CBSE Class 12 Maths

Question Bank

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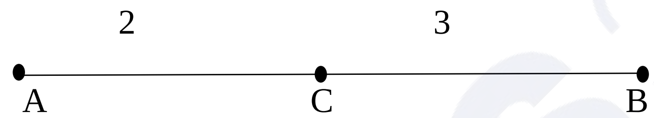
CONSTRUCTION

(Marks 4)

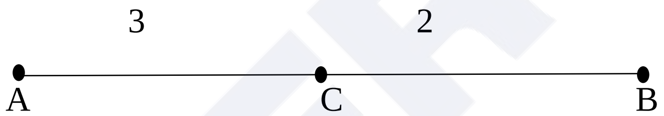
➢ ***Division of a line segment and construction of a Triangle***

Division of a line Segment:

Here we will divide a given line segment AB (say) in two segments "AC" and "CB" (say), such that C divides "AB" internally in the given ratio. e.g. If AC:CB = 2:3, then C divides AB internally in the ratio 2:3.



Here, AC = $\frac{2}{3}$ AB and CB = $\frac{3}{5}$ AB.

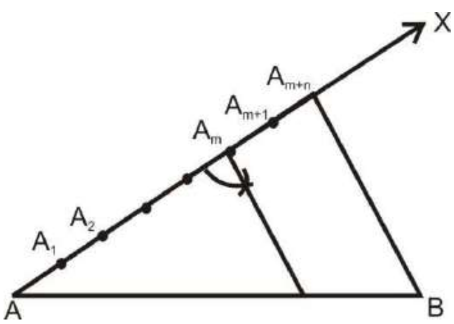


And if the ratio is 3:2, then AC = $\frac{3}{5}$ AB and CB = $\frac{2}{5}$ AB.

Construction – 1:

➢ ***To divide a line segment internally in the given ratio.***

To divide a line segment AB (Say) internally in the given ratio m : n, where m and n are both positive integers. We use the following steps:



Step – 1 : Draw the given *line segment AB* and any *ray AX*, making an *acute angle* with the *line segment AB*. This *ray AX* can be *draw above* or *below AB*.

Step – 2 : Mark $m + n = p$ points ($A_1, A_2, \ldots A_m, \ldots A_P$) on the *ray AX*, such that $AA_1 = A_1A_2 = \ldots A_{P-1}, A_P$.

Step – 3 : Join BA.

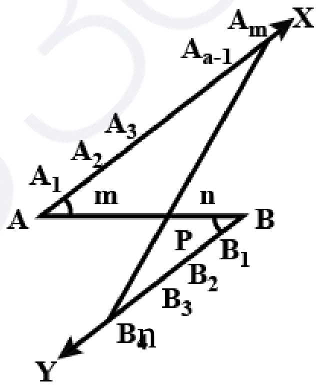
Step – 4 : Through the *point* A_m, draw a line *parallel to* A_PB (by making an *angle equal to* $<AA_PB$ *at A*) which *intersects* the *line segment AB at point C*. Thus *C divides* the *line segment AB internally* in the ratio *m : n* i.e. *AC:AB = m : n*.

Alternate Method of Construction

To divide a line segment AB (Say) internally in the given ratio m : n, where m and n are both positive integers. We use the following steps:

Step – 1 : Draw the given *line segment AB (say)* and *any ray AX* making an *acute angle* with the *line segment AB.*

Step – 2 : Draw *another ray BY // AX* by making $<ABY = < BAX$.



Step –3 : *Mark off m points* $A_1, A_2, \ldots A_n$ *on AX and n* points $B_1, B_2 \ldots . B_n$ *on BY* such that $AA_1 = A_1A_2 = \ldots . A_{m-1}A_m = BB_1 = B_1B_2 = \ldots B_{n-1}B_n$.

Step – 4 : Join A_mB_n which *intersects line segment AB* at the *point C*. Now, *C is required point* which divides *line segment "AB" internally* in the *ratio m : n.*

Construction – 2 :

- ***To construct a triangle similar to a given triangle as per the given scale factor.***

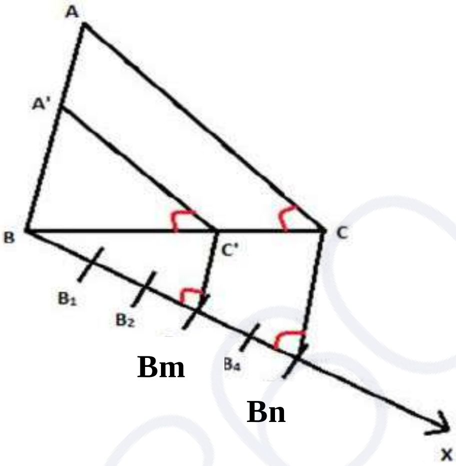
Scale Factor: *The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is called the scale factor*. Generally, it is written as $\frac{m}{n}$ which may be less than "1" or greater than 1.

Thus two different situations to construct a triangle depends on scale factor arise which are discussed below:

a) If $\frac{m}{n} < 1$ or "m" < n then the sides of the triangle to be constructed will be smaller than corresponding sides of the given triangle.

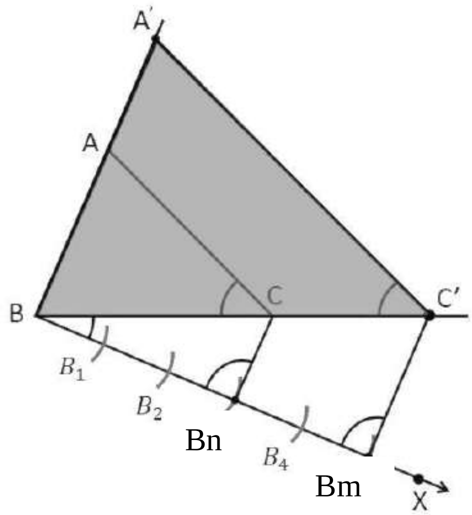
b) If $\frac{m}{n} > 1$ or m > n, then the sides of the triangle to be constructed will be larger than the corresponding sides of the given triangle.

Condition – 1 When $\frac{m}{n} < 1$ or m < n. To construct $A'BC' \sim ABC$, we use the following steps.



- ☞ Take BC as base and draw Δ ABC of given measurement.
- ☞ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ☞ Find the value of m and n, then mark off n points (the greater of m & n in $\frac{m}{n}$) on BX such that $BB_1 = B_1B_2 \ldots = B_{m1}B_n$).
- ☞ Join B_nC and draw a line through Bm (m being smaller of m & n in $\frac{m}{n}$) parallel to B_nC to intersect BC at C' . Then $B_mC' \parallel$ "B_nC".
- ☞ Draw a line through C' parallel to CA which intersects BA at A'. Then $C'A' \parallel CA$.
- ☞ Then A' B C' is the required triangle similar to given Δ ABC as per scale factor $\frac{m}{n}$ <1.

Condition – 2 When $\frac{m}{n} > 1$ or m > n. To construct $\Delta A'BC' \sim \Delta ABC$, when $\frac{m}{n} > 1$, we use the following steps.



- ❖ Take BC as base and draw a Δ ABC of given measurement.
- ❖ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ❖ Find the value of m and n, then mark off m points (the greater of m & n in $\frac{m}{n}$) $B_1B_2 \ldots B_m$ on BX such that $BB_1 = B_1B_2 \ldots = B_{m-1}B_m$).
- ❖ Join B_nC (n being smaller of m & n in $\frac{m}{n}$) and draw a line through $B_m \parallel BC$ to intersect the extended line segment BC at C'.
- ❖ Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.
- ❖ Then A' B C' is the required triangle.

IMPORTANT CONSTRUCTIONS OF A TRIANGLE IN DIFFERENT CASES

Before constructing a triangle similar to a given triangle say Δ ABC, we have to construct the Δ ABC. For this, let us review some constructions in different cases which are given below:

CASE – 1 : When three sides are given.

We use the following steps of construction:

Step – 1 : First draw base BC.

Step – 2 : By taking B and C as centers, draw two arcs of radius CA and BA respectively which intersects each other at A.

Step – 3 **:** Join AB and AC.

Then ABC is a required triangle.

CASE – 2 : When base and alternate of a triangle are given.

Let the base of Δ ABC be BC = m cm and Alternate is n cm. Then, we use the following steps.

Step – 1 : Draw the base BC = m cm and draw its perpendicular bisector OQ (say) which intersects BC at P (say).

Step – 2 : By taking P as center, draw an arc of radius n cm which intersects the line segment PO or extended line segment of PO at A.

Step – 3 : Join AB and AC.

Then ABC is a required triangle.

CASE – 3 : When two sides and angle between them are given.

Let the two sides of Δ ABC be AB = a cm, BC = b cm and angle between them i.e. <ABC = x^0. Then, we use the following steps.

Step – 1 : Draw base “AB” = “a” cm

Step – 1 : Draw a ray “BX” making an angle x^0 at B and cut off BC = b cm from BX.

Step – 3 : Join “AC”. Thus Δ ABC is required triangle.

CASE – 3 : When one side and two angles are given.

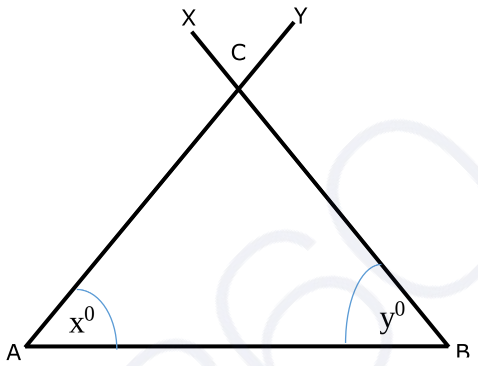
Let one side of triangle ABC be AB = a cm and < A = x^0 and < B = y^0

Step – 1 : Draw base AB = a cm

Step – 2 : Draw two rays AX and BY making an angle x^0 at "A" and y^0 at B which intersects each other at C.

Step – 3 : Join AC and CB.

Then ABC is a required triangle.



CONSTRUCTION OF TANGENTS TO A CIRCLE

Tangent: ***A tangent to a circle is a straight line which touches the circle at a point. This point is called point of contact and the radius through the point of contact is perpendicular to the tangent.*** The number of tangents drawn to a circle from a point depends on the position of the point with respect to circle.

a) If a point lies on the circle, then only one tangent at this point can be drawn.

b) If a point lies inside the circle, then no tangent can be drawn.

c) If a point lies outside the circle, then two tangents from this point can be drawn.

Construction of a tangent to a circle at a point lies on it. (By two methods)

Method – 1 : By using the Centre of Circle.

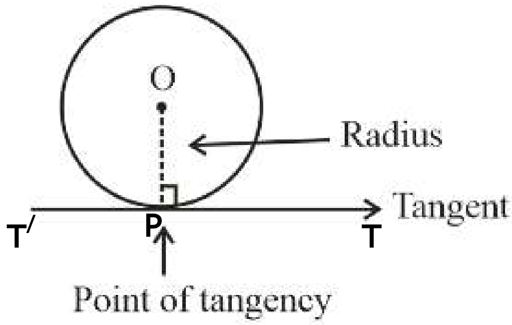
Step – 1 : Take a point O as center and draw a circle of given radius.

Step – 2 : Take a point P on the circle, at which we want to draw a tangent.

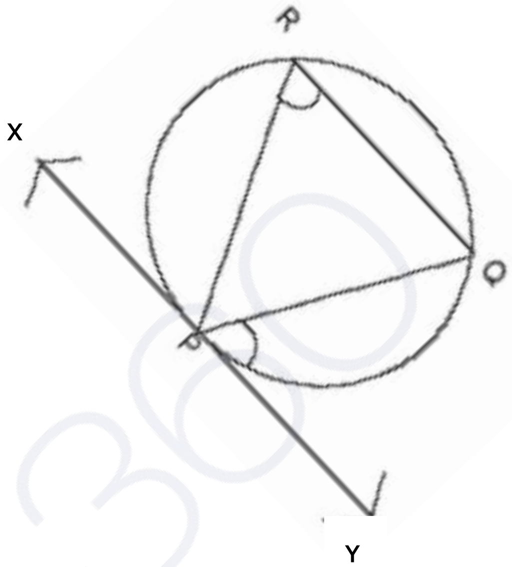
Step – 3 : Join OP, which is the radius of Circle.

Step – 4 : Take OP as base and construct < OPT = 90^0 at P.

Step – 5 : Produce "TP to T/ to get the required tangent TPT/



Method – 1I : ***Without using the center of Circle.***



Step – 1 : Draw a circle of given radius r cm and take a point P (at which we want to draw a tangent) on the circle.

Step – 2 : Draw any chord PQ through the given point P on the circle.

Step – 3 : Take a point R in either the major arc or minor arc and join PR and QR.

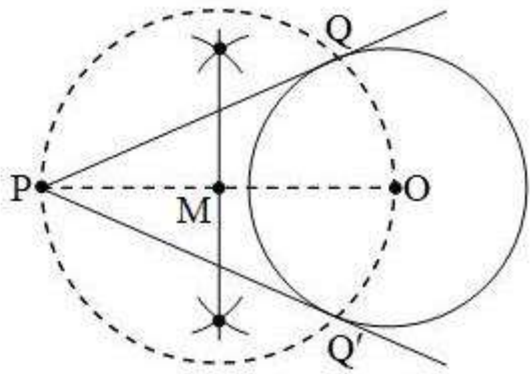
Step – 4 : Taking PQ as base, construct < QPY equal to < PRQ and on the opposite side of "R".

Step – 5 : Produce YP to X to get the tangent XPY.

Construction of Tangents to a circle from a point outside the circle:

If a point lies outside the circle, then there will be two tangents to the circle from this point. These tangents can be drawn in two cases which are given below:

CASE – 1 : When the center of circle is given.



Step – 1 : Draw a circle with center O and take a point P outside it.

Step – 2 : Join OP and bisect it. Let its mid-point be M, then MP = MO.

Step – 3 : Taking M as center and MO or MP as radius draw a dotted circle which intersects the given circle at Q and Q' (say).

Step – 4 : Join PQ and PQ'.

Thus PQ and PQ' are the required tangents drawn to the circle from the external point P. Here we observe that $PQ = PQ'$.

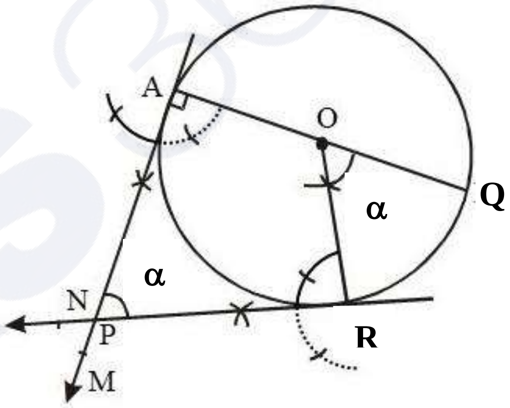
Case – II : When the center of the circle is unknown.

Step – 1 : First draw the circle and then two non-parallel chords of the circle.

Step – II : Draw the perpendicular bisectors of both chords which intersect each other at a point say O. Then this point O gives the center of given circle. Now use the steps given in case – 1 to draw the tangents

Construction of tangents to a circle when angle between them is given

Sometimes, angle between two tangents (or pair of tangents) is given and we have to draw the tangents. Then, we use following steps of constructions:



Step – 1 : First, draw the given circle with center O and radius r cm.

Step – 2 : Draw any diameter say AOQ of this circle.

Step – 3 : Make given angle α at center O with OQ (say) as base which intersects the circle at point R (say) or draw the radius or meet the circle at R such that < QOR = α

Step – 1 : Now, draw perpendiculars to OA at A" and OR at R which intersects each other at a point say P.

The AP and RP are the required pair to tangents to given circle inclined at angle α.

EXERCISE

Q.1 Divide a line segment of length 9.6 cm in the ratio of 5:3. Measure the two parts.

Q.2 Draw a line segment of length 7.7 cm and divide it in the ratio 3:4.

