. For the frequency $f_1 = 320\,\mathrm{Hz}$:

$$\lambda_1 = \frac{v}{f_1} = \frac{340\,\mathrm{m/s}}{320\,\mathrm{Hz}} \approx 1.0625\,\mathrm{m}$$

2. For the frequency $f_2 = 340\,\mathrm{Hz}$:

$$\lambda_2 = \frac{v}{f_2} = \frac{340\,\mathrm{m/s}}{340\,\mathrm{Hz}} = 1\,\mathrm{m}$$

Finally, we find the difference in wavelengths:

$$\Delta\lambda = \lambda_1 - \lambda_2 = 1.0625\,\mathrm{m} - 1\,\mathrm{m} = 0.0625\,\mathrm{m}$$

Therefore, the difference in the wavelengths of the sound waves is 0.0625 meters.

Q. 14. Calculate the change in angular momentum of electron when it jumps from third orbit to first orbit in hydrogen atom.

Solution:

$L = n\hbar$

For the hydrogen atom:
- The angular momentum in the third orbit ($n = 3$) is:
$L_3 = 3\hbar$

- The angular momentum in the first orbit ($n = 1$) is:
$L_1 = 1\hbar$

The change in angular momentum (ΔL) when the electron jumps from the third orbit to the first orbit is:
$\Delta L = L_1 - L_3$
$\Delta L = 1\hbar - 3\hbar$
$\Delta L = -2\hbar$

Therefore, the change in angular momentum is $-2\hbar$.

SECTION- C

Q. 15. A circular coil of wire is made up of 200 turns, each of radius $10\ \mathrm{cm}$. If a current of $0.5\ \mathrm{A}$ passes through it, what will be the magnetic field at the centre of the coil?

Solution:

$B = \frac{\mu_0 N I}{2R}$

Given the values:
- $N = 200$ turns,
- $I = 0.5$ A,
- $R = 10$ cm $= 0.1$ m,

We can plug these values into the formula to calculate the magnetic field:

$B = \frac{4\pi \times 10^{-7} \times 200 \times 0.5}{2 \times 0.1}$

Simplifying:

$B = \frac{4\pi \times 10^{-7} \times 100}{0.1}$
$B = \frac{4\pi \times 10^{-5}}{0.1}$
$B = 4\pi \times 10^{-4}$

So, the magnetic field at the center of the coil is:

$B = 4\pi \times 10^{-4}\ \mathrm{T}$

Q. 16. Define the photoelectric effect and explain the experimental set-up of the photoelectric effect.

Solution:

Definition of the Photoelectric Effect:

The photoelectric effect is the phenomenon where electrons are emitted from the surface of a material (usually a metal) when it absorbs electromagnetic radiation (such as light). The emitted electrons are known as photoelectrons. This effect demonstrates the particle nature of light, as it shows that light can transfer energy in discrete packets called photons.

Experimental Setup of the Photoelectric Effect

The typical experimental setup to observe the photoelectric effect involves the following components:

1. **Light Source**: A monochromatic light source (like a mercury vapour lamp) is used to emit light of a specific wavelength or frequency. The light can be directed and focused onto the surface of the material.

2. **Metal Surface (Photocathode)**: The light is shined onto a clean metal surface, usually within a vacuum tube to prevent interference from air molecules. Commonly used metals include zinc, potassium, and caesium, which have low work functions (the minimum energy needed to emit an electron).

3. **Electron Detector (Anode)**: Positioned opposite the metal surface, the anode collects the emitted photoelectrons. The anode is part of an electric circuit that allows the measurement of the photoelectric current.

4. **Vacuum Tube**: The entire setup is enclosed in a vacuum tube to avoid the scattering of emitted electrons by air molecules.

5. **Ammeter**: Connected in the circuit to measure the current produced by the emitted electrons.

6. **Voltage Source**: A variable voltage source is connected between the photocathode and the anode. It can apply a stopping potential (negative voltage) to stop the electrons from reaching the anode, which helps in measuring the kinetic energy of the emitted electrons.

Process

1. When light of sufficient frequency shines on the metal surface, it transfers energy to the electrons.
2. If the energy of the photons is greater than the work function of the metal, electrons are emitted from the surface.
3. These emitted electrons are attracted to the positively charged anode, creating a current that can be measured by the ammeter.
4. By varying the frequency of the incident light and the stopping potential, one can study the relationship between the kinetic energy of the emitted electrons and the frequency of the incident light, verifying the equation given by Einstein for the photoelectric effect:

Q. 17. Define the current gain α_{DC} and β_{DC} for a transistor. Obtain the relation between them.

Solution:

Definition of Current Gain α_{DC} and β_{DC} for a Transistor

α_{DC} (Common Base Current Gain)

$\alpha_{DC} = \frac{I_C}{I_E}$

In an ideal transistor, α_{DC} is close to 1, typically between 0.95 and 0.99.