Q.3 Construct a triangle similar to given triangle ABC, where AB = 6 cm, BC = 7 cm and AC = 8 cm with its sides equal to $\frac{3}{4}$ of the corresponding sides of triangle ABC.

Q.4 Construct a triangle similar to a given triangle ABC whose sides are 6 cm, 7 cm and 8 cm with its sides equal to $\frac{5}{3}$ of the corresponding sides of a triangle ABC.

Q.5 Draw an equilateral triangle ABC of each side 4 cm. Construct a triangle similar to it and of scale factor $\frac{3}{5}$. Is the new triangle also an equilateral? *(Yes No)*

Q.6 Draw two tangents to a circle of radius 4 cm from a point "P" at a distance of 7 cm from its center.

Q.7 Draw two tangents from the end points of the diameter of a circle of radius 4 cm. Are these tangents parallel? *(Yes / No)*

Q.8 Draw a circle of radius 1 cm. From a point P, 2.2 cm apart from the center of the circle.

Q.9 Construct a pair of tangents to circle whose radius is 6.5 cm are inclined to each other at angle of 30^0.

Q.10 Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of 45^0.

Q.11 Construct a Δ ABC in which BC = 13 cm, CA = 5 cm, AB = 12 cm. Draw its incircle and measure it radius.

Q.12 Draw a pair of tangents to circle of radius 3 cm that are inclined to each other at an angle of 50^0.

Q.13 **Value Based Questions**

a) Sanjeev have a piece of cloth of 8 cm long. He decided to divide this piece into two "A" and "B" internally in the ratio of 3:4.

i) Draw a construction of above problem.

ii) If Sanjeev gave 4^{th} part of the piece of cloth to the person "A", then what value is violated by Sanjeev?

Q14 Geometrically divide line segment of length 8.4 cm in the ratio of 5:2.

Q.15 Construct a Δ ABC similar to a given isosceles Δ PQR with QR = 6 cm, PR=PR=5 cm such that each side is $6/7^{th}$ of the corresponding sides of Δ PQR.

Q.16 Draw a circle of diameter 8 cm, from a point "P" 7 cm away from its center. Construct a pair of tangents to the circle.

Q.17 Draw a right angled triangle ABC, in which BC = 12 cm, AB = 5 cm < B= 90^0. Then construct a triangle similar to it and of scale factor 2/3, is the new triangle also a right angle triangle.

Q.18 Draw a circle of radius 4 cm. Construct a pair of tangents on it, so that they are inclined at 60^0.

Q.19 Draw a circle of radius 4 cm. Take two points P & Q on one of its extended diameters, each at a distance of 9 cm from its center. Draw tangents to the circle from these two points "P & Q".

Q.20 Construct a triangle ABC, in which AB = 5 cm, <A = 300 and AC = 6 cm. Then construct a triangle whose sides are 4/3 of the corresponding sides of triangle ABC.

Q.21 Draw a triangle ABC with BC = 7 cm , < B = 450 & < C = 600. Then construct another triangle whose sides are 3/5 times of the corresponding sides of triangle ABC& justify your construction.

1 Mark Questions

Q1. A line segment of length 20 cm is divided in the ratio of 2:3, the measure of the two parts in the given ratio respectively would be

a) 8 cm, 12 cm b) 4 cm, 8 cm c) 4 cm, 12 cm d) 4cm, 9 cm

Q.2 A line segment of length 25 cm in divided in the ratio2:3, the measure of the two parts in the given ratio respectively would be

a) 15 cm, 10 cm b) 10 cm, 15 cm c) 9 cm, 6 cm d)None

Q.3 To construct a pair of tangents to a circle at an angle of 600 to each other, it is to draw tangents at end points of those two radii of the circle, the angle between them should be

a) 100^0 b) 90^0 c) 180^0 d) 120^0

Q.4 A pair of tangents can be constructed from a point to a circle of radius 3.5 cm situated at a distance of ………….. from the center

a) 3.5 cm b) 2.5 cm d) 5 cm d) 2 cm

Q.5 To construct a triangle ABC and then a triangle similar to it whose sides are 2/3 of the corresponding sides of the first triangle. A ray "AX is drawn where multiple points at equidistance are located. The last point to which point "B" will meet the ray AX will be

a) A_1 b) A_2 c) A_3 d) A_4

Q.6 To draw a pair of tangents to a circle which are inclined to each other at an angle of 45^0. It is required to draw tangents at the end points of those two radii of the circle, the angle between

a) 135^0 b) 155^0 c) 160^0 d) 120^0

Q.7 How many tangents can be drawn from a point out to a circle?

a) 1 b) 2 c) 3 d) None

Q.8 The lengths of two tangents drawn from a point outside to a circle are ……….. (equal /not equal).

ANSWER

Q.1	(a)	Q.2	(b)	Q.3	(d)	Q.4	(c)
Q.5	(c)	Q.6	(a)	Q.7	(b)	Q.8	equal

CLASS X Real numbers:-

EUCLIDS Division Lemma and Algorithm

Euclid's division lemma tells us about divisibility of integers. It states that any positive integer "**a'** can be divided by any other positive integer **'b'** in a such a way that it leaves a remainder r (where r<b), We may recognize this as the usual long division process ;

Euclid's division lemma provides a stepwise procedure to compute the HCF of any two +ve integers which is known as Euclid's division algorithm.

Euclid's division lemma : states that, for any two positive integers say **a and b ,** there exists two unique whole numbers say **q and r**, such that

a=bq + r , where 0 ≤ r < b

Here a = Dividend , b=Divisor , q=Quotient and r = Reminder
It can be written as
Dividend= (Divisor ×Quotient) +Reminder
Note : A Lemma is a proven statement used for proving another statement.
Euclid's Division Algorithm :
Euclid's division algorithm is a technique to complete the highest common factor (HCF) of two or three given +ve integers.

According to this, the HCF of any two positive integers a and b with a> b is obtained as follows.

Step I Apply the division lemma to find q and r where a≠ bq + r, 0≤ r < b.

Step II If r=0 , the HCF is b If r≠0 apply Euclid's lemma to b and r .

Step III Continue the process till the remainder is zero .The divisor at this stage will be HCF (a,b)

Methods of Finding HCF of three numbers by using Euclid's Division Algorithm

Example : Find the HCF of 180, 252 and 324 by using Euclid's Division Lemma.

Step I Arrange the given three numbers be a, b, c such that a>b>c .

Given numbers are 180 , 252 and 324.

324 > 252 > 180.

Step II HCF of two numbers a and b by EDA say d
Now on applying EDL for 324 and 252 . we get
324 = (252 x 1) +72
Here 72 ≠ 0
So again apply EDL with dividend 252 and new divisor 72, we get
252= 72 x 3 + 36

Here remainder = 36 ≠ 0

So again applying EDL with new dividend 72 and new divisor 36, we get
72 = 36 x 2 + 0.
Here R = 0.
So HCF of 324 and 252 is 36.

Step III:- Now , find the HCF of two numbers (remaining numbers) and d
(HCF of a and b by EDA.
So applying EDL for 180 and 36 .
We get 180 = 36 ×5 +0
R=0
So HCF of 180 and 36 is 36
Step IV The HCF obtained in step 3 is required HCF of given three numbers
Hence HCF of 324 , 252 and 180 is 36.

Fundamental Theorem of Arithmetic Prime and Composite numbers’

A number is called a prime number if it has no factor other than 1 and the number itself
e.g. 2,3,5,7,13,19…. are prime numbers.
A number is called a composite number if it has at least one factor other than 1 and itself .
e.g. 4,6,8,10,…. 188 are composite numbers.

Note:- 1 is neither prime nor composite.
2, is the smallest prime number . It is the only even number.
The smallest composite even number is 4 and smallest composite odd number is 9.
Prime number have only two factors 1 and itself.
Factor of a Number
If a number divides another number exactly (without leaving any remainder), than the number which divides is called a factor of the number that has been divided.

e.g 1,2,3,4,6,12 are the factors of 24.

Fundamental Theorem Of Arithmetic

According to the Fundamental theorem of arithmetic ,every composite number can be written (factorized) as the product of primes and this factorization is unique , apart from the order in which the prime factors occur.

Fundamental Theorem of Arithmetic is also called Unique factorization Theorem.

Composite number = Product of prime numbers.

OR

An integer greater than 1 , either be a prime no or can be written as a product of prime factors.

HCF and LCM
By Prime Factorization Method:

The method of finding HCF and LCM of two or more positive numbers by using Fundamental theorem of arithmetic is called prime factorization method .In this method we first express the given two or more no's as the product of prime factors separately ,then

1. HCF of two or more numbers = Product of the smallest power of each common prime factor involved in the numbers.
2. LCM of two or more numbers = Products of the greatest power of each prime factor involved in the numbers , with highest power.

Relation between numbers and their HCF and LCM .

1. For any two +ve integers a and b, the relation between these numbers and their HCF and LCM is

$$HCF(a, b) \times LCM(a, b) = a \times b$$

$$HCF(a, b) = \frac{a \times b}{LCM(a, b)}$$

Or

$$LCM(a, b) = a \times b \ \frac{a \times b}{HCF(a, b)}.$$

2. For any three positive integers a , b and c , the relation between these numbers and their HCF and LCM is

$$HCF(a, b, c) = \frac{a \times b \times c \times LCM(a, b, c)}{LCM(a, b) \times LCM(b, c) \times LCM(c, a)}$$

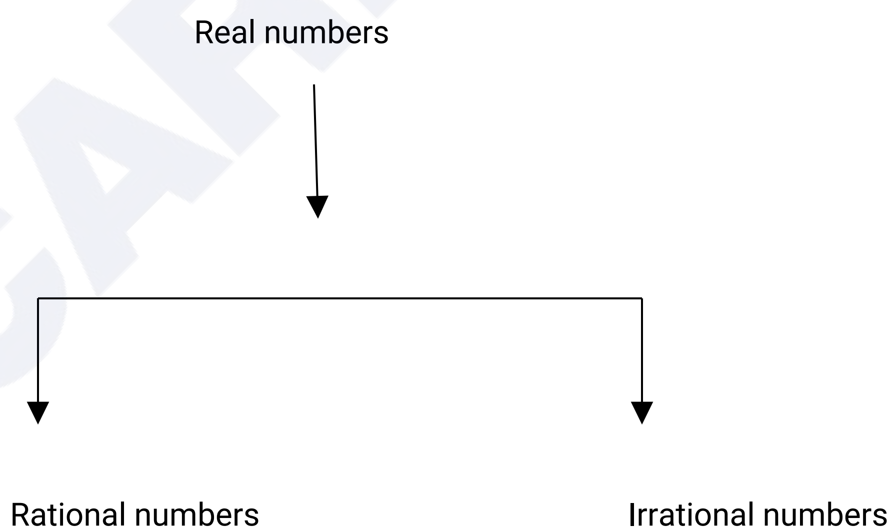
Or

$$LCM(a, b, c) = \frac{a \times b \times c \times HCF(a, b, c)}{HCF(a, b) \times HCF(b, c) \times HCF(c, a)}$$

Revisiting Rational and Irrational Numbers , Decimal Expansion of Rational numbers.

Real numbers:

A number which is either rational or irrational is called a real number . Different components of real numbers can be understood with the help of following flow diagram



$-3, 0, 7, \frac{11}{2}, \frac{-3}{5}, \frac{4}{7}....$ $\sqrt{2},\ 5\sqrt{3},\ \sqrt{6},\ 2\sqrt{5}......$

Integers

.. -2,-1,0,1,2....

Non integers

$\frac{11}{2}, \frac{-5}{7}, \frac{-3}{5}$

Negative integers

.....-3, -2 , -1

Whole no's (non-negative integers)

0, 1 , 2 , 3 .....

Zero

1

Natural numbers

Positive Integers

1, 2 , 3 , ...

Rational numbers:- A number that can be expressed as $\frac{p}{q}$ where p, q are integers and q≠0 is called a rational number .

e.g $\frac{3}{5}$ $\frac{7}{9}$, $\frac{13}{15}$, $\frac{-7}{3}$ etc..

Irrational numbers : A number that cannot be expressed in the form $\frac{p}{q}$ where p , q are integers and q ≠0 is called irrational number.

e.g $\sqrt{2}$, $\sqrt{3}$, $\sqrt{15}$, π , $\frac{\sqrt{2}}{\sqrt{3}}$,0.1011011101111.......etc.

Co prime integers: A pair of integers having no common factors other than 1 (or -1) are called co prime integers.

e.g (1, 3) , (-2 , 5) , (6, 35) etc are co prime integers.

Theorem : Let p be a prime number and a be a positive integer .If p divides a^2 then p divides a

Given : Let p be a prime number and a be a positive integer such that p divides a^2.

To prove : P divides a.

Proof: We know that every positive integer can be expressed as the product of primes

So let a = p_1 , p_2 , …..p_n.

Where p_1, p_2, p_3 …p_n are primes not necessarily all distinct.

Then a^2 = (p_1 , p_2 … p_n) p_1 , p_2 , … p_n).

a^2 = (p_1, p_2 … p_n)

Now let p divides a^2 .

P is a prime factor of a^2.

P is one of p_1, p_2….p_n.

Since by using uniqueness of the fundamental theorem of arithmetic the only prime factor of a^2 are p_1 , p_2 … p_n

P divides a | a = p_1, p_2 … p_n|

Thus p divides a^2 | p divides a. Hence proved.

Questions : 1 Marks

Q1. Are the following statements true or false ?

(i) The number 3^n can end with the digit 0 for any $x \in N$.

Q2. (ii) Every +ve odd integer is of the form 2q +1 where q is some whole number.

Q3. Every composite number can be expressed as a product of

(A) Co primes (B) Primes (C) Twin primes (D) None .

Q4. The decimal expansion if a rational number is always

(A) Non – terminating (B) Non – terminating and non repeated (C) Terminating or non – terminating repeated (D) None

Q5. Prime factors if 4050 is

(A) $2 \times 3^3 \times 5$ (B) $2 \times 3^4 \times 5$ (C) $2 \times 3^4 \times 5^2$ (D) $2 \times 3^4 \times 5^3$

Q6. If P^2 is even integer then p is an

(A) Odd integer (B) Even integer (C) Multiple of 3 (D) None

Q7. ……………. Is a proven statement which is used to prove another statement .

(A) Lemma (B) Axiom (C) Theorem (D) Algorithm

Q8. The number $0.\overline{57}$ in the $\frac{p}{q}$ form q≠0 is

(A) $\frac{26}{45}$ (B) $\frac{13}{27}$ (c) $\frac{57}{99}$ (D) $\frac{13}{29}$

Questions : 2 Marks , 3 Marks , and 4 Marks.

1. A number when divided by 53 gives 34 as quotient and 21 as remainder .Find the number .
2. In Euclid's division lemma a= bq +r , where 0≤ r < b.What is a?
3. Use Euclid's division algorithm to find the HCF of (i) 650 and 1170. (ii) 870 and 225.
4. The product of two consecutive positive integers is divisible by 2. Is this statement true or false. Give reasons .
5. Use EDA to find the HCF of the following three no's 441 , 567, and 693.
6. Prove that $\sqrt{3}$ is an irrational numbers?
7. Prove that $\sqrt{2}$ is an irrational number?
8. Prove that $6+\sqrt{2}$ is irrational number.
9. Show that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number?
10. What is the condition for the decimal expansion of a rational number to terminate ? Explain with an example.
11. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

12. Write 98 as product of its prime factors.

13. If *a* is rational and $\sqrt{b}$ is irrational, then prove that (a+ ($\sqrt{b}$) is irrational.

14. Check whether 4^n can end with the digit 0 for any natural number n.

15. Sate Unique Factorization Theorem .

16. Check whether 3^n can end with digit 0 for any natural number n.

17. In EDL , the value of r , when a +ve integer a is divided by 3 are 0 and 1 only . Is this statement true or false ? Justify your answer.