β_{DC} (Common Emitter Current Gain)

$\beta_{DC} = \frac{I_C}{I_B}$

Relation between α_{DC} and β_{DC}

The relationship between α_{DC} and β_{DC} can be derived from their definitions. We start with the basic current relationships in a transistor:

$I_E = I_B + I_C$

From the definition of α_{DC}:

$\alpha_{DC} = \frac{I_C}{I_E}$

$I_C = \alpha_{DC} I_E$

Since $I_E = I_B + I_C$, substituting I_C from the above equation:

$I_E = I_B + \alpha_{DC} I_E$

Rearranging to solve for I_E:

$I_E(1 - \alpha_{DC}) = I_B$

$I_E = \frac{I_B}{1-\alpha_{DC}}$

Now substituting I_E back into the definition of β_{DC}:

$\beta_{DC} = \frac{I_C}{I_B} = \frac{\alpha_{DC} I_E}{I_B} = \frac{\alpha_{DC}\left(\frac{I_B}{1-\alpha_{DC}}\right)}{I_B} = \frac{\alpha_{DC}}{1-\alpha_{DC}}$

Therefore, the relation between α_{DC} and β_{DC} is:

$\beta_{DC} = \frac{\alpha_{DC}}{1-\alpha_{DC}}$

Similarly, we can express α_{DC} in terms of β_{DC}:

$\alpha_{DC} = \frac{\beta_{DC}}{\beta_{DC}+1}$

Q. 18. Define the surface energy of the liquid. Obtain the relation between the surface energy and surface tension.

Solution:

Definition of Surface Energy of a Liquid:

Surface energy is the energy required to increase the surface area of a liquid by a unit amount. It is a measure of the cohesive forces at the surface of the liquid. The molecules at the surface of a liquid experience an imbalance of intermolecular forces compared to those in the bulk, leading to a higher energy state. The surface energy quantifies this additional energy.

Relation between Surface Energy and Surface Tension

Surface tension is the force per unit length acting at right angles to an imaginary line drawn on the surface of the liquid. It is responsible for the liquid surface behaving like a stretched elastic membrane.

To establish the relation between surface energy (γ) and surface tension (σ), consider a liquid film of surface area A. If the area of the surface is increased by a small amount ΔA, the work done (W) to increase the surface area is given by:

$$W = \gamma \Delta A$$

Surface tension (σ) is defined as the force per unit length. If l is the length over which the force acts, the work done to increase the surface area can also be expressed as:

$$W = \sigma \Delta l$$

For an infinitesimal increase in surface area, the increase in length Δl is related to the change in surface area ΔA. For a liquid film with two surfaces, the relation is:

$$\Delta A = 2l\Delta l$$

Combining the two expressions for work done:

$$\gamma \Delta A = \sigma \Delta l$$

Substituting $\Delta A = 2l\Delta l$:

$$\gamma(2l\Delta l) = \sigma \Delta l$$

$$\gamma = \frac{\sigma}{2}$$

Thus, the surface energy per unit area is equal to the surface tension:

$$\gamma = \sigma$$

Q. 19. What is an isothermal process? Obtain an expression for work done by a gas in an isothermal process.

Solution:

Definition of an Isothermal Process

An isothermal process is a thermodynamic process in which the temperature of the system remains constant. For an ideal gas undergoing an isothermal process, the internal energy remains constant because the internal energy of an ideal gas is a function of temperature only. This implies that any heat added to the system is entirely used to do work by the gas, and conversely, any work done on the system results in the release of heat.

Expression for Work Done by a Gas in an Isothermal Process

For an ideal gas, the work done during an isothermal process can be derived using the ideal gas law:

$$PV = nRT$$

where:
- P is the pressure,
- V is the volume,
- n is the number of moles,
- R is the universal gas constant,
- T is the temperature.

The work done W by the gas during an isothermal expansion or compression from an initial volume V_i to a final volume V_f can be found by integrating the pressure with respect to volume. For an isothermal process, T is constant, so:

$P = \frac{nRT}{V}$

The work done is:

$W = \int_{V_i}^{V_f} P\, dV$

Substituting the expression for P:

$W = \int_{V_i}^{V_f} \frac{nRT}{V}\, dV$

Since n, R, and T are constants, they can be taken out of the integral:

$W = nRT \int_{V_i}^{V_f} \frac{1}{V}\, dV$

The integral of $\frac{1}{V}$ is the natural logarithm $\ln(V)$:

$W = nRT[\ln(V)]_{V_i}^{V_f}$

$W = nRT\,(\ln(V_f) - \ln(V_i))$

Using the properties of logarithms, we can combine the terms:

$W = nRT \ln\left(\frac{V_f}{V_i}\right)$

Therefore, the work done by the gas in an isothermal process is:

$W = nRT \ln\left(\frac{V_f}{V_i}\right)$

Q. 20. Derive an expression for equation of stationary wave on a stretched string. Show that the distance between two successive nodes or antinodes is $\lambda/2$.

Solution:

Derivation of the Equation for a Stationary Wave on a Stretched String

A stationary wave, also known as a standing wave, is formed when two waves of the same frequency and amplitude traveling in opposite directions superpose. Consider two harmonic waves traveling in opposite directions along a stretched string.

Let the equations of the two waves be:
$y_1 = A\sin(kx - \omega t)$
$y_2 = A\sin(kx + \omega t)$

where:
- A is the amplitude,
- k is the wave number ($k = \frac{2\pi}{\lambda}$),
- ω is the angular frequency,
- x is the position along the string,
- t is the time,
- λ is the wavelength.

The resultant displacement y at any point x and time t is the sum of the displacements due to the two waves:
$y = y_1 + y_2$
$y = A\sin(kx - \omega t) + A\sin(kx + \omega t)$

Using the trigonometric identity for the sum of sine functions:
$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

we get:
$y = 2A\sin(kx)\cos(\omega t)$

This is the equation of a stationary wave. Here, $2A\sin(kx)$ represents the amplitude of the stationary wave which varies with position x, and $\cos(\omega t)$ represents the time variation of the wave.

Distance Between Two Successive Nodes or Antinodes

Nodes are points on the string where the displacement is always zero. For nodes:
$2A\sin(kx) = 0$
$\sin(kx) = 0$

Since $k = \frac{2\pi}{\lambda}$, the condition for nodes is:
$kx = n\pi$
$\frac{2\pi}{\lambda}x = n\pi$
$x = n\frac{\lambda}{2}$

where n is an integer (0, 1, 2, 3, ...).

Antinodes are points on the string where the displacement is maximum. For antinodes:
$2A\sin(kx) = \pm 2A$
$\sin(kx) = \pm 1$

The condition for antinodes is:
$kx = (n + \frac{1}{2})\pi$

Q. 21. Derive an expression for the impedance of an LCR circuit connected to an AC power supply. Draw phasor diagram.

Solution:

Derivation of Impedance of an LCR Circuit

An LCR circuit consists of an inductor L, a capacitor C, and a resistor R connected in series with an AC power supply.

The impedance Z of an LCR circuit is given by combining the resistance, inductive reactance, and capacitive reactance.

1. Resistance (R):
   - The resistance R has an impedance of R (purely real).

2. Inductive Reactance (X_L):
   - The inductive reactance is $X_L = \omega L$, where $\omega = 2\pi f$ is the angular frequency of the AC source.

3. Capacitive Reactance (X_C):
   - The capacitive reactance is $X_C = \frac{1}{\omega C}$.

Impedance Calculation

The impedance Z of the series LCR circuit is the sum of the resistance and the net reactance (which is the difference between the inductive and capacitive reactances):

$$Z = R + j(X_L - X_C)$$

Where j is the imaginary unit. Substituting the expressions for X_L and X_C:

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The magnitude of the impedance $|Z|$ is given by:

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

This is the impedance of the LCR circuit.

Phasor Diagram

In a phasor diagram, the voltages across the resistor, inductor, and capacitor are represented as vectors (phasors). The phasor diagram helps visualize the phase relationships between the voltages and current.

1. Resistor (R): The voltage across the resistor is in phase with the current.
2. Inductor (L): The voltage across the inductor leads the current by 90 degrees.
3. Capacitor (C): The voltage across the capacitor lags the current by 90 degrees.

Here's a simple sketch of the phasor diagram:

```
    V_L = I jωL
       ↑
       |
       |
       |
```

|________ V_R = I R (In phase with I)
|
|
|
V_C = I (-j / ωC)

- V_R is horizontal, in phase with the current I.
- V_L is vertical and leads the current I by 90 degrees.
- V_C is vertical and lags the current I by 90 degrees.

The resultant voltage V is the vector sum of V_R, V_L, and V_C.