18. For what value of n, $2^n \times 5^n$ ends with 5 ?

19. The product of three consecutive positive integers is divisible by 6. Is this statement true or false? Justify your answer .

20. 21. 22. Without actually performing the long division , State whether the following rational numbers will have a terminating decimal expansion or not .Also write the terminating decimal expansion , if exist

(i) $\frac{3}{8}$ (ii) $\frac{31}{343}$ (iii) $\frac{14588}{625}$

23. If $\frac{13}{125}$ is a rational number , find the decimal expansion of it, which terminates .

24. Explain, why (3 x 5 x 7) + 7 is a composite number?

25. Express 1001 as a product if its prime factors?

26. If HCF (6, a) = 2 . LCM (6, a) = 60. Than find the value of **a**

27.State Euclid's Division Lemma .

28. Find two rational numbers and two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$.

Answers of Objectives

01.True

02.True

03. Primes

04. C

05. a

06.b

07. a

08. c

Chapter 2

POLYNOMIALS

INTRODUCTION:

Expressions like 4x+2, $2y^2-3y+4$, $5x^3-4x^2+x-\sqrt{2}$; $7x^6-\frac{3}{2}x^4+4x^2+x-8$ etc are called Polynomials.

General form of a Polynomial: An algebraic

Expression of the form $P(x) = a_0+a_1+a_2+a_2x^2+a_3x^3+$- - - - - - - - - - - - $+a_nx^n$ where

$a_n \neq 0$ is called a Polynomial in variable x of degree n.

Here a_0, a_1, a_2, a_3- - - - - - - -, a_n are real numbers and each power of x is a non-negative Integer.

The highest Power in x of the Polynomial is called the degree of the Polynomial.

The representation of the Polynomial in the ascending or descending order of the powers of the polynomial is called the standard form of the Polynomial.

It is the degree of the Polynomial that classifies Polynomials. The different types of the Polynomial are as follows

- A Polynomial of degree Zero is called a Constant Polynomial.
- A Polynomial P(x) =ax+b of degree 1 is called a linear Polynomial.
- A Polynomial $P(x) = ax^2+bx+c$ of degree 2 is called quadratic Polynomial.
- A Polynomial $P(x) = ax^3+bx^2+cx+d$ of degree 3 is called a cubic Polynomial.
- A Polynomial $P(x) = ax^4+bx^3+cx^2+dx+e$ of degree 4 is called a bi-quadratic Polynomial

We also classify Polynomials on the basis of number of terms

1. **Zero Polynomial**: if all the terms of a Polynomial are zero`s e.g. $0.x^n+0.x^{n-1}+0.x^{n-2}+0.x^{n-3}+$- - -0.x+0 then it is called a zero Polynomial. The degree of a zero Polynomial is not defined.

2. **Monomial**: A Polynomial of one single term is called a monomial e.g. 2x, $\frac{3}{4}x^2y$; $2x^2yz$ etc.

3. **Binomial**: A Polynomial having two terms is called a binomial e.g. $2x+1$; $x+y$; $3x^2y+z$; $3xy+2x$ etc.

4. **Trinomial**: A Polynomial having three terms is called a Trinomial e.g. $x+y+z$; $2x-3y-4$; $2x^2y+3xy^2-4xy$ etc.

5. **Quadrinomial:** A Polynomial having four terms is called a quadrinomial e.g. $2x+3y+5z+6$; $3x-y+5z-xy$ etc. And so on.

In general a Polynomial means having many terms say 5, 6,7 ,10 etc

Consider the Polynomial, $P(x) = x^2-3x-4$. Putting x=2 in the Polynomial we get $P(2) = 2^2-3(2)-4= 4-6-4= -6$ the value " -6" is obtained by replacing x by 2 in x^2-3x-4 is the value of Polynomial x^2-3x-4 at x=2.

- If P(x) is a Polynomial in x and if k is any real number then the value obtained by replacing x by k in P(x) is called the value of p (x) at x=k and is denoted by P (k).

Again consider $P(x) = x^2-3x-4$.

$P(-1) = (-1)^2-3(-1)-4 = 1+3-4 = 0$

$P(4) = 4^2-3(4)-4 = 16-12-4 = 16-16 = 0$ Here -1 and 4 are called the Zero`s of the Polynomial.

Zero of Polynomial: A real number k is said to be a zero of a Polynomial P(x) if P (k) = 0

- Geometrically, the zeros of a Polynomial P(x) are precisely the x coordinates of of the points, where the graph of y = P(x) intersects the x axis.
- A quadratic Polynomial can have at most two zeros and a cubic Polynomial can have at most three zeros.
- In general a Polynomial of degree 'n' has at most 'n' zeros.

Relation between zeros and co-efficient of Polynomial

1. For Linear Polynomial $ax+b$; $a\neq0$

$x=-\frac{b}{a}$ here zero of a linear Polynomial is $K = -\frac{b}{a} = -\frac{\text{constant}}{\text{co-efficient of } x}$

2. For quadratic Polynomial : ax^2+bx+c ; $a\neq0$

If α, β are zeros of this quadratic Polynomial.

Then the sum of zeros = $\alpha + \beta = \frac{\text{co-efficient of x}}{\text{co-effiecient } x^2} = -\frac{b}{a}$

Product of zeros= $\alpha\beta$= constant term/co-efficient $x^2 = \frac{c}{a}$

3. Cubic Polynomial : ax^3+bx^2+cx+d ; $a \neq 0$

   Let α, β, Y be the zeros of cubic Polynomial

   Then sum of zeros = $\alpha+\beta+\gamma = -\frac{\text{co-efficient of } x^2}{\text{cofficient of } x^3} = -\frac{b}{a}$

   Product of zeros taken two at a time = $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{\text{co-efficient of x}}{\text{cofficient of } x^3} = \frac{c}{a}$

   Product of zeros = $\alpha\beta Y = \frac{\text{constant term}}{\text{cofficient of } x^3} = \frac{d}{a}$

FORMATION OF POLYNOMIALS:

- A quadratic Polynomial whose zeros are α and β is given by $P(x) = x^2-(\alpha+\beta)x+\alpha\beta$.

  i.e. $P(x) = x^2$-(sum of zeros)x+ Product of zeros
- A cubic Polynomial whose zeros are α ,β, γ is given by

  $P(x) = x^{3-} (\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x- \alpha\beta r$
- For more general form of a Polynomial development.

  Example: let the sum and product of zeros of a quadratic Polynomial be $\sqrt{2}; \frac{1}{3}$

Sol: let ax^2+bx+c be a quadratic Polynomial.

Let α, β, be its zeros

$\therefore$. Sum of zeros = $\alpha+\beta = -b/a = \sqrt{2} = \frac{3\sqrt{2}}{3}$ Making denominators same

Product of zeros $=\alpha\beta = \frac{c}{a} = \frac{1}{3} = \frac{1}{3}$

Setting a= 3, the common denominator of the above two equations

$\therefore -b = 3\sqrt{2}$ => $b = -3\sqrt{2}$ and c=1

So one quadratic Polynomial which fits the given condition is $3x^2-3\sqrt{2}\,x+1$

But we can set a= any multiple of 3 say = 3K, we can check other quadratic Polynomials fitting above conditions will be of form K $(3x^2-3\sqrt{2}x+1)$

Similar relation holds between the zeros of a cubic Polynomial and its co-efficient and for other higher degree Polynomials as well.

The zeros of a quadratic Polynomial ax^2+bx+c; $a\neq0$ are precisely the x - coordinates of the points where the parabola representing $y= ax^2+bx+c$ intersects the x-axis

In fact, for any quadratic polynomial ax^2+bx+c , $a\neq0$ the graph of the corresponding equation $y = ax^2+bx+c$ has one of the two shapes either open upwards like $\cup$ or open downwards like $\cap$ depending whether $a> 0$ or $a<0$ these curves are called Parabolas.

(For graphs see the text book)

Extreme values of quadratic Polynomials:

Let ax^2+bx+c be a quadratic Polynomial: $a \neq0$

Then $ax^2+bx+c= a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) = a\left\{\left(x+\frac{b}{2a}\right)^2+\left(\frac{c}{a}-\frac{b^2}{4a^2}\right)\right\} = a\left\{\left(x\frac{+b}{2a}\right)^2+\frac{4ac-b^2}{4a}\right\}$

<u>Case I</u> when $a>0$ then $ax^2+bx+c \geq a\frac{4ac-b^2}{4a^2} = \frac{4ac-b^2}{4a}$

$\therefore$ Minimum value of ax^2+bx+c is $\frac{4ac-b^2}{4a}$

Which occurs when $x = -\frac{b}{2a}$

<u>Case II</u> When $a<0$, then $ax^2+bx+c \leq \frac{4ac-b^2}{4a}$

Example: Let $P(x) = x^2-5x+6$ Here $a=1$, $b= -5$ $c = 6$

$a > 0$, Minimum value of ax^2+bx+c is $\frac{4ac-b^2}{4a}$

$\therefore$ Minimum value of $x^2 -5x + 6$ is $\frac{4\times1\times6-(-5)^2}{4\times1} = \frac{24-25}{4} = -\frac{1}{4}$

Which occurs at = $-\dfrac{b}{2a} = -\left(\dfrac{-5}{2(1)}\right) = \dfrac{5}{2}$

Example 2 Let P(x) = $-x^2+5x+6$ be the quadratic Polynomial. Here a < 0; a = −1, b= 5; c= 6

The max. Value = $\dfrac{4ac-b^2}{4a} = \dfrac{4(-1)6-5^2}{4(-1)} = \dfrac{-24-25}{4} = \dfrac{-49}{-4} = \dfrac{49}{4}$

Which occurs at x= $-\dfrac{b}{2a} = -\dfrac{5}{2(-1)} = \dfrac{5}{2}$

DIVISION ALGORITHM FOR POLYNOMIALS:

If p(x) and g(x) are two polynomials with g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that

P(x) = g(x) × q(x) +r(x)

Where r(x) = 0 or degree of r(x) ∠ degree of g(x).

- If r(x) = 0 then g(x) is a factor of p(x)

•Dividend = Divisor × Quotient + Reminder

Factorization:

A. By splitting middle term

Example 1: Let P(x) = $x^2-5x + 6$

Sol: P(x) = $x^2-2x-3x+6$

= (x) (x−2) −3 (x−2)

∴ P(x) = (x−2) (x−3)

$2\,\big|\,\underline{6}$
$\quad\;\; 3$

(−2) + (−3) = −5

and

(−2) × (−3) = 6

Example 2: Let $P(x) = x^2+5x+6$

$P(x) = x^2+2x+3x+6$

$= x(x+2) + 3(x+2)$

Hence $P(x) = (x+2)(x+3)$

2 6
3

$2+3 = 5$

$2\times3 = 6$

Example 3: Let $P(x) = 4x^2+8x+3$

Sol: $P(x) = 4x^2+2x+6x+3$

$= 2x(2x+1) +3(2x+1)$

$P(x) = (2x+1)(2x+3)$

$4 \times 3 = 12$

2 | 12
3

$2 + 6 = 8$

$2 \times 6 = 12$

B. Factorization using Reminder Theorem:

Statement: if the polynomial f(x) is divided by x−a, then the reminder is equal to f(a).

If f(a) = 0 , the reminder when f(x) is divided by x−a is zero . Then f(x) is divisible by x−a i, e. (x−a) is a factor of f(x).

Example: Let polynomial $f(x) = x^2-4x+4$

Here $f(2) = 2^2-4(2) +4= 4-8+4=0$

$\therefore$ X−2 is a factor of x^2-4x+4.

The other factor of the expression can be found out .similarly or by taking out x−2 as a factor of expression (long division method can be used)

- It is wise to explain reminder theorem once more in order to explain the procedure of factorizing a polynomial of degree more than or equal to 3.
- For a quadratic polynomial $P(x)=ax^2+bx+c$; $a\neq0$ $D= b^2-4ac$

a) If D= 0; P(x) is a perfect square, having two equal factors e.g. x^2-4x+4

Here $D= (-4)^2-4(1)(4) = 16-16= 0$

We write $x^2-4x+4 = x^2-4x+4 = x^2-2\times2\times x+2^2 = (x-2)^2$

$= (x-2)(x-2)$

b) If D> 0 (+ve) ; P(x) will have two different factors

c) If D< 0 (− ve); No Linear factors possible for P(x)

e.g. x^2+x+1; Here $D= 1^2-4(1)(1) = -3<0$

=> x^2+x+1 cannot be factorised into linear factors.

- If $P(x) = ax^2+bx+c$ and D=0

Then $ax^2+bx+c = a\left(x\frac{+b}{2a}\right)^2$

VERY SHORT ANSWER TYPE QUESTIONSS

Q1:- if one of the zero of quadratic polynomial x^2+3x+K is 2, then the value of K is

(a) 10 (b) −10 (c) 5 (d) −5

Q2:- A quadratic polynomial whose zeros are −3 and 4 is

(a) X^2-x+12 (b) x^2+x+12 (c) x^2-x-12 (d) $x^2-7x-12$

Q3 The relation between the zeros and co-efficient of the quadratic polynomial as ax^2+bx+c is

(a) $\alpha + \beta = \frac{c}{a}$ (b) $\alpha+\beta = \frac{b}{a}$ (c) $\alpha+\beta = \frac{-c}{a}$ (d) $\alpha+\beta= \frac{-b}{a}$

Q4 The zeros of the polynomial $x^2+7x+10$ are

(a) 2 and 5 (b) −2 and 5 (c) −2 and −5 (d) 2 and −5

Q:-5 The relationship between zeros and co-efficient of the quadratic polynomial ax^2+bx+c is

(a) $\alpha\times = \frac{c}{a}$ (b) $\alpha\times = \frac{-b}{a}$ (c) $\alpha\beta = \frac{-c}{a}$ (d) $\alpha\beta = \frac{b}{a}$

Q6 α, β are the zeros of the polynomial $f(x) = x^2+x+1$ then $\frac{1}{\alpha}+\frac{1}{\beta} =$

(a) 0 (b) 1 (c) −1 (d) None of these

Q7 If the sum of zeros of the polynomial $f(x) = 2x^2-3kx^2+4x-5$ is 6, then value of k is

(a) 2 (b) 4 (c) −2 (d) −4

Q8:- The zeros of a polynomial P(x) are precisely the x−coordinates of the points, where the graph of y=P(x) intersects the

(a) x−axis (b) y-axis (c) origin (d) none of these.

Q9:- A quadratic polynomial can have at most -------Zeros

(a) 0 (b) 1 (c) 2 (d) 3

Q10:- A polynomial of degree n has at most ----- zeros

(a) 0 (b) n+1 (c) n (d) n−1

Q11:- which of the following is not a polynomial?

(a) $\sqrt{3}\,x^2-2\sqrt{3}\,x+3$ (b) $\frac{3}{2}x^3-5x^2-\frac{1}{\sqrt{2}}x-1$ (c) $x+\frac{1}{x}$ (d) $5x^2-3x+\sqrt{2}$

Q12:- On dividing x^3+3x^2+3x+1 by x+1 the reminder is

(a) 0 (b) 1 (c) 2 (d) 3

Q13:- a quadratic polynomial whose sum and product of zeros are −5 and 6 is

(a) x^2-5x-6 (b) x^2+5x-6 (c) x^2+5x+6 (d) None of these

Q14:- which are the zeros of $P(x)=x^2+3x-10$

(a) 5, −2 (b) −5, 2 (c) −5, −2 (d) none of these.

Q15:- A real number K is called a zero of polynomial f(x) then

(a) f(k) = −1 (b) f(k) = 1 (c) f(k)=0 (d) f(k)= −2

Q16:- which of the following is polynomial:

(a) $x^2+\frac{1}{x}$ (b) $2x^2-3\sqrt{x}+1$ (c) $x^2+x^{-2}+7$ (d) $3x^2-3x+1$

Q17:- if the sum of the zeros of the polynomial $3x^2-kx+6$ is 3 then the value of k is

(a) 3 (b) −3 (c) 6 (d) 9

S.A Type: 2 Marks each

Q1:- if $2x-1$ and $x+2$ are the length and breadth of a rectangle in centimetres. Find its area A(x).