In conclusion, the impedance Z of an LCR circuit is:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Q. 22. Calculate the wavelength of the first two lines in Balmer series of hydrogen atom.

Solution:

To calculate the wavelengths of the first two lines in the Balmer series of the hydrogen atom, we use the Balmer formula. The Balmer series corresponds to electron transitions from higher energy levels $n \geq 3$ to the $n = 2$ level.

The wavelength λ of light emitted in the Balmer series is given by the Rydberg formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where:
- R_H is the Rydberg constant, approximately $1.097 \times 10^7 \, \mathrm{m}^{-1}$.
- n is the principal quantum number of the higher energy level.

First Line of the Balmer Series (n=3 to n=2)

For the first line (n=3 to n=2):

$$\frac{1}{\lambda_1} = R_H \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = R_H \left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda_1} = R_H\left(\frac{9-4}{36}\right)$$

$$\frac{1}{\lambda_1} = R_H\left(\frac{5}{36}\right)$$

$$\lambda_1 = \frac{36}{5R_H}$$

$$\lambda_1 = \frac{36}{5 \times 1.097 \times 10^7}\,\mathrm{m}$$

$$\lambda_1 = 656.3\,\mathrm{nm}$$

Second Line of the Balmer Series (n=4 to n=2)

For the second line (n=4 to n=2):

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_2} = R_H\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\frac{1}{\lambda_2} = R_H\left(\frac{4-1}{16}\right)$$

$$\frac{1}{\lambda_2} = R_H\left(\frac{3}{16}\right)$$

$$\lambda_2 = \frac{16}{3R_H}$$

$$\lambda_2 = \frac{16}{3 \times 1.097 \times 10^7} \text{ m}$$

$$\lambda_2 = 486.1 \, \text{nm}$$

Q. 23. A current carrying toroid winding is internally filled with lithium having susceptibility $\chi = 2.1 \times 10^{-5}$. What is the percentage increase in the magnetic field in the presence of lithium over that without it?

Solution:

To find the percentage increase in the magnetic field in the presence of lithium with a given magnetic susceptibility $\chi = 2.1 \times 10^{-5}$, we need to compare the magnetic field with and without lithium.

Magnetic Field Without Lithium:

The magnetic field inside a toroid without any magnetic material (just air or vacuum) is given by:

$$B_0 = \mu_0 n I$$

where:
- B_0 is the magnetic field without the magnetic material.
- μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7} \, \text{H/m}$).
- n is the number of turns per unit length.
- I is the current through the winding.

Magnetic Field With Lithium:

When the toroid is filled with a magnetic material with magnetic susceptibility χ, the relative permeability μ_r of the material is given by:

$$\mu_r = 1 + \chi$$

The magnetic field inside the toroid with lithium is then:

$$B = \mu_0 \mu_r n I = \mu_0 (1 + \chi) n I$$

Percentage Increase in the Magnetic Field:

The percentage increase in the magnetic field is calculated as:

$$\text{Percentage Increase} = \left(\frac{B - B_0}{B_0} \right) \times 100\%$$

Substituting B and B_0:

$$\text{Percentage Increase} = \left(\frac{\mu_0(1+\chi)nI - \mu_0 nI}{\mu_0 nI}\right) \times 100\%$$

Simplifying the expression:

$$\text{Percentage Increase} = \left(\frac{\mu_0 nI(1+\chi) - \mu_0 nI}{\mu_0 nI}\right) \times 100\%$$

$$\text{Percentage Increase} = \left(\frac{\mu_0 nI + \mu_0 nI\chi - \mu_0 nI}{\mu_0 nI}\right) \times 100\%$$

$$\text{Percentage Increase} = \left(\frac{\mu_0 nI\chi}{\mu_0 nI}\right) \times 100\%$$

$$\text{Percentage Increase} = \chi \times 100\%$$

Given $\chi = 2.1 \times 10^{-5}$:

$$\text{Percentage Increase} = 2.1 \times 10^{-5} \times 100\%$$

$$\text{Percentage Increase} = 2.1 \times 10^{-3}\%$$

Q. 24. The radius of a circular track is $200 \mathrm{~m}$. Find the angle of banking of the track, if the maximum speed at which a car can be driven safely along it is $25 \mathrm{~m/sec}$.

Solution:

To find the angle of banking of a track given the radius and the maximum speed at which a car can be driven safely, we can use the concept of banking of a road in circular motion.

Given:
- Radius of the track, $r = 200$ m
- Maximum speed, $v = 25$ m/s

Formula for Banking Angle

The banking angle θ can be found using the formula:

$$\tan\theta = \frac{v^2}{rg}$$

where:
- v is the speed of the vehicle,
- r is the radius of the circular track,
- g is the acceleration due to gravity (approximately 9.8 m/s^2).

Calculation

Substitute the given values into the formula:

$$\tan\theta = \frac{25^2}{200 \times 9.8}$$

Simplify the expression:

$$\tan\theta = \frac{625}{1960}$$

$$\tan\theta = \frac{625}{1960} \approx 0.3189$$

To find the angle θ, take the arctangent of both sides:

$$\theta = \tan^{-1}(0.3189)$$

Using a calculator:

$$\theta \approx 17.65^\circ$$

Q. 25. Prove the Mayer's relation : $C_p - C_v = \frac{R}{J}$

Solution:

To prove Mayer's relation, which states $C_p - C_v = \frac{R}{J}$, we start with the basic definitions and relationships in thermodynamics.

Definitions and Relationships
Heat capacities:
- C_p: Heat capacity at constant pressure
- C_v: Heat capacity at constant volume

Ideal Gas Law:

$$PV = nRT$$

where:
- P is the pressure
- V is the volume

- n is the number of moles
- R is the universal gas constant
- T is the temperature

First Law of Thermodynamics:

$$dQ = dU + PdV$$

where:
- dQ is the heat added to the system
- dU is the change in internal energy
- PdV is the work done by the system

Proof

At Constant Volume:
- At constant volume, $dV = 0$.
- Therefore, the heat added at constant volume is:

$$dQ_v = dU$$

- The heat capacity at constant volume is:

$$C_v = \left(\frac{dQ}{dT}\right)_v = \left(\frac{dU}{dT}\right)_v$$

At Constant Pressure:
- At constant pressure, the heat added is:

$$dQ_p = dU + PdV$$

- The heat capacity at constant pressure is:

$$C_p = \left(\frac{dQ}{dT}\right)_p$$

Using the first law of thermodynamics and the definition of enthalpy $H = U + PV$, we have:

$$dH = dU + PdV + VdP$$

For a constant pressure process, $dP = 0$, so:

$$dH = dU + PdV$$

Thus, the heat added at constant pressure is:

$$dQ_p = dH$$

$$C_p = \left(\frac{dH}{dT}\right)_p$$

Relation between C_p and C_v:

For an ideal gas, the internal energy U is a function of temperature only. Thus, the change in internal energy can be expressed as:

$$dU = C_v dT$$

The enthalpy H is given by:

$$H = U + PV$$

For an ideal gas, using the ideal gas law $PV = nRT$:

$$H = U + nRT$$

Taking the differential:

$$dH = dU + nRdT$$

Using $dU = C_v dT$:

$$dH = C_v dT + nRdT$$

$$dH = (C_v + nR)dT$$

Therefore:

$$C_p = C_v + nR$$

Since n is the number of moles and R is the gas constant, we can write:

$$C_p - C_v = nR$$

In terms of one mole of gas ($n = 1$):

$$C_p - C_v = R$$

Introducing the Mechanical Equivalent of Heat:

The mechanical equivalent of heat J relates the unit of energy in terms of work (joules) to the unit of energy in terms of heat (calories). Therefore:

$$C_p - C_v = \frac{R}{J}$$

This is Mayer's relation:

$$C_p - C_v = \frac{R}{J}$$

Thus, we have proven Mayer's relation.

Q. 26. An alternating voltage is given by $e = 8 \sin 628.4\text{t}$. Find
(i) peak value of e.m.f.
(ii) frequency of e.m.f.
(iii) instantaneous value of e.m.f. at time $t = 10\text{ ms}$.

Solution:

Given the alternating voltage $e = 8\sin(628.4t)$, we need to find:

1. The peak value of the e.m.f.
2. The frequency of the e.m.f.
3. The instantaneous value of the e.m.f. at $t = 10$ ms.

1. Peak Value of e.m.f.
The peak value of an alternating voltage $e(t) = E_0 \sin(\omega t)$ is E_0.

In this case, $E_0 = 8$.

Thus, the peak value of the e.m.f. is 8 volts.

2. Frequency of e.m.f.
The general form of the alternating voltage is $e(t) = E_0 \sin(\omega t)$, where ω is the angular frequency.

We are given $\omega = 628.4\,\text{rad/s}$.

The frequency f is related to the angular frequency ω by:

$$\omega = 2\pi f$$

So,

$$f = \frac{\omega}{2\pi} = \frac{628.4}{2\pi}$$

$$f \approx \frac{628.4}{6.2832} \approx 100\,\text{Hz}$$

Thus, the frequency of the e.m.f. is 100 Hz.