Q2:- if $2x-1$ and $x+2$ are the length and breadth of a rectangle in centimetres. For what value of x it will become a square?

Q3 :- if $A(x) = 2x^2-7x+6$ be the area function of a rectangle .find its length and breadth.

Q4 :- Fill in the blanks:

A quadratic polynomial ax^2+bx+c can be factorised into

(i) two ---------- if D>0

(ii) Two ------------ if D=0

(iii) Cannot have two linear factors if D is --------

Q5:- if $P(x) = 3x^3-2x^2+6x-5$; Find P (2)

Q6:- Draw the graph of polynomial $f(x) = x^2-2x-8$ Read the zeros of polynomial from the graph.

Q7:- Draw the graph of polynomial $f(x) = -4x^2+4x-1$

Read the zeros from the graph.

Q8:- Find the quadratic Polynomial whose zeros are

$\frac{3-\sqrt{3}}{5}$ and $\frac{3+\sqrt{3}}{5}$

Q9:- Match column 'A' with column 'B'

A	B
(i) P(x) be polynomial, g(x) the Divisor q(x) the quotient and r(x) The reminder then division algorithm Is written as	(a) P(x) and g(x) are of same degree
(ii) Remainder r(x) is zero if	(b) less than degree of g(x)
(iii) If P(x) is divided by g(x) and degree of quotient is zero Relation between P(x) and g(x)	(c) P(x) = g(x) × q(x) + r(x)
(IV) Degree of r(x) is always	(d) g(x) is factor of P(x)

Q10:- If 2 and -3 are zeros of the polynomial $x^2+(a+1)x-b$. Then find the values of a and b.

Q11:- On dividing x^3-3x^2+x+2 by a polynomial g(x) , the quotient and remainder were $x-2$ and $-2x+4$ respectively .Find g(x).

Q12:- If the product of zeros of a polynomial ax^2-6x-6 is 4. Find the values of 'a' .

Q13:- Write a quadratic polynomial, sum of whose zeros are $2\sqrt{3}$ and their product is 2.

Q14:- Find the sum and product of zeros of $P(x) = 2(x^2-3)+x$.

L.A Type 3 marks each

Q1. Find the zeros of the quadratic polynomial $6x^2-7x-3$ and verify the relationship between the zeros and the co-efficient.

Q2. Find a quadratic polynomial, the sum and product of whose zeros are $\sqrt{2}$ and $-\frac{3}{2}$

Q3. If one zero of the quadratic polynomial x^2+3x+k is 2, then find the value of k.

Q4. Given that one of the zeros of the cubic polynomial ax^3+bx^2+cx+d is zero, then find the product of the other two zeros.

Q5. If one of the zeros of the cubic polynomial x^3+ax^2+bx+c is -1, then find the product of the other two zeros.

Q6. If one of the zero`s of the quadratic polynomial $(k-1)x^2+kx+1$ is -3, then find value

of k.

Q7. If the zero`s of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3, then find the values of 'a' and 'b'

Q8. If α,β are zero`s of the quadratic polynomial $x^2-(k+6)x+2(2k-1)$. Find value of k if $\alpha+\beta=\frac{1}{2}\alpha\beta$

Q9. If polynomial $6x^4+8x^3-5x^2+ax+b$ is exactly divisible by the polynomial $2x^2-5$, then find the values of a and b

Q10. Find a cubic polynomial whose zero`s are 3, $\frac{1}{2}$ and -1.

Q11. Verify that 5, -2 and $\frac{1}{3}$ are the zero`s of the polynomial $P(x)=3x^2-2x^2-5x+6$.

Q12. Find the quotient and remainder when $4x^3+2x^2+5x-6$ is divided by $2x^2+3x+1$

Q13. On dividing x^4-5x+6 by a polynomial g(x), the quotient and remainder were $-x-2$ and $-5x+10$ respectively. Find g(x).

Q14. Given that $\sqrt{2}$ is zero of the cubic polynomial $6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$ .find its other two zero`s.

Q15. If α,β are the zeros of polynomial $f(x)=6x^2+x-2$ find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

Q16. Find value of k so that x^2+2x+k is a factor of $2x^4+x^3-14x^2+5x+6$.

VERY LONG ANSWER TYPE (4 marks each)

Q1. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, then remainder comes out to be x+a. Find k and a.

Q2 If α,β are the zero`s of the quadratic polynomial $P(x)=x^2-3x-2$,then find a quadratic polynomial whose zero`s are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

Q3 If α,β are the zero`s of quadratic polynomial $P(x)=2x^2-5x+7$, then find a quadratic polynomial whose zero`s are $2\alpha+3\beta$ and $3\alpha+2\beta$.

Q4. If α,β are the zero`s of polynomial $f(x)=x^2-P(x+1)-c$, show that $(\alpha+1)(\beta+1)=1-c$

Q5. What must be subtracted from $8x^4+4x^3-2x^2+7x-8$. So that the resulting polynomial is exactly divisible by $4x^2 + 3x + 2$.

Q6. What must be added to the polynomial $4x^4+2x^3-2x^2+x-1$ so that the resulting polynomial is exactly divisible by x^2+2x-3

Q7. Find all the zero`s of the polynomial $x^4-6x^3-26x^2+138x-35$, if two of its zero`sare $2+\sqrt{3}$ and $2-\sqrt{3}$.

Q8. Find values of a and b so that $x^4+x^3-8x^2+ax+b$ is divisible by x^2+1.

Q9. If the polynomial $f(x) = x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x+a$ find k and a .

Q10. If α and β are the zero`s of the quadratic polynomial $f(x) =x^2-2x-8$, then find the value of (i) $\alpha-\beta$ (ii) $\alpha^2+\beta^2$.

Q11. Divide $3x^2-x^3-3x+5$ by $x-1-x^2$ and verify the division algorithm.

Q12. Find the zero`s of the quadratic polynomial $f(x) =abx^2+(b^2-ac)x-bc$ and verify the relationship between the zero`s and its co-efficients.

Q13. Find a quadratic polynomial, the sum and product of whose zero`s are $\sqrt{2}$ and $\frac{1}{3}$ respectively. Also find its zero`s

ANSWERS

VERY SHORT ANSWER TYPE

Q1 (b) Q2 (c) Q3 (d) Q4 (c) Q5 (a) Q6 (c) Q7 (b) Q8 (a) Q9 (c) Q10 (c)

Q11 (c) Q12 (a) Q13 (c) Q14 (b) Q15 (c) Q16 (d) Q17 (d)

SHORT ANSWER TYPE.

Q1 A (x)=$2x^2+3x-2$ Q2 x=2 Q3 L=2x-3 ; B=x−2

Q4. (i) Distinct real linear factors (ii) Equal real linear factors

(iii) D is less than zero Q5 23 Q6 4 and −2

Q7. $\frac{1}{2}, \frac{1}{2}$ Q8. $25x^2-30x+6$ Q9 (i) (c) (ii) (d) (iii) (a) (iv) (b)

Q10. a=0; b= −6 Q11. X^2-x+1 Q12. a=$-\frac{3}{2}$ Q13. $X^2-2\sqrt{3}\ x+2$ Q14. Sum= $-\frac{1}{2}$ product= −3

LONG ANSWER TYPE.

Q1 $\frac{3}{2}$ and $-\frac{1}{3}$ Q2. $2x^2-2\sqrt{2}\ x-3$ Q3. K= −10 Q4. $\frac{c}{a}$ Q5. a-b-1 Q6. K=$^4/_3$

Q7. a=0; b= -6 Q8. K=7 Q9. a= -20; b= -25 Q10. $2x^3-5x^2-4x+3$ Q12. 2x-2, R=9x-4

Q13. g(x) = -x+2 Q14 $-\frac{\sqrt{2}}{2}$ and $-2\frac{\sqrt{2}}{2}$ Q15. $\frac{-25}{12}$ Q16. K= $-\frac{27}{7}$

VERY LONG ANSWER TYPE

Q1. K=5 and a=35 Q2 $16x^2-9x+1$ Q3. $2x^2-25x+82$ Q5. 12x-2 Q6. 61x-65

Q7. -5 and 7 Q8 a=1 and b=7 Q9. K=5 and a= -5 Q10. (i) 6 (ii) 20

Q11. $-x^3+3x^2-3x+5 = (-x^2+x-1)(x-2)$ Q12 $\frac{c}{b}$ and $\frac{-b}{a}$ Q13. $3x^2-3\sqrt{2}\ x+1$; Zero`s are $\frac{3\sqrt{2} \pm \sqrt{6}}{6}$

CHAPTER -3

PAIR OF LINEAR EQUATIONS IN TWO VARABLES

- Introduction
- Methods to find the solution pair of linear equations
- Substitution method
- Elimination method
- General method (cross multiplication method)
- Eliminating constant method
- Application of linear equations in solving practical problems.

Introduction

- An equation of the form ax + by + c = 0, where a, b, c are real numbers $a^2 + b^2 \neq 0$ is called a linear equation in two variables x and y.
- The numbers a and b are called co-efficient of the variables of equation ax + by + c = 0 , and the number c in called the constant of the equation ax + by + c = 0.
- Two linear equations is the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equation is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers

Such that $a_1^2+b_1^2 \neq 0$; $a_2^2+b_2^2 \neq 0$

Consistent System: A system of simultaneous linear equations is said to be consistent, if it has at least one solution.

Inconsistent system: A system of simultaneous linear equations is said to be inconsistent, if it has no solution.

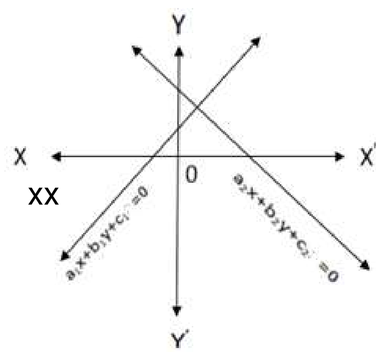
Geometrical (i.e. graphical) method of solution of a pair of linear equations: The graph of a pair of linear equations in two variables is represented by two :.

Only one of the following three possibilities can happen.

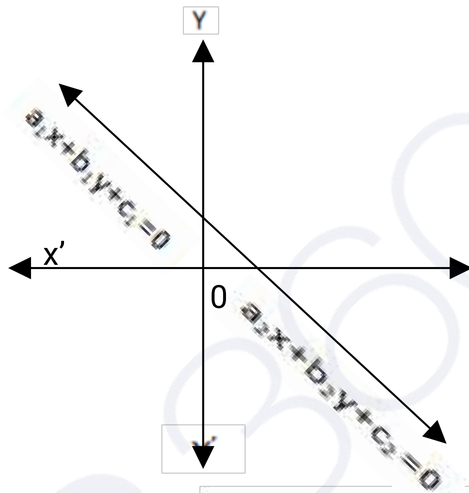
(i) The two lines will intersect at one point

(ii) The two lines will not intersect i.e. they are parallel.

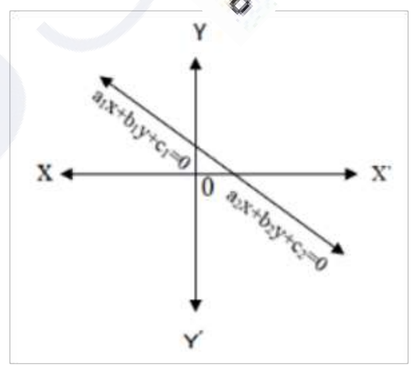
(iii) The two lines will be coincident.



1. If the lines intersect at a point, then that point gives The unique solution of two equations. In this case, The pair of equation is consistent.



2. if the lines coincides then there are infinitely many solutions. Each point on the line being a solution. In this case the pair of equations is dependent (consistent)



3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equation is inconsistent. Algebraic interpretation of pair of linear equations in two variables. The pair of linear equations be these lines.

$a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$

(a) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations has exactly one solution i.e (unique solution)

(b) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solution.

(c) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations have no solution.

S. no	Pair of lines	Compare the ratios	Graphical representation	Algebraic interpretation
1.	$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting Lines	Unique solution
2.	$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident line	Infinitely many solutions
3.	$a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,	Parallel lines	No solution

ALGEBRAIC METHODS FOR SOLVING A PAIR OF LINEAR EQUATIONS:-

(A) Substitution Method:

Stepwise method of solving linear equations by (substitution method)

$a_1x+b_1y+c_1=0$___(i)

$a_2x+b_2y+c_2=0$____(ii)

step 1:

Find the value of one variable say 'y' in terms of the other variable i.e *x* from either equation, whichever is convenient.

Say $y=-\frac{a_1}{b_1}x \quad \frac{c_1}{b_1}$ (iii)

Step 2:

Substitute this value of y in the other equation say (II) and reduce it into an equation in one variable i.e interms of *x*, which can be solved.

Step 3:

Substitute value of x got in step 2 in III equations and get value of y.

In this way we get vales of x and y.

Important: sometimes while doing step 2, we get statements with no variable. If this statement is true, we conclude that the pair of linear equations has infinitely many solutions.

If the statement is False, then the pair of linear equations have no solutions i.e they are inconsistent.

ELIMINATION METHOD:

Following are the steps to solve the pair of linear equations by eliminations method:

Step 1 :_ first multiply both the equations by some suitable non-zero constants to make the co-efficient of one variable (either *x* or *y*) numerically equal.

Step 2:- Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to step 3.

If in step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions. However

If in step 2, we obtain a false statement involving no variable, then the original pair of equations has no solutions i.e. it is inconsistent.

Step 3:- solve the equation in one variable (x or y) so obtained to get its value.

Step 4:- substitute this value of *x* (or y) in either of the original equations to get the value of other variable.

C) CROSS MULTIPLICATION METHOD.

Let the pair of linear equations be $a_1x + b_1y + c_1 = 0$____(1)

$a_2x + b_2y + c_2 = 0$____(2)

Step 1:- Multiplying equation (1) by b_2 and equation (2) by b_1 to get

$b_2a_1x + b_2b_1y + b_2c_1 = 0$ ____(3)

$b_1a_2x + b_1b_2y + b_1c_2 = 0$______(4)

Step 2:- subtracting equation (4) from (3) we get.

$(b_2a_1 - b_1a_2)x + (b_2b_1 - b_1b_2)y + (b_2c_1 - b_1c_2) = 0$

i.e. $(b_2a_1 - b_1a_2)x = b_2c_1 - b_1c_2$

so $x = \frac{b_1c_2 - b_2c_1}{b_2a_1 - b_1a_2}$ provided $b_2a_1 - b_1a_2 \neq 0$ __________(5)

Step 3 substituting this value of x in (1) or (2) we get

$Y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$ ___________________(6)

Now two cases arise:

Case I:- $a_1b_2 - a_2b_1 \neq 0 \Rightarrow a_1b_2 \neq a_2b_1 \Rightarrow \frac{a_1}{a_2} \quad \frac{b_1}{b_2}$

$\Rightarrow$ pair of linear equations has unique solution

Case II:- $a_1b_2 - a_2b_1 = 0 \Rightarrow a_1b_2 = a_2b_1 \Rightarrow \frac{a_1}{a_2} \quad \frac{b_1}{b_2} = K$ (suppose)

$\therefore$ $a_1 = ka_2$ and $b_1 = kb_2$

Substituting the values of a_1, b_1 in equation (1) we get

$K(a_2x + b_2y) + c_1 = 0$ ___________(7)

We see (7) and (2) can both be satisfied only if

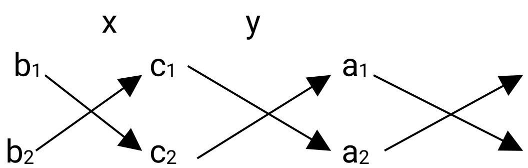
$C_1 = Kc_2$ i.e $\frac{c_1}{c_2} = K$ thus if $c_1 = Kc_2$. Any solution of equation (2) will satisfy the equation (1)

And vice versa so if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = K$, then there are infinitely many solutions to the pair of linear equations. If $c_1 \neq Kc_2$ then any solution of equation (1) will not satisfy equation (2) and vice versa.