3. Instantaneous Value of e.m.f. at $t = 10$ ms
The instantaneous value of the e.m.f. $e(t)$ at any time t is given by:

$$e(t) = 8\sin(628.4t)$$

Substitute $t = 10$ ms $= 0.01$ seconds:

$$e(0.01) = 8\sin(628.4 \times 0.01)$$

$$e(0.01) = 8\sin(6.284)$$

Since 6.284 radians is approximately 2π radians, and $\sin(2\pi) = 0$:

$$e(0.01) = 8\sin(6.284) \approx 8\sin(2\pi) = 8 \times 0 = 0$$

Thus, the instantaneous value of the e.m.f. at $t = 10$ ms is 0 volts.

SECTION- D

Solution:

To address the question about transformers, we'll go through the definition, construction, working principle, and derive the equation for a transformer.

What is a Transformer?

A transformer is an electrical device used to transfer electrical energy between two or more circuits through electromagnetic induction. It changes the voltage level of alternating current (AC) in a circuit without changing its frequency.

Construction of a Transformer

A typical transformer consists of:

1. **Core:**
   - Made of laminated silicon steel to reduce eddy current losses.
   - Provides a path for the magnetic flux.
2. **Windings:**
   - **Primary Winding:** Connected to the input AC source.
   - **Secondary Winding:** Connected to the output load.
   - The number of turns in the windings determines the voltage transformation ratio.
3. **Insulation:**
   - Used to isolate the windings from each other and the core.

Working Principle of a Transformer

The transformer works on the principle of electromagnetic induction, specifically mutual induction. When an AC voltage is applied to the primary winding, it creates an alternating magnetic flux in the core. This changing magnetic flux induces an electromotive force (emf) in the secondary winding according to Faraday's law of electromagnetic induction.

Derivation of Transformer Equations

The emf induced in the windings can be derived using Faraday's law. Let:

- NpN_pNp be the number of turns in the primary winding.
- NsN_sNs be the number of turns in the secondary winding.
- VpV_pVp be the voltage in the primary winding.
- VsV_sVs be the voltage in the secondary winding.
- IpI_pIp be the current in the primary winding.
- IsI_sIs be the current in the secondary winding.

Q. 28. Using the geometry of the double slit experiment, derive the expression for fringe width of interference bands.

Solution:

To derive the expression for the fringe width of interference bands using the geometry of the double-slit experiment, we need to analyze the interference pattern formed by coherent light passing through two closely spaced slits.

Geometry of the Double-Slit Experiment

Slit Separation and Screen Distance:
- Let d be the separation between the two slits.
- Let D be the distance between the slits and the screen where the interference pattern is observed.
- Let λ be the wavelength of the light used.

Path Difference:
- Consider two points S_1 and S_2 as the positions of the slits.
- Light waves from these slits travel different distances to a point P on the screen.
- The path difference between the two waves reaching point P is given by:

$$\Delta x = S_2P - S_1P$$

- For constructive interference (bright fringe), the path difference should be an integer multiple of the wavelength:

$$\Delta x = n\lambda$$

where n is an integer (0, 1, 2, ...).

Small Angle Approximation:
- When $D \gg d$, the angle θ between the central axis and the point P is small.
- For small θ:

$$\sin\theta \approx \tan\theta \approx \frac{y_n}{D}$$

where y_n is the distance from the central maximum to the n-th bright fringe on the screen.

Derivation of Fringe Width

Path Difference and Angle:
- The path difference Δx can also be expressed in terms of θ:

$$\Delta x = d\sin\theta$$

Constructive Interference Condition:
- Combining the conditions for constructive interference and the path difference:

$$d\sin\theta = n\lambda$$

- Using the small angle approximation:

$$d\frac{y_n}{D} = n\lambda$$

Position of Bright Fringes:
- Solving for y_n:

$$y_n = \frac{n\lambda D}{d}$$

- The distance y_n represents the position of the n-th bright fringe from the central maximum.

Fringe Width (Separation Between Consecutive Bright Fringes):
- The fringe width β is the distance between two consecutive bright fringes (say y_{n+1} and y_n):

$$\beta = y_{n+1} - y_n$$

$$\beta = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d}$$

where:
- λ is the wavelength of the light used,
- D is the distance between the slits and the screen,
- d is the separation between the two slits.

This formula shows that the fringe width is directly proportional to the wavelength and the distance to the screen, and inversely proportional to the slit separation.

Q. 29. Distinguish between an ammeter and a voltmeter. (Two points eaeh).
The displacement of a particle performing simple harmonic motion is $\frac{1}{3}$ rd of its amplitude. What fraction of total energy will be its kinetic energy?