Therefore the pair linear equations have no solutions We can write the solution given by equation (5) and (6)

In the form: $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$

In remembering the above result, the following diagram may be helpful to you



The arrows between the two numbers indicate that they are to be multiplied and the second product is to be subtracted from the first.

For solving a pair of linear equations by this method we will follow the following steps.

<u>Step 1</u>:- Write the given equation in the form (1) and (2)

<u>Step 2</u>:- Taking the help of the diagram above, write equations as in given (3)

<u>Step 3</u>:- Find x and y, provided $a_1b_2 - a_2b_1 \neq 0$

<u>N:B</u>; Step 2 above gives you an indication of why this method is called the cross-multiplication method.

D) CONSTATNT ELIMINATION METHOD:

Steps to solve the pair of linear equations by constant elimination method

$$a_1x + b_1y + c_1 = 0 \_\_\_\_ (I)$$

$$a_2x + b_2y + c_2 = 0 \_\_\_\_ (II)$$

<u>Step 1</u>:- Dividing equation (1) by equation (2)

say $\dfrac{a_1x + b_1y}{a_2x + b_2y} = \dfrac{c_1}{c_2}$

<u>Step 2</u>:- Taking cross multiplications we get.

$a_1c_2x + b_1c_2y = a_2c_1 + b_2c_1y$ | consistants got

& $(a_1c_2 - a_2c_1)x = (b_2c_1 - b_1c_2)y$ | eliminated

Step 3:- equating this equation as (LCM of co-efficients of and y) K

e.g $(a_1c_2 - a_2c_1)x = (b_2c_1 - b_1c_2)y = (a_1c_2 - a_{2} - a_2c_1)(b_2c_1 - b_1c_2)K$

∴ $x = (b_2c_1 - b_1c_2)K$ and $y = (a_1c_2 - a_2c_1)K$ ____________(3)

Using these valued of x and y in (1) {or (2)}

We get

$a_1 (b_2c_1 - b_1c_2)k + b_1(a_1c_2 - a_2c_1)k = c_1$

or $[a_1b_2c_1 - a_1b_2c_2 + a_1b_1c_2 - a_2b_1c_1]k = c_1$

Or $k = \frac{c_1}{(a_1 b_2 - a_2 b_1)c_1} = \frac{1}{(a_1 b_2 - a_2 b_1)}$ provided $a_1b_2 - a_2b_1 \neq 0$

$\therefore$ From (3) $x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$ and $y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$

Important: Sometimes we may cross multiply and all terms cancel and we get say 0=0 i.e. statement in time , the system has infinitely many solutions or sometimes we may get 7=9 etc, the statement is false so the system has no solution.

Example 1:- let $3x + 2y = 7$ ________(1)

$3x - y = 1$ ______(2)

Dividing (1) by (11) we get $\frac{3x+2y}{3x-y} = \frac{7}{1}$

Taking cross multiplication we get

$21x-7y = 3x+2y$ or $18x = 9y$ or $2x=y$

Let $2x=7y=3x+2y$ or $18x=9y$ or $2x=y$

Using these values of x and y in (1) get

$3k + 2 (2K) = 7$ or $7K = 7$ or $K \frac{7}{7} = 1$

Hence $x = k = 1$ and $y = 2k = 2x1 = 2$ so

$x = 1$

$Y = 2$

<u>Example 2</u>:- solve for x and y: $3x + 2y = 5$_____(I)

$6x + 4y = 10$____(II)

Solution: Dividing (1) by (ii) and cross multiply we get.

$30x + 20y = 30x + 20y$ or $0 = 0$

The statement is true. Hence the given system of equations have infinitely many solutions.

<u>Example 3</u>:- Solving : $3x + 2y = 7$____(1)

$6x + 4y = 12$____(II)

Solution : Dividing (1) by (II) and cross multiply we get.

$42x + 28y = 36x + 24y \Rightarrow 42x - 36x = 24y - 28y$

$\Rightarrow$ $6x = -4y \Rightarrow 3x = -2y$

Taking $3x = -2y = 6k$

Or $x = 2k$ and $y = -3k$

| just to avoid fractional values of K we take (LCM of 2,3)k

Using these values of x and y in (I) equation we get.

$3(2k) + 2(-3k) = 7 \Rightarrow 6k - 6k = 7$

$\Rightarrow$ $0 = 7$. The statement is false

Hence the system has no solution.

Example 4:- Solving equations $3x - y = 0$___(I)

$8x - 2y = 2$___(II)

From (1) we take $3x = y = 3k$ (suppose)

$\therefore$ $x = K; y = 3k$, using in (II) equation

$8k - 2(3k) = 2 \Rightarrow 8k - 6k = 2 \Rightarrow 2k = 2$

$\Rightarrow k = 1$

Hence ~~x~~ = k = 1 and $y = 3k = 3(1) = 3$

i.e $x = 1$ and $y = 3$

Note: if $a_1x + b_1y = 0$

And $a_2x + b_2y = o$

Then trivial solution is $x = 0$; $y = 0$

N:B Apply all other methods to the above examples and see if the answers are same.

Objective type: VSA Type (one mark each)

Q1:- Match column A with column B

	A		B
(i)	A pair of equation is consistant	(a)	The lines will always be parallel.
(ii)	Equations $3x + 4y = 18$ $4x + \frac{16}{3}y = 24$ ha	(b)	the lines will be Intersecting or conincident
(iii)	A pair of equation is inconsistant	C)	infinite number of solutions.

Q 2. Write true or false as the case may be.

(i) The lines $3x - 2y = 4$ and $2x + 3y = 18$ are intersecting lines.
(ii) $2 + 3y = 5$ and $6x + 9y = 18$ are intersecting lines.
(iii) $2 + 3y = 5$ and $2x + 3y = 9$ are not parallel lanes.

Q 3. Fill in the blanks.

(i) The dependent equation from ………………..lines
(ii) One line is parallel to x-axis, another is parallel to y-axis, then these two lines are ……..to each other.
(iii) A linear equation in two variables represents a ……..

Q 4. Choose the most appropriate answer among 4 given alternatives.
A pair of equations is consistent, then the lines will be.
(a) Parallel (b) Always coinicident (c) Always intersecting
(d) intersecting or conincident.

Q 5. The pair of equations $y = o$ and $y = -7$ has

(a) One solutions (b) two solutions (c) infinitely many solutions
(d) No solution.

Q 6. The pair of equation $x = a$ and $y = b$ graphically represents the lines which are
(a) parallel (b) intersecting at (a,b) (c) coincident
(d) Intersecting at (b,a)

Q 7. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solution is
(a) 3 (b) – 3 (c) – 2 (d) no value.

Q 8. The pair of equations $5x - 15y = 8$ and $3x - 9y = \frac{24}{5}$

has

(a) infinite solutions (b) unique solution (c) no solution

(d) one solution.

Q 9. The sum of the digits of two digit number is 9. If 27 is added to it, the digits of the numbers get reversed. The number is

(a) 36 (b) 72 (c) 63 (d) 25

Q 10. The solutions of the equation $x + y = 14$ and $x - y = 4$ is

(a) $x = 9$ and $y = 5$ (b) $x = 5$ and $y = 9$

(c) $x = 7$, $y = -7$ (d) $x = 10$, $y = 4$

Q 11. The value of K for which the system of equations $x - 2y = 3$ and $3x + ky = 1$ has a unique solution is

(a) $K = -6$ (b) $K \neq -6$ (c) $K = 0$ (d) no value.

Q12. A pair of equations is inconsistent, then the lines will be.

(a) parallel (b) Always coincident (c) Always intersecting

(d) intersecting or coincident

Q13. The value of K for which the system of equations $Kx - y = 2$ and $6x - 2y = 3$ has a unique solution is

(a) $k = -3$ (b) $k \neq -3$ (c) $k = 0$ (d) $k \neq 0$

Q 14. The value of K for which the system of equations $2x + 3y = 5$ and $4x + ky = 10$ has infinitely many solutions is

(a) $k = -3$ (b) $k \neq -3$ (c) $k = 6$ (d) none of these

Q 15. Sum of two numbers is 35 and their difference is 13 then the numbers are.

(a) 24 and 12 (b) 24 and 11 (c) 12 and 11 (d) none of these

Q 16. The value of K for which the system of equations $x + 2y = 3$ and $5x + ky + 7 = 0$ has no solution is.

(a) K = 10 (b) K = 6 (c) K = 3 (d) K = 1

Q 17. The value of K for which the system of equations $3x + 5y = 0$ and $Kx + 10y = 0$ has a non-zero solution is

(a) K = 0 (b) K = 2 (c) K = 6 (d) K = 8

Q 18. The sum of the digits of a two digit number is 12. The number obtained by intersecting its digits exceeds the given number by 18. Then the number is

(a) 72 (b) 75 (c) 57 (d) none of these

Q 19. If (6,k) is a solution of the equation $3x + y - 22 = 0$, then the valuce of k is:

(a) 4 (b) −4 (c) 3 (d) −3

Q 20. The pair of equations $2x + 3y = 7$ and $k + \frac{9}{2}y = 12$ have no solutions, then the value of k is

(a) $\frac{2}{3}$ (b) − 3 (c) 3 (d) 3/2

SHORT ANSWER TYPE (2 MARKS EACH)

Q1. Solve for x and y:

$$11x + 15y + 23 = 0$$
$$7x - 2y - 20 = 0$$

Q 2. $2x + y = 7$ and $4x - 3y + 1 = 0$

Q 3. $2x + 5y = \frac{8}{3}$ and $3x - 2y = \frac{5}{6}$

Q 4. $3x - 5y - 19 = 0$ and $-7x + 3y + 1 = 0$

Q 5. Find the value of k so that the system of equations has no solution

$3x - y - 5 = 0$; $6x - 2y - k = 0$

Q 6. Find the value of k, so that the following system of equations has a non-zero solution

$3 + 5y = 0$; $kx + 10y = 0$

Q 7. Find the value of k, so that the system of equations has no solution:

$x - 2y = 3$; $3x + ky = 1$

Q 8. Find the value of k, so that the system of equation has no solution.

$kx + 3y = 3$; $12x + ky = 6$

Q 9. Find the values of k so that the system of equations has a unique solution:

$x - 2y = 3$ and $3x + ky = 1$

Q 10. For what value of k the pair of linear equations has infinite number of solutions.

$kx + 3y = 2k + 1$; $2(k+1)x + 9y = 7k + 1$

Q 11. Solve the system of linear equations graphically.

$x + 2y = 3$; $4x + 3y = 2$

Q 12. Solve the system of linear equations graphically.

$2x - 3y - 17 = 0$; $4x + y - 13 = 0$

Shade the region bounded by the above lines and x-axis.

Q 13. The sum of two numbers is 137 and their difference is 43. Find the numbers.

Q 14. The sum of two natural numbers is 8 and the sum of their reciprocals is $\frac{18}{15}$. Find the the numbers.

Q 15. The sum of the digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.

Q 16. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and denominator, it becomes $\frac{5}{6}$. Find the fraction.

<u>LONG ANSWER TYPE (3 MARKS EACH)</u>

Q 1. Find the value of k, so that the system of equations has no solution:

$(3k + 1)x + 3y - 2 = 0$

and $(x^2 + 1)x(x - 2)y - 5 = 0$

Q 2. Find the value of K so that the systems of equations has a unique solution:

$kx + 3y = k - 3$

and $12x + ky = K$

Q 3. For what value fo k, the pair of linear equations has infinite number of solutions:

$x + (2k - 1)y = 4$ and $kx + 6y = k+6$

Q 4. Solve the pair of linear equations.

$\frac{x}{3} + \frac{y}{2} = 3$; $x - 2y = 2$

Q 5. Find the value of a and b for which system of linear equations has an infinite number of solutions.

$(a - 1)x + 3y = 2$; $6x + (1 - 2b)y = 6$

Q 6. Solve the system of linear equations graphically

$2x - 5y + 4 = 0$; $2x + y - 8 = 0$. Find the points where these lines meet the y – axis.

Q 7. Mutton is sold at Rs 535 per kg without offal and Rs 495 per kg alongwith 100gms offal. Find the cost of 1kg of an offal.

Q 8. The cost of one quintal of paddy is Rs 1200 and cost of one quintal rice is Rs 2000. Over head costs to convert paddy into rice is Rs 400. A one quintal paddy gives 70 kg rice and 30 kg husk. Find the cost of 1 quintal of husk?

Q 9. The sum of twice the first and thrice the second is 92 and four times the first exceeds seven times the second by 2. Find the numbers.

Q 10. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference between the digits is 3, then find the number.

Q 11. Five years ago Nuri was thrice old as Sony. Ten years later, Nuri will be twice as old as Sonu. Find the present age of Nuri and Sony.

Q 12. In a $\triangle$ ABC , $\angle C = 3 \angle B = 2 (\angle A + \angle B)$. Find the angles.

Q 13. Find the four angles of a cyclic quadrilateral ABCD in which
$\angle A = (2x - 1)^{\circ}$; $\angle B \; (y+5)^{\circ}$; $\angle C = (2y+15)^{\circ}$; $\angle D = (4x - 7)^{\circ}$

Q 14. The cost of 5 oranges and 3 apples is Rs 35 and the cost of 2 oranges and 4 apples is Rs 28. Find the cost of an orange and an apple.

<u>LONG ANSWER TYPE (4 MARKS EACH)</u>

Q1. Solve for x and y
$x + y = 5xy$; $3 + 2y = 13xy$; $x \neq 0$; $y \neq 0$

Q 2. Solve: $\frac{x}{3} + \frac{y}{4} = 11$; $\frac{5x}{6} - \frac{y}{3} + 7 = 0$

Q 3. $\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}$

$\frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2$ solve it.

Q 4. Solve for —and y
$\frac{44}{x+y} + \frac{30}{x-y} = 10$; $\frac{55}{x+y} + \frac{40}{x-y} = 13$; where $x + y \neq 0$ and $x - y \neq 0$

Q 5. Solve for x and y

$(a - b)x + (a + b) = a^2 - 2ab - b^2$
$(a + b)(x + y) = a^2 + b^2$

Q 6. Solve for x and y : $ax - by = a^2 + b^2$
$x + y = 2a$

Q 7. Solve for x and y
$2(ax - by) + (a + 4b) = 0$
$2(bx + ay) + (b - 4a) = 0$

Q 8. The sum of a two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2. Find the number. How many such numbers are there?

Q 9. The denominator of a fraction is greater than its numerator by 11. If 8 is added to both its numerator and denominator, it reduces to $\frac{1}{2}$. Find the fraction.

Q 10. The present age of a women is 3 years more than three times the age of her daughter. Three years hence, the woman's age will be 10 years more than twice the age of her daughter. Find their present ages.

Q 11. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down stream. Determine the speed of the stream and that of the boat in still water.

Q 12. The area of a rectangle remains the same if the length is increased by 7m and the breadth is decreased by 3 m. The area remains unaffected, if the length is decreased by 7 m and the breadth is increased by 5m . find the dimensions of the rectangle.

Q 13. A and B each has some money. If A gives Rs 30 to B then B will have twice the money left with A. But if B gives Rs 10 to A then A will have thrice as much as is left with B. How much money does each have.

ANSWERS

VERY SHORT ANSWER TYPE

Q 1. (i) → (b) (ii) → (c) (iii) → (a)
Q 2. (i) True (ii) False (iii) False
Q 3. (i) coincident (ii) perpendicular (iii) line.
Q 4. (d) Q 5 (d) Q 6 (b) Q7 (d) Q8 (c) Q9 (a)
Q10 (a) Q11 (b) Q12 (a) Q 13 (C) Q14 (C)
Q15 (b) Q16 (a) Q17 (C) Q18 (c) Q19 (a)
Q 20 (c)

SHORT ANSWER TYPE

Q 1. x = 2 ; y = 3 Q 2. x = 2 ; y = 3 Q 3. x $\frac{1}{2}$; y= $\frac{1}{3}$
Q 4. x – 2 ; y – 5 Q 5. k = 10 Q 6. k = 6
Q 7. k = – 6 Q8. k = – 6 Q 9. k ≠ – 6
Q 10. k = 2 Q 11. x = –1 ; y = 2 Q 12. x = 4 ; y = $\frac{47}{7}$
Q 13. x= 90 ; y = 47 Q 14. Two natural numbers are 3 and 5
Q 15. 57 Q 16. $\frac{7}{9}$

lONG ANSWER TYPE

Q1. K = – 1 Q2. K ≠ 6 Q3. K = 2 Q4. x = 6;y = 2
Q5. a = 3 ; b = – 4 Q 6. X = $\frac{10}{3}$; y = $\frac{4}{3}$ Q 7. Rs 135. Q8. Rs 666.66
Q 9. Numbers are 25 and 14 Q 10. 36
Q 11. Nuri = 50 years sonu= 20 years

Q 12. $\angle A = 20^\circ$, $\angle B = 40^\circ$, $\angle c = 120^\circ$.

Q 13. $\angle A = 65^\circ$, $\angle B = 55^\circ$, $\angle C\ 115^\circ$, $\angle D = 125^\circ$

Q 14. Cost of orange = Rs 4 cost of an apple = Rs5

VERY lONG ANSWER TYPE

Q1. $x = \frac{1}{2}$; $y = \frac{1}{3}$ Q 2. x = 6 ; y = 36 Q3. x = 2; y =1

Q 4. x= 8 , y = 3 Q 5. $x = a + b$, $y = \frac{-2ab}{a+b}$

Q6. $x = a + b$, $y = a - b$ Q 7. $x = \frac{-1}{2}$,y =2

Q 8. 42 Q9. $\frac{3}{14}$ Q10. Woman = 33yrs, Daughter=10 yrs.

Q11. Speed of water = 3km/h, speed of boat = 8 km / h

Q 12. Length = 28 m, Breadth = 15 m

Q 13. A has Rs 62 , B has Rs 34.

Chapter -4
Quadratic Equation

- Introduction
- Methods to find the solution of Quadratic Equations.
- Relation between the Roots and the co-efficient.
- Nature of the Roots.
- Formation of Quadratic Equation when the Roots are known.
- Equations Reducible to Quadratic Form.
- Application of Quadratic Equations in solving practical problems.

Introduction: A polynomial of the form $P(x)= ax^2+bx+c$, where the highest index of power of the variable x is 2 and a, b, c are real numbers; $a\neq0$ is called a quadratic polynomial.

Equation: An algebraic equation is an equality involving constants and variables. The values of the expression on L.H.S and R.H.S are equal.

Quadratic Equation: An equation P(x) =0 , where P(x) is a quadratic polynomial is called a quadratic equation. The general form of quadratic equation is $ax^2+bx+c=0$, where a, b , c are real constants, $a\neq0$ and x is a real variable.

Equation and Identity: The main difference between equation and an identity is that equation is true for the number of values of unknowns equal to the degrees of the variable involved.

If it is true for the number of values more than degree of an equation, then it will turn out to be an identity.

Roots of Quadratic equation:

A quadratic equation has only two roots. The value of x satisfying a quadratic equation known as the roots of the quadratic equation. Thus if α and β be the zero`s of quadratic polynomial P(x), then they are the roots of the quadratic equation P(x)=0

Methods of finding the solution of Quadratic equations

1. Factorisation Method

2. Method of completing the square.
3. Quadratic Formula method
4. Converting all quadratic equations to the form $ax^2+c=0$ (Through Examples only)

1. FACTORISATION METHOD:

- Write the given quadratic equation in standard form $ax^2+bx+c=0$
- Splitting Middle term `b` i.e. find numbers α and β such that sum of α + β = b and product α β = α c.
- Write the middle term as αx+βx and factorise the quadratic equation , let factors be (lx+p) (mx+q) = 0
- Now equate each factor to zero and find values of x.
- These values of x are required roots of the given quadratic equation.

A. Method of completing the square.

Solution: Let quadratic equation in standard form be

$ax^2+bx+c=0$ Transfer c on R.H.S

or $ax^2+bx=-c$ Dividing both sides by `a`

or $x^2+\frac{b}{a}x=-\frac{c}{a}$ | Adding square of half of the Co-efficient of x i.e. $(\frac{b}{2a})^2$ on both sides.

or $x^2+\frac{b}{a}x+(\frac{b}{2a})^2 = -\frac{c}{a}+(\frac{b}{2a})^2$

or $x^2+2.(\frac{b}{2a})x+(\frac{b}{2a})^2 = \frac{b^2}{4a^2}-\frac{c}{a}$

or $(x+\frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$ | By writing L.H.S as a perfect square

Taking square root on both sides.

$$\text{Or } x + \frac{b}{2a} = \pm \frac{\sqrt{b^2-4ac}}{4a^2}$$

$$\text{Or } x = -\frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a} = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$$

B. Method of completing the square.

Solution: let $ax^2+bx+c= 0$ be a Quadratic equation | Transfer term `c` on R.H.S

or $ax^2+bx = - c$ | multiplying b/s by 4a

$\therefore$ $4a^2x^2+4abx = - 4ac$

Adding b^2 on both sides.

Or $(2ax)^2+2\,(2ax).b +b^2 = - 4ac+b^2$

Make and write L.H.S as perfect square

$(2ax+b)^2= b^2 - 4ac$

Taking square root on both sides.

Or $\sqrt{(2ax+b)^2} = \pm\sqrt{b^2-4ac}$

Or $2ax+b = \pm\sqrt{b^2-4ac}$

Or $2ax = -b \pm \sqrt{b^2-4ac}$

$$\text{Or } x = -\frac{b\pm\sqrt{b^2-4ac}}{2a} = -\frac{b\pm\sqrt{D}}{2a}$$

3. Quadratic Formula Method:

- Firstly , write the given quadratic equation in standard form $ax^2+bx+c= 0$
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D=b^2-4ac$. If value of D is negative, there is no real solution

i.e. solution does not exist. If value of D≥ 0, the roots is real i.e. solution exists.

❖ Put the value of a, b and D in quadratic formula $x= -\frac{b\pm\sqrt{D}}{2a}$ and get the required roots/ solution.

4. <u>Converting all quadratic equations to the form</u> $ax^2+c=0$

A quadratic equation has four different forms viz .

1) $ax^2=0$ 2) $ax^2+bx = 0$ (3) $ax^2+c=0$ (4) $ax^2+bx+c= 0$

Where a, b , c are real numbers and a≠0

Case (I): $ax^2=0$ => $x^2 = \frac{0}{a}$ or x.x =0

⇨ Both roots are zero`s i.e. x= 0 , 0

Case (II) : $ax^2+bx= 0$ or $x(ax+b)= 0$

Either x=0 or ax+b=0

⇨ Either x=0 or $x = -\frac{b}{a}$

Case (III) : $ax^2+c=0$ => $x^2 = -\frac{c}{a}$

Or $x= \pm\frac{\sqrt{-c}}{a}$. if c is negative, a is positive. Roots are real.

Or if `a` is negative, `c` is positive , Roots are real . Otherwise roots are imaginary.

Case (IV) : $ax^2+bx+c =0$

Let us illustrate its solution through examples.

Example 1: $x^2-6x+8=0$

We will convert this into case (III)

Suppose roots lie around mean of roots.

Mean of roots=$\frac{\text{sum of roots}}{\text{number of roots}} = -(-\frac{6}{2}) = \frac{6}{2} = 3$

Let roots be x= mean+h=3+h , where h is zero , +ve or –Ve real number.

Substitute value of x in given equation

$(3+h)^2 - 6(3+h)+8= 0$

Or $9+h^2+6h-18-6h+8=0$

Or $h^2-1=0$ => $h^2=1$ => $h = \pm\sqrt{1} = \pm 1$

Hence roots are x= 3+h = 3± 1 = 3+1 or 3-1

i.e. x= 4 , 2

Example 2: Let $2x^2-10x+12=0$ be a given quadratic equation. Solve it

Solution: $2x^2-10x+12=0$ dividing both sides by 2

$\therefore$ $x^2-5x+6 = 0$

Let roots be $x= \text{mean}+h = -(\frac{-5}{2})+h = \frac{5}{2}+h$

Substitute in given equation

$(\frac{5}{2}+h)^2 - 5(\frac{5}{2}+h) +6 = 0$ or $\frac{25}{4}+h^2+5h - \frac{25}{2} -5h+6=0$

Or $h^2 - \frac{25}{4} +6=0$ or $h^2 - \frac{1}{4} = 0$ => $h^2 = \frac{1}{4}$

Taking sq root ; $h = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

Roots are $= \frac{5}{2}+h = \frac{5}{2} \pm \frac{1}{2} = \frac{5-1}{2}$ or $\frac{5+1}{2} = 2, 3$

Example 3: Solve $x^2-4x+5 =0$

Solution : let $x= \text{mean}+h = \frac{4}{2}+h = 2+h$.

Substitute in given equation we get

$(2+h)^2-4(2+h)+5=0$ or $4+h^2+4h-8-4h+5=0$

Or $h^2+1=0 \Rightarrow h = \pm\sqrt{-1} \notin R$

⇨ Given equation have no real roots

i.e. it has no solution.

Sum and Product of roots: Relation between roots and co-efficient.

❖ Solution of quadratic equation $ax^2+bx+c=0$

$$\text{Is } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

Where $D = b^2-4ac$ is called discriminant.

$$\text{So two roots are } \frac{-b+\sqrt{D}}{2a} \text{ and } \frac{-b-\sqrt{D}}{2a}$$

$$\text{Sum of two roots} = \frac{-b+\sqrt{D}}{2a} + \frac{-b-\sqrt{D}}{2a} = \frac{-2b}{2a}$$

$$= \frac{-b}{a} = -\left\{ \frac{\text{co-efficient of } x}{\text{co-efficient of } x^2} \right\}$$

$$\text{Product of roots} = \left[\frac{-b+\sqrt{b^2-4ac}}{2a}\right]\left[\frac{-b-\sqrt{b^2-4ac}}{2a}\right]$$

$$= (-b)^2 - \frac{\left(\sqrt{b^2-4ac}\right)^2}{4a^2} = \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2}$$

$$= \frac{c}{a} = \frac{\text{constant term}}{\text{co-efficient of } x^2}.$$

Nature of Roots: In $ax^2+bx+c=0$; $a \neq 0$ a, b, c are real numbers.

$D = b^2-4ac$ called the Discriminant. There are four cases:

Case (I) if D=0 , then quadratic equation has two equal real roots I.e. $x = \frac{-b}{2a}$

and

$\frac{-b}{2a}$.

Case (II) when D>0 and is a perfect square, then the roots are rational (real) and distinct.

Case (III) when D>0 and not a perfect square then the roots are irrational (real) and distinct.

Case (IV) when D<0 , then the roots are not real I.e. Real roots does not exist.

FORMATION OF A QUADRATIC EQUATION:

Let $ax^2+bx+c= 0$ be a quadratic equation

Then $x^2+\frac{b}{a}x + \frac{c}{a} =0$ by dividing b/s by a

Then $x^2 - (\frac{-b}{a})x + \frac{c}{a} = 0$

Or $x^2- (\alpha+\beta) x +\alpha\beta = 0$ | where α ,β are roots of the equation

So any quadratic equation can be formed as

x^2- (sum of roots)x + product of roots = 0

Or simply remember $x^2- Sx+P = 0$

Where `S` is sum of roots `P` the product of roots.

Very Short Answer Type Question (1 mark)

Q1. The roots of equation $x^2-3x-10 = 0$ are (a) -2 and 5 (b) 2 and 5 (c) -2 and -5 (d) 2 and -5

Q2. Both the roots of equation $ax^2+bx+c=0$ are positive

If (a) S> 0 and P> o (b) S< 0 and P< 0

(c) S< 0 and P> 0 (d) S> 0 and P< 0

Q3. Both the roots of equation $ax^2+bx+c = 0$ are negative if (a) S>0 and P>0 (b) S<0 and P<0

(c) S<0 and P>0 (d) S>0 and P<0

Q4. Sum of roots of equation $\sqrt{2}\,x^2+7x+5\sqrt{2}=0$ is

(a) $\frac{7}{\sqrt{2}}$ (b) $-\frac{7}{2\sqrt{2}}$ (c) $\frac{-7\sqrt{2}}{2}$ (d) $\frac{7\sqrt{2}}{2}$

Q5. Product of roots for equation $\sqrt{2}\,x^2+7x+5=0$ is

(a) $\frac{-5}{\sqrt{2}}$ (b) $\frac{-5\sqrt{2}}{\sqrt{2}}$ (c) $\frac{5\sqrt{2}}{\sqrt{2}}$ (d) $\frac{5}{2\sqrt{2}}$

Q6. If P and q are the roots of equation $x^2-px+q=0$

Then (a) P=1 ; q= -2 (b) P= -2 ; q= 0 (c) P= 0 ; q= 1 (d) P= 1 ; q = 0

Q7. If the equation $x^2+4x+k=0$ has real and equal roots then

(a) K= 4 (b) K<4 (c) K> 4 (d)-2 ≤ k≤2

Q8. If the equation $x^2+4x+k=0$ has real and distinct roots then

(a) K<4 (b) K>4 (c) K ≤ 4 (d) K≥ 4

Q9. If the equation $x^2-ax+1=0$ has two distinct roots then

(a) |a| = 2, (b) |a| > 2, (c) |a|<2 (d) none of these.

Q10. A root of a quadratic equation which does not satisfy it is called (a) Real Root (b) Un real root (c) Extraneous root (d) un defined root.

Q11. The equation $\frac{(x-a)(x-b)}{(c-a)(c-b)} .+ \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} = 1$

Where a ≠ b ≠ c is true for values x= a , x= b and x= c

Then this equation will be called

(a) Quadratic equation (b) Cubic equation (c) Identity (d) None of these.

Q12 (a) Match the statement under A with the statement under B

A B

(i) x^2+x+1 (a)	Opens downwards in parabolic graph And intersects x-axis at two points
(ii) $x^2+2x+1= 0$	(b) The graph in a parabola opens upwards and intersects x-axis at two points.
(iii) $x^2-5x+6=0$	(c) the graph opens upwards and do not touch or intersect x-axis .
(iv) $-x^2+2x+3=0$	(d) The graph opens downwards and do not touch or intersect x-axis
(v) $-x^2-x-1=0$	(e)The parabola graph of equation touches x-axis at a single point.

Q12 (b) A polynomial equation of degree n has at the

most -------------roots (fill in the blanks)

Q12 (c) The -------- of quadratic polynomial ax^2+bx+c and the ------------ of a quadratic equation $ax^2+bx+c=0$ are the same. (Fill in the blanks)

Q13. In a polynomial P(x) , in variable x ,such that p(h)= 0 , then for P(x) = 0; h is a root .

True / False

Q14. The graph of a quadratic equation is always a parabola

Q15. The graph of a Quadratic equation which intersect x-axis at one or two points are called roots of quadratic equation. True/ false.

Q16. The parabola graph of a quadratic equation which intersects y-axis at one or two points then these points are called solution or roots of equation . True/false

Q17. If the two linear factors of a quadratic equation are taken as length and breadth of a rectangle then the values of x at which length or breadth of rectangle is zero (i.e area of rectangle is zero) are called roots of quadratic equation. True/False

Q1. Find the roots of equation $2x^2-7x+3 =0$

Q2. Find the value of K for which the equation $x^2+kx+4=0$ and $x^2-8x+k=0$ will have both real and equal roots.

Q3. Find the value of k for which the quadratic equation $2x^2+kx+3=0$ has real and equal roots.