Solution:

Distinguish between an Ammeter and a Voltmeter

Ammeter:
1. Function: An ammeter is an instrument used to measure the electric current flowing through a circuit. It is connected in series with the circuit elements to measure the current accurately.
2. Internal Resistance: An ammeter has a very low internal resistance so that it does not significantly alter the current flowing through the circuit.

Voltmeter:
1. Function: A voltmeter is an instrument used to measure the electric potential difference (voltage) between two points in a circuit. It is connected in parallel with the circuit elements across which the voltage is to be measured.
2. Internal Resistance: A voltmeter has a very high internal resistance to ensure that it draws a minimal amount of current from the circuit, thereby not significantly affecting the circuit's operation.

Kinetic Energy in Simple Harmonic Motion

The displacement x of a particle performing simple harmonic motion is given as $\frac{1}{3}$ of its amplitude A.

We need to find the fraction of the total energy that is kinetic energy when the displacement is $\frac{A}{3}$.

Total Energy in Simple Harmonic Motion

The total energy E in simple harmonic motion is given by the sum of kinetic energy (KE) and potential energy (PE):

$$E = \frac{1}{2}kA^2$$

where k is the spring constant and A is the amplitude.

Potential Energy

The potential energy PE at displacement $x = \frac{A}{3}$ is:

$$PE = \frac{1}{2}kx^2$$

$$PE = \frac{1}{2}k\left(\frac{A}{3}\right)^2$$

$$PE = \frac{1}{2}k\frac{A^2}{9}$$

$$PE = \frac{1}{18}kA^2$$

The kinetic energy KE is the difference between the total energy and the potential energy:

$$KE = E - PE$$

$$KE = \frac{1}{2}kA^2 - \frac{1}{18}kA^2$$

$$KE = \frac{1}{2}kA^2\left(1 - \frac{1}{9}\right)$$

$$KE = \frac{1}{2}kA^2\left(\frac{9}{9} - \frac{1}{9}\right)$$

$$KE = \frac{1}{2}kA^2\left(\frac{8}{9}\right)$$

$$KE = \frac{4}{9}\left(\frac{1}{2}kA^2\right)$$

Since $\frac{1}{2}kA^2$ is the total energy E:

$$KE = \frac{4}{9}E$$

The fraction of the total energy that is kinetic energy when the displacement is $\frac{1}{3}$ of its amplitude is $\frac{4}{9}$.

Q. 30. Draw a neat labelled diagram of Ferry's perfectly black body. Compare the rms speed of hydrogen molecules at $227^{\circ}\mathrm{C}$ with rms speed of oxygen molecule at $127^{\circ}\mathrm{C}$. Given that molecular masses of hydrogen and oxygen are 2 and 32 respectively.

Solution:

Ferry's perfectly black body is an idealized physical object that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. A black body in thermal equilibrium emits electromagnetic radiation called black-body radiation.

Here's a labeled diagram of Ferry's perfectly black body:

```plaintext
 ______________________
|                      |
|      Black body      |
|      cavity          |
|______________________|
      |      |
      |      |
  Small hole  (1)
```

1. Black Body Cavity: The internal walls are coated with a material that absorbs all incident radiation.
2. Small Hole (Aperture): This is the only opening through which radiation can enter or leave the cavity. The hole acts as an ideal absorber since radiation entering the hole has a very low probability of escaping.

2. Comparing RMS Speeds of Hydrogen and Oxygen Molecules

The root mean square (RMS) speed of gas molecules is given by:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where:
- v_{rms} is the root mean square speed.
- k is the Boltzmann constant.
- T is the temperature in Kelvin.
- m is the mass of one molecule of the gas.

Converting Temperatures to Kelvin

- Temperature of hydrogen gas: $T_H = 227^{\circ}C = 227 + 273 = 500\,K$
- Temperature of oxygen gas: $T_O = 127^{\circ}C = 127 + 273 = 400\,K$

Molar Masses and Mass of Molecules

- Molar mass of hydrogen (H_2): $M_H = 2\,\mathrm{g/mol}$
- Molar mass of oxygen (O_2): $M_O = 32\,\mathrm{g/mol}$

The mass of a single molecule m can be found by dividing the molar mass M by Avogadro's number N_A:

$$m_H = \frac{M_H}{N_A} = \frac{2\,\text{g/mol}}{6.022 \times 10^{23}\,\text{mol}^{-1}} = \frac{2 \times 10^{-3}\,\text{kg}}{6.022 \times 10^{23}}$$

$$m_O = \frac{M_O}{N_A} = \frac{32\,\text{g/mol}}{6.022 \times 10^{23}\,\text{mol}^{-1}} = \frac{32 \times 10^{-3}\,\text{kg}}{6.022 \times 10^{23}}$$

RMS Speed Calculation

Using the formula $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$:

For hydrogen (H_2):

$$v_{\text{rms,H}} = \sqrt{\frac{3k \cdot 500}{m_H}}$$

For oxygen (O_2):

$$v_{\text{rms,O}} = \sqrt{\frac{3k \cdot 400}{m_O}}$$

We don't need the exact numerical values of k and m because we can compare the speeds by ratio.