Q4. If α,β are the roots of the quadratic equation $4x^2+3x+7=0$, then find the value $\frac{1}{\alpha}+\frac{1}{\beta}$

Q5. Find the value of k for which x=1 is a root of the quadratic equation $kx^2+x-6=0$

Q6. If x=1 and x=2 are roots of quadratic equation $px^2+3x+q=0$, then find the value of p and q.

Q7. If $x^2-5x+1=0$; find the value of $x+\frac{1}{x}$?

Q8. One root of the quadratic equation $x^2-5x+p=0$ is 2 .find the other root.

Q9. Find the value of p so that the quadratic equation $x^2+5px+16=0$ have no real roots.

Q10. The equation $x^2+4x+k=0$ has real and distinct roots, then find value of k

Q11. Define degree of an equation in one variable.

Q12. Find the sum and product of roots of quadratic equation $\sqrt{5}x^2+3x-5=0$

Q13. Find the value of K if x= -2 is the root of the equation $2x^2+kx-6=0$.

Q14. Fill in the blanks.

(i) A quadratic equation $ax^2+bx+c=0$ has two --------------- if $b^2-4ac>0$

(ii) Two equal roots -------------------------------- if D= 0

(iii) Two equal roots if -------------------------- $= \frac{b}{4a}^2$

15. Find the value of λ so that the equation $2x^2+\lambda x+3=0$ has equal roots.

Q16. Find value of k for which the quadratic equation $kx^2-6x-2=0$ has two equal roots.

Q17. Match column A with column B

A	B

(i) x^2+x+1 (a) Not a quadratic equation

(ii) $(x+2)^2 = x^2+4x+4$ (b) equation has no real roots

(iii) $(x+2)^2 = x^2+3x+1$ (c) sum of roots=0

(iv) $x^2-6=0$ (d) This equation is an identity

LONG ANSWER TYPE (3 MARKS)

Q1. If the equation $(a^2+b^2)x^2 - 2(ac+bd)x+c^2+d^2 = 0$ has equal roots, then show that ad=bc.

Q2. Find two numbers whose sum is 27 and product is 182.

Q3. The sum of a number and its reciprocal is $\frac{5}{2}$ find the number.

Q4. The sum of the reciprocal of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Q5. A shopkeeper buys a number of books for Rs 80. If he had bought 4 more books for the same amount, each book would have cost Rs 1 less. How many books did he buy ?

Q6. Out of a group of cameras in a jungle, $\frac{7}{2}$ times the square root of number is playing in the jungle. The remaining two camels are drinking water from the stream. What is the total number of camels?

Q7. If the list price of a toy is reduced by Rs 2, a person can buy 2 toys for Rs 360.find the original price of the toy.

Q8. 300 apples are distributed equally among a certain number of students. Had there been 10 more students each would have received one apple less. Find number of students.

Q9. If the roots of the equation $(a-b)^2x+(b-c)x+(c-a)= 0$ are equal . Prove that b+c=2a.

Q10. If the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)= 0$ are real and equal .Show that either a=0 or $a^3+b^3+c^3=3abc$

Q11. Find two consecutive odd positive integers, sum of whose squares is 290.

Q12. A two digit number is such that the product of its digits is 12 when 36 is added to the number, the digits are reversed. Find the number.

Q13. The sum of ages of a boy and his brother is 25 years and the product of their ages in years is 126.Find their ages.

Q14. The side of a larger square is double the side of smaller square .the difference between areas is 432 sq cms. The sum of their perimeters is 144 cm.Find the sides of two squares.

Q15. John and jivanti together have 45 marbles .Both of them lost 5 marbles each and the product of the number of marbles they now have is 124.Find the number of marbles they had to start with

16. The length and breadth of a rectangle are the factors of the polynomial $A(x) = 2x^2-7x+6$ This area $A(x) = 0$ if length or breadth of a rectangle is zero .find the value of x when length or breadth of a rectangle is zero.

VERY LONG ANSWER TYPE QUESTIONS

Q1. Find the roots of the quadratic equation (if they exist)

By method of completing square. $2x^2-5x+3=0$

Q2. Solve the quadratic equation by completing square method $\frac{1}{2}x^2-(\sqrt{2}+1)x + \sqrt{2} = 0$

Q3. Using quadratic formula to solve the quadratic equation $a^2b^2x^2-(4b^4-3a^4)x-12a^2b^2=0$

Q4. Solve the quadratic equation by using quadratic formula. $3a^2x^2+8abx+4b^2=0$; $a\neq0$

Q5. Prove that both the roots of the equation $(x-a)(x-b)+(x-c)+(x-c)(x-a)= 0$ are real but they are equal only when $a=b=c$

Q6. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2) = 0$ has equal roots. Prove that $c^2=a^2(1+m^2)$

Q7 if α ,β are the roots of equation $x^2-5x+6= 0$. Form the equation whose roots are $3\alpha+2\beta$ and $2\alpha+2\beta$. Also explain the double answer.

Q8. Find three consecutive positive integers such that the sum of the square of the first and the product of the other two is 154.

Q9. A two digit number is four times the sum of digits and twice the product of its digits Find the number.

Q10. A motor boat whose speed is 9km/h in still water , goes 15 km downstream and come back in a total time of 3 hours 45 minutes. Find the speed of the stream.

Q11. A passenger train takes 2 hours less for a journey of 300 km, if its speed is increased by 5km/h from its usual speed. Find its usual speed.

Q12. An aeroplane left 30 minutes later than its schedule time and in order to reach its destination 1500 km away in time .it had to increase its speed by 250 km/h from its usual speed. Determine its usual speed.

Q13. The length of the hypotenuse of aright triangle exceeds the length of the base by 2cm and exceeds twice the length of the altitude by 1 cm .Find the length of each side of the triangle.

Q14. If two pipes function simultaneously, a reservoir will be filled in 12 hours. one pipe fills the reservoir 10 hours faster than the other. How many hours will the second pipe take to fill the reservoir ?

Q15. In a class test , the sum of Rahim`s marks in Mathematics and English is 40 .Had he got 3 marks more in mathematics and 4 marks less in English , the product of the marks would have been 360.find his marks in two subjects separately.

Q16 A teacher attempting to arranges the students for mass drill in the form of a solid square found that 24 students were left. When he increased the size of the square by 1 student, he found that he was short of 25 students. Find the number of students.

Q 17. One fourth of the herd of camles were grazing in the jungle. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

Q 18. In a class test, the sum of marks by obtained by Ather in Mathematics and Science is 28. Had he got 3 more marks in Mathematics and 4 marks less in Science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained by him in the two subjects separately.

Q 19. Rs 250 were divided equally among a certain number of children. If there were 25 more children each would have received 50 paise less. Find the number of children.

Q 20. A man buys a number of pens for Rs 80. If he had bought 4 more pens for the same amount, each pen would have cost him Rs 1 less. How many pens did he buy?

Very short Answers

Q 1. (a) Q2. (a) Q 3. (c) Q 4. (c) Q5. (c) Q 6. (d)

Q 7. (a) Q 8. (a) Q 9. (b) Q 10. (c) Q 11. (c)

Q 12. (a) (i) → c (ii) → e (iii) → b (iv) → a (v) → d

Q 12. (b) : n Q 12. (c) zeros, Roots.

Q 13. True Q 14. True. Q 15. True Q 16. False

Q 17. True

Short Answer Type

Q 1. $\frac{1}{2}$, 3 Q 2. 16 Q 3. $\pm 2\sqrt{6}$ Q4. $\frac{-3}{7}$ Q5. K = 5

Q6. p = -1, q = -2. Q7. 5 Q 8. 3 Q 9. $\frac{-8}{5} < p < \frac{8}{5}$

Q10 . $-4 < k < 4$ Q 11. The highest power of the variable

Q 12. Sum = $\frac{-3\sqrt{5}}{5}$ product = $-\sqrt{5}$

Q13. K = 1 Q 14. (i) Distinct real root (ii) Equal roots (iii) $c = \frac{b^2}{4a}$

Q 15. $\lambda = \pm 2\sqrt{6}$ Q16. $K = \frac{-9}{2}$ Q 17. (i) ___c; (ii) ___(d)

(iii) ___(a) (iv)____(c)

LONG ANSWER TYPE QUESTIONS

Q2. 13, 14 Q3. $-\frac{1}{2}$, 2 Q4. 7 years Q 5. 16 Q6. 16

Q 7. 18 Q8. 50 Q11. 11 and 13 Q12. 26 Q13. 18 years and 7 yrs.

Q 14. 24cm, 12cm Q15. 36 and 9 marbles Q16. 2 ; $\frac{3}{2}$

VERY LONG ANSWER TYPE QUESTIONS

Q1. 1 ; $\frac{3}{2}$ Q2. $\sqrt{2} + \sqrt{3} + 1$ and $\sqrt{2} - \sqrt{3} + 1$

Q3. $\frac{4b^2}{a^2}$; $\frac{3a^2}{b^2}$ Q4. $\frac{-2b}{3a}$; $\frac{-2b}{a}$

Q7. $x^2 - 22x + 120 = 0$ or $x^2 - 23x + 130 = 0$

Q8. 8,9,10 Q9. 36 Q10. 3 km /h

Q11. 25km /h Q12. 750 km /h Q13. Sides of rt $\triangle$ are 8 m, 15cm ,17cm.

Q14. 30 hours Q 15. Math 21, English 19 or Math = 12, English 28

Q16. 600 students. Q17. 36 camels

Q18. Math = 12, Science 16 or math =9, Science 19

Q19. 100 children Q20. 16 pens

ARITHMATIC PROGRESSION(A.P)

5.1:- INTRODUCION:-

In our daily life, we come across different situations where we need to work following a set pattern. For example, for a particular investment over the year we require an addition of fixed amount every year, so that we get a fixed amount for our use. An insect covers a particular distance every minute and distance keeps reducing every minute uniformly will the insect be able to reach destination? These are few daily life examples which need to be worked out and to work these, we will learn about the concept of sequences and in particular arithmetic sequence before that let's learn some terminology.

5.2:- SEQUENCE

A sequence is an arrangement of numbers in a definite order and according to some rule.

e.g. (a) 1,2,3,4,………. Is a sequence where each successive item is 1 greater than the preceding term.

(b) 1,4,9,16………..is a sequence where each term is the square of successive natural numbers and so on.

5.3:- TERMS OF A SEQUENCE:-

The different mumbers / number in a sequence are called terms.

e.g in the sequence 1,2,3,4…………….., η,……………….

1 is the I^{st} term and is denoted by a_1 or t_1 or T_1

2 is the2^{nd} term and is denoted by a_2 or t_2 or T_2

3 is the 3^{rd} term and is denoted by a_3 or t_3 or T_3 and so on.

η is the ηth tern and is denoted by $a_η$ or $t_η$ or $T_η$.

General Term:- $η^{th}$ tern of a sequence is called the General term of the sequence.

e.g in the sequence 1,4,9,16………., $η^2$……..

i.e 1^2, 2^2, 3^2, 4^2,………… $η^2$………

General term = n^{th} term = n^2 = a_n

If a_1, a_2, a_3, …………a_n ,……be a sequence, it can be resented by <a_n> or {a_n}

5.4:- PROGRESSION:- Sequence following definite patterns are called progression. E.g the following sequence are progression.

(i) 3,7,11,15,……… (ii) ½, $^1/_5$, $^1/_8$, $^1/_{11}$,……….

(iii) 2,4,8,16……….

5.5:- SERIES:- If the terms of the sequence are connected together by +ve or –ve sign, we get a series

For example

(i) 1+3+5+…………

(ii) 2+4+8+…………

(iii) $^1/_3$ + $^1/_6$ + $^1/_9$ +……… are all series of the Sequences

(i) 1,3,5,………….

(ii) 2,4,8,………….

(iii) $^1/_3$, $^1/_6$, $^1/_9$,……… respectively

5.6:- FINITE SEQUENCE and INFINITE SEQUENCE:-

If last term of a sequence is known, then it is a finite sequence. If last term is not known, then it is an infinite sequence.

e.g 1,2,3,5……………99 is a finite sequence.

2,4,6……… is an infinite sequence.

Note:- If we know any three consecutive terms of a sequence, we can find the rest of the terms of the sequence.

5.7:- TYPES OF PROGRESSION:-

There are four types of progression

(i) Arithmatic progresson (AP)

(ii) Geometric Progression(GP)

(iii) Harmonic Progression(HP)

(iv) Arithmatice Geometric prog (AGP).

(i) Arithmatic Progression (A.P). Sequence of no's such that the difference of any two successive terms of the sequence is a constant called common difference (C.D)

e.g 1,4,7,10,……….is an A.P with C.D= 3, and so on

(ii) Geometric Progression (G.P):- Sequence of non-zero numbers such that the quotient (ratio) of any two successive terms of the sequence is a constant (called common Ratio (C.R)

E.g 2,4,8,16………… is a G.P with C.R = 2

(iii) Harmonic Progression (H.P):- Sequence of numbers such that their reciprocal form an Arithmatic progression (A.P)

e.g $^1/_3$, $^1/_6$, $^1/_9$, ……..is a H.P, as the reciprocals of the terms of this progression i.e 3,6,9…………… from an A.P

(iv) Arithmetic Geometric Progression (AGP):- Sequence of nonzero numbers which is both an A.P as well as G.P . e.g 4,4,4,4………is an AGP as this sequence is both an AP with C.D zero and G.P as well with C.R equal to 1

5.8:- ARITHMATIC PROGRESSION (A.P)

As already discussed a sequence ,finite or infinite is said to be an Arithmatic Progression (A.P), if the difference of a term and the previous term is always same(constant)

Thus, a sequence $a_1, a_2, \ldots\ldots, a_n$ is said to form an A.P of

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots\ldots = d$ (say)

i.e if $a_n - a_{n-1} =$ for n = 2,3,4 ………………

The constant'd' (which is independent of n) is called the common difference (C.D)

of an A.P. the first term is usually denoted by 'a'.

e.g :- (i) 2,4,6,8 -------- is an A.P with a=2, d=2

(ii) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$, ---------- is an A.P with $a=\frac{1}{6}$, $d=\frac{1}{6}$

(iii) 0,- 3,-6,------is an A.P with a=0 . d= − 3

5.9 General Term OR the n^{th} term of An A.P :-

If 'a' and'd' be the first term and C.D of an A.P, then the general form of an A.P

is a, a+d, a+2d, a+3d,-----------

i.e $a_1 = a = a + (1 - 1)d$

$a_2 = a + d = a + (2 - 1)d$

$a_3 = a + 2d = a + (3 - 1)d$

$a_4 = a + 3d = a + (4 - 1)d$

– – – –

– – – –

$a_n = a + (n - 1)d$

Therefore, n^{th} term or the General term of A.P is $a_n = a + (n -$

Note: - If in the general term, we know any three quantities, then the fourth one can be easily determined by using above formula.

5.10:- nth TERM FROM THE END OF AN A.P:-

To find the nth term from the end of the A.P

a , a+d , a+2d, - - - - - - -,(a+(n− 3)d, a(n− 2)d, a(n− 1)d = l

Where a= Ist term , d=C.D , l= last term , reverse the A.P, we get the reversed A.P as

l=a(n− 1)d, a+(n− 2)d, a+(n− 3)d, - - - - - -,a+2d, a+d, a

in which ,now

First term (a) =l=a+(n-1)d, C.D (d) = − d and then we find the nth term from the beginning of the reversed A.P , we get

nth term from end = l+(n− 1)(-d)

=> nth term from the end = l-(n-1)d

5.11 PROPERTIES OF AN A.P :-

(i) If a constant is added/subtracted to each term of an A.P, the resulting sequence is also an A.P

(ii) if the term of an A.P are divided or multiplied by a non zero constant , then the resulting sequence is also an A.P

5.12 ARITHMETIC MEAN (A.M):- if three numbers a,A,b from an A.P , then 'A' is said to be an Arithmetic Mean (AM) of 'a' and 'b' and is given by

$$A=\frac{a+b}{2}$$

e.g. A.M of 4 and 16 is

$$A = \frac{4+16}{2} = \frac{20}{2} = 10$$

(i.e. 4 , 10,16 form A.P)

5.13 :-   SUM OF FIRST 'n' TERM OF AN A.P :-

Let S_n = a+(a+d) + (a+2d) + - - - - - - - -+a+(n-1)d (= l)represents the sum of the first 'n' terms of an A.P with first term a, C.D=d and last term (l)= a+(n-1)d then

$S_n = \frac{n}{2}$ [2a+ (n-1) d] this form can be used when n, a and d are known

=> $S_n=\frac{n}{2}(a+a+(n-1)d)$

=> $S_n=\frac{n}{2}(a+l)$ we can use this form when we know the values of 'n' 'a' 'l'.