Ratio of RMS Speeds

$$\frac{v_{\text{rms,H}}}{v_{\text{rms,O}}} = \sqrt{\frac{3k \cdot 500}{m_H}} \div \sqrt{\frac{3k \cdot 400}{m_O}}$$

$$\frac{v_{\text{rms,H}}}{v_{\text{rms,O}}} = \sqrt{\frac{500 \cdot m_O}{400 \cdot m_H}}$$

Substituting $m_H = \frac{2}{6.022 \times 10^{23}}$ and $m_O = \frac{32}{6.022 \times 10^{23}}$:

$$\frac{v_{\text{rms,H}}}{v_{\text{rms,O}}} = \sqrt{\frac{500 \times 32}{400 \times 2}}$$

$$\frac{v_{\text{rms,H}}}{v_{\text{rms,O}}} = \sqrt{\frac{500 \times 16}{400}}$$

$$\frac{v_{\text{rms,H}}}{v_{\text{rms,O}}} = \sqrt{20}$$

$$\frac{v_{\text{rms,H}}}{v_{\text{rms,O}}} = \sqrt{5} \approx 2.236$$

- The RMS speed of hydrogen molecules at 227°C is approximately 2.236 times the RMS speed of oxygen molecules at 127°C.

Q. 31. Derive an expression for energy stored in a charged capacitor. A spherical metal ball of radius $15\ \text{cm}$ carries a charge of $2\mu\text{C}$. Calculate the electric field at a distance of $20\ \text{cm}$ from the center of the sphere.

Solution:

Expression for the Energy Stored in a Charged Capacitor

The energy (U) stored in a charged capacitor can be derived as follows:

Capacitance (C):
The capacitance of a capacitor is given by:

$$C = \frac{Q}{V}$$

where Q is the charge on the capacitor and V is the voltage across the capacitor.

Energy (U):
The work done in charging the capacitor to a charge Q and voltage V is stored as the energy in the capacitor.

The differential work done to move a small charge dq from one plate to another is:

$$dW = V\,dq$$

Since $V = \frac{q}{C}$:

$$dW = \frac{q}{C}\,dq$$

Total Energy Stored:
To find the total energy, integrate this expression from 0 to Q:

$$U = \int_0^Q \frac{q}{C}\,dq$$

$$U = \frac{1}{C} \int_0^Q q\, dq$$

$$U = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q$$

$$U = \frac{1}{C} \cdot \frac{Q^2}{2}$$

$$U = \frac{Q^2}{2C}$$

This is one form of the expression for the energy stored in a charged capacitor. Other forms, using the relationship $Q = CV$, are:

$$U = \frac{1}{2} C V^2$$

or

$$U = \frac{1}{2} Q V$$

Calculate the Electric Field at a Distance of 20 cm from the Center of the Sphere

Given:
- Radius of the spherical metal ball: $r = 15\,\text{cm} = 0.15\,\text{m}$
- Charge on the sphere: $Q = 2\,\mu\text{C} = 2 \times 10^{-6}\,\text{C}$
- Distance from the center: $R = 20\,\text{cm} = 0.2\,\text{m}$

The electric field E at a distance R from the center of a charged sphere (for $R > r$) is given by:

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$$

where ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$).

Substitute the given values:

$$E = \frac{1}{4\pi \times 8.854 \times 10^{-12}} \frac{2 \times 10^{-6}}{(0.2)^2}$$

Calculate the denominator first:

$$4\pi \times 8.854 \times 10^{-12} = 1.11265 \times 10^{-10}$$

Now calculate $(0.2)^2$:

$$(0.2)^2 = 0.04$$

Thus:

$$E = \frac{2 \times 10^{-6}}{1.11265 \times 10^{-10} \times 0.04}$$

$$E = \frac{2 \times 10^{-6}}{4.4506 \times 10^{-12}}$$

$$E = 4.49 \times 10^{5}\,\mathrm{N/C}$$