5.14 :- TO FIND a_n when S_n of an A.P is given :-

From given S_n, find S_{n-1} by substituting (n-1) instead of 'n' in S_n and then apply

$$a_n = S_n - S_{n-1}$$

If $S_n = n^2-1$, then

$S_{n-1}=(n-1)^2-1= n^2+\cancel{1}-2n-\cancel{1}$

$= n^2-2n$

$\therefore a_n=S_n-S_{n-1}$

=> $a_n=(n^2-1)-(n^2-2n)$ $=(n^2-2n)$ $=\cancel{n^2}-1-\cancel{n^2}+2n$

=> $a_n = 2n-1$

5.15 :-   TO SELECT THE TERMS OF AN A.P :-

(i) When we have to select three terms in A.P , we take, a– d, a, a+d

(ii) When we have to select four terms in A.P we take,

a-3d, a-d, a+d, a+3d

Note:- Gauss was the first mathematician who was associated with finding the sum of first 100 natural numbers.

(A) Multiple Choice questions

Choose the correct answer from the given four options:

1. In an A.P , if d=-4, n=7 , a_n= 4, then a is

(A) 6 (B) 7 (C) 20 (D) 28

2. In an A.P , if a=3.5, d=0 , n= 101 then a_n will be

(A) 0 (B) 3.5 (C) 103 (D) 104.5

3. The list of numbers -10, -6, -2, 2, - - - - -is

(A) an A.P with d= -16

(B) an A.P with d=4

(C) an A.P with d=-4

(D) not an AP

4. The 11^{th} term of the AP:

-5, -5/2, 0, 5/2, - - - - - - - is

(A) -20 (B) 20 (C) -30 (D) 30

5. The first four terms of an AP, whose first term is -2 and common difference is -2 are

(A) -2, 0, 2, 4

(B) -2, 4, -8, 16

(C) -2, -4, -6, -8

(D)-2, -4, -8, -16

6. The 21st term of the AP whose first two terms are -3 and 4 is

(A) 17 (B) 137 (C) 143 (D) -143

7. If the 2nd term of an AP is 13 and the 5th term is 25, what is 7th term?

(A) 30 (B) 33 (C) 37 (D) 38

8. Which term of the AP: 21, 42, 63, 84, - - - - ---- is 210?

(A) 9th (B) 10th (C) 11th (D) 12th

9. If the common difference of an AP is 5, then what is $a_{18}-a_{13}$?

(A) 5 (B) 20 (C) 25 (D) 30

10. What is common difference of an AP which is $a_{18}-a_{14}=32$?

(A) 8 (B) -8 (C) -4 (D) 4

11. Two AP`s have the same common difference .the first term of one of these is -1 and that of other is -8. Then the difference between their 4th terms is

(A) -1(B) -8 (C) 7 (D) -9

12. If 7 times the 7th term of an AP is equal to 11th times, its 11th term, then its 18th term will be

(A) 7(B) 11 (C) 18(D) 0

13. The 4th term from the end of AP: -11, -8, -5, - - - - -- 49 is

(A) 37(B) 40 (C) 43 (D) 58

14. The famous mathematician associated with finding the sum of first 100 natural numbers is

(A) Pythagoras (B) Newton (C) Gauss (D) Euclid

15. If the first term of an AP is -5 and the common difference is 2, then the

sum of first 6 terms is

(A) 0 (B) 5 (C) 6 (D) 15

16. The sum of first 16 terms of the AP: 10, 6,2,-----is

(A) -320 (B) 320 (C)-352 (D) -400

17. In an AP if a=1, a_n =20 and S_n =399, then n is

(A) 19 (B) 21 (C) 38 (D) 42

18. The sum of first five multiples of 3 is

(A) 45 (B) 55 (C) 65 (D) 75

19. The 10th term of the AP: 5, 8, 11, 14,--------is

(A) 32 (B) 35 (C) 38 (D) 185

20. In an AP if a= -7.2, d= -3.6, a_n = 7.2 then n is

(A) 1(B) 3 (C) 4 (D) 5

Short Answer Questions

1. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.

2. Determine the AP whose fifth term is 19 and difference of eighth term from the thirteenth term is 20,

3. The 26th, 11th and last term of an AP are 0, 3 and -1/5, respectively. Find the common difference and the number of terms

4. The sum of 5th and 7th terms of AP is 52 and 10th term is 46.Find the AP.

5 . Find the 20th term of AP whose 7th term is 24 less than the 11th term ,first term being 12 .

6. If the 9^{th} term of an AP is zero, prove that its 29^{th} term is twice its 19^{th} term

7. Find whether 55 is a term of AP : 7, 10, 13,--------or not. if yes find which term

8. Determine k so that k^2+4k+8, $2k^2+3k+6$, $3k^2+4k+4$ are consecutive term of an AP.

9. Split 207 into three parts such that these are in AP and product of the two smaller parts is 4623. (Main concept staking 3 terms as a-d, a, a+d with their sum = 207)

10. The angles of a triangle are in AP .the greatest angle is twice the least .Find all the angles of the triangle

[Hint sum of interiors <'s = 180. Taking d 3 <'s as a-d, a, a+d]

11. If the nth term of the two AP`s : 9,7 ,5, ----- and 24, 21, 18,---- are the same .find the value of n. Also find that term.

12. If the sum of 3^{rd} and 8^{th} terms of an AP is 7 and sum of 7^{th} and 14^{th} terms is -3,

Find the 10^{th} term

13. Find the 12^{th} term from the end of the AP: -2, -4,-6,--0-------,-100

14. Which term of the AP : 53.48,43, ------ is the first negative term?

[Hint let a_n be first –ve term ∴ $a_n < 0$]

15. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

[required AP is 11, 15, 19,……………299]

16. Find the sum of the two middle terms of the AP: -4/3, -1, -2/3, --------- $4\frac{1}{3}$

[hint find no. of terms then median of n] n = 18 9^{th} (+) 10^{th} = 3

17. The first term of an AP is -5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

18 Which term of the AP, -2,-7, - 12…..will be -77? Find the sum of this AP upto the term – 77

19. if a_n = 3-4n, show that

a_1, a_2, a_3 ……form an AP. Also find S_{20}

20. In an AP, if S_n = n (4n + 1) find the AP [Hint a_n = S_n-S_{n-1}] ,

21. In an AP, if Sn = $3n^2$ + 5n and ak = 164. Find the value of K.

22. If Sn denotes the sum of first n terms of an AP, prove that

S_{12} = 3(S_8-S_4)

23. Find the sum of first 17 terms of an AP whose 4th and 9th terms are – 15 and – 30. respectively

24. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256. Find the sum of first 10 terms.

25. Find the sum of all 11 terms of an AP whose middle most term is 30.

26. Find the sum of last ten terms of the AP. 8, 10, 12,…..126.

27. Find the sum of first seven numbers which are multiples of 2 as well as of 9.

[Hint: Take the LCM of 2 and 9]

28. How many terms of the AP:
-15, -13, -11, ….. are needed to make the sum -55? Explain the reason for double answer.

29. If the numbers n – 2, 4n – 1 and 5n + 2 are in AP, Find the value of n.

30. Find the value of the middle most terms (s) of the AP: -11, --7, -3,…..49.

[C] Long Answer Questions

1. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its twenty terms.

2. Find the

(i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5

(iii) sum of those integers from 1 to 500 which are multiples of 2 or 5

[Hint (iii) : These numbers will be: multiples of 2+ multiples of 5- multiples of 2 as well as of 5]

3. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.

4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

5. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9

(ii) not divisible by 9

[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9)

6. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2:3. Find the ratio of the 5^{th} term to the 21^{st} tem, and also the ratio of the sum of the first five terms to the sum of the first 21 term.

7. Solve the equation

$-4 + (-1) + 2 + \ldots + x = 437$ [Hint take $a_n = x$, then $s_n = 437$]

8. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first installment of Rs 1000. If he increase the installment by Rs 100 every month. What amount will be paid by him in the 30th installment? What amount of loan does he still have to pay after the 30th installment?

9. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at interval of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flags at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

ANSWERS

Multiple Choice Question

Question	Ans	Question	Ans	Question	Ans	Question	Ans	Question	Ans

01	D	02	B	03	B	04	B	05	C
06	B	07	B	08	B	09	C	10	A
11	C	12	D	13	B	14	C	15	A
16	A	17	C	18	A	19	A	20	D

Short Question Answers

01. a = -1, b +15 c =31

02. 3, 7, 11, ----------

03. d = -1/5, n = 27

04. 1 , 6 , 11,16……….

05. 126

06. ____________

07. Yes, 17^{th} term

08. K = 0

09. 67,69,71

10. 40°, 60°, 80°,

11. n=16, term = - 21

12. a_{10}= -1

13. a_{12} = - 78

14. a_{12} = -2

15. n = 73

16. 3

17. (d = 10, n = 6)

18. [n = 16, S_{16} = -632]

19. [S_{20} = -780]

20. 5, 13, 21, 29…….

21. K = 27

22. ________

23. . S_{17} = -510

24. S_{10} = 100

25. S_{11} = 330

26. (S_{10} = 1170)

27. (S_7 = 504)

28. n = 11, 5

29. n=1

30. (n = 16, 8^{th} , 9^{th})

Long Answer Question

01. 970

02. i)12250, ii) 12750, iii) 75250

03. 3

04. 3,7,11

05. i) 1683 , ii) 13167

06. $a_5:a_{21}$ = 1:3

$S_5: S_{21}$ = 5: 49

07. $\varkappa$ = 50

08. (Rs 44500)]

09. (26 m)

Class 10th **Mathematics**

Topic: Triangles

Triangle:

A three-sided closed figure is called triangle.

Similarity:

Two figures are said to be similar if they have the same shape. Any two circles, squares, rhombuses are always similar

Congruency:

Two figures are said to be congruent if they have same shape and same size. Any two circles of same radius, two squares of same side, two rhombuses of same side are congruent.

Similarity in Triangles:

Two triangles are said to be similar if:

a) their all the three corresponding pairs of sides are equal.
b) their corresponding angles are equal.
c) the ratio of their corresponding sides is same.
d) Any two angles of one triangle are equal to the corresponding angles of another triangle.

Objectives

Q.1. Two triangles are said to be similar if:

a) only one corresponding angle is equal

b) Two corresponding sides are equal

c) Two corresponding angles are equal

d) None of these

Q.2. A diagonal in a square divide it into two ………….. triangles:

a) Similar

b) Congruent

c) Similar as well as congruent

d) None of these

Q.3. The diagonals of a square divide it into ………… Similar triangles.

a) Two

b) Three

c) One

d) Four

Q.4. The perpendicular bisector in an isosceles right-angled triangle divides it into…………

a) Two obtuse angled triangles

b) Two isosceles right-angled triangles but not similar

c) Two isosceles right-angled triangles which are not similar

d) Two isosceles right-angled congruent triangles

Q.5. Which one of the following is not true?

a) Two circles are always similar

b) Two squares area always similar

c) Two rectangles are always similar

d) Two right-angled isosceles triangles are always similar.

Q.6. Two squares are always …………

(Similar/Congruent)

Q.7. Two circles are always …………..

(Similar/Congruent)

Q.8. If $\Delta ABC \simeq \Delta DEF$ and $\Delta DEF \simeq \Delta GHI$, then ΔABC and ΔGHI are also similar triangles. (True/False)

Q.9. If two squares are similar, then their sides are always equal. (True/False)

Q.10. If the radii of two circles are 5cm and 7cm respectively. The circles are similar. (true/false)

Q.11. The median in an equilateral triangle divides it into two triangles which are ……….

Similar but not right-angled/ Similar and right-angled

Q.12. The areas of two similar triangles are in the ratio 4:9, then the ratio of their any two corresponding sides is:

a) 4:9

b) 2:3

c) 16:81

d) 1:2

Q.13. The area of two similar triangles is $100cm^2$ and $200cm^2$. If one side of triangle having area $100cm^2$ is 10cm, then the corresponding side in second triangle is:

a) 10cm

b) 15cm

c) 20cm

d) Cannot be found

Q.14. Two triangles of equal area will always be similar.

(true/False)

Q.15 Two triangles of equal area will always be congruent.

(true/false)

Q.16. Two similar triangles are always equal in area.

(true/false)

Q.17. Two congruent triangles are always equal in area.

(true/false)

Q.18. Two rectangles having one corresponding side same are always similar. (True/False)

Q.19. Two right-angled triangles are always similar.

(true/False)

Q.20. If the ratio of corresponding sides of two triangles is equal, then the triangles are ……….

(Similar/Congruent)

Q.21. Sides of ΔABC are 3cm, 4cm and 5cm. Sides of ΔPQR are 6cm, 8cm and 10cm. Then ΔABC and ΔPQR are:

(Similar/Congruent)

Very Short Answer Type Questions

Q.1. Draw a triangle ABC and draw another line segment DE ∥ BC

Q.2. The ratio of the areas of two similar triangles ……..

(complete the statement of the theorem)

Q.3. Find the hypotonus of a right-angled triangle which has other two sides are 3cm and 4cm.

Q.4. Draw a triangle PQR in which ST is a line segment joining the mid points of the sides PQ and PR. Observe of the line segment ST is parallel to QR

Q.5. If the ratio of the areas of two similar triangles is 4:9, show that the ratio of their any two corresponding pairs of sides is 2:3.

Q.6. Take a rectangular paper. Fold it along any of the diagonal. Check if the folded parts superimpose each other.

Q.7. Draw a rhombus and its two diagonals. Observe if the diagonals are perpendicular to each other.

Q.8. Give the names of any three pairs of objects available in your home which are similar but not congruent.

Q.9. Give the names of any three pairs of objects available in your home which are similar as well as congruent.

Q.10. Give the names of any three pairs of objects available in your home which are neither similar nor congruent.

Q.11. categorise the given objects as similar or congruent or else:

(a) A pair of spoons of same size and same shape

(b) A pair of papers from the same book

(c) A pair of shoes

(d) A pair of cylindrical mugs of different size

Q.12. Draw two triangles which are similar but not congruent.

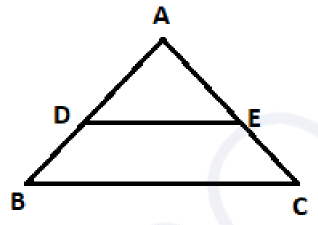
Q.13. Draw two circles which are congruent as well as similar

Q.14. Draw two circles which similar but not congruent.

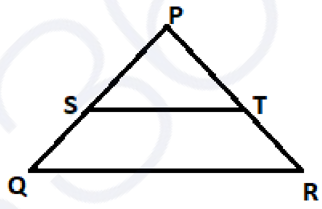
Q.15. Are all rhombuses similar? If so, draw a pair of rhombuses which are congruent as well similar.

Short Answer Type Questions

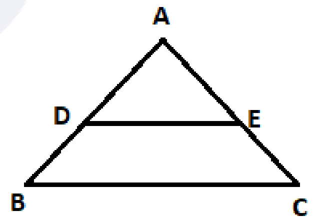
Q.1. In the given figure, DE∥BC, AD=2, BD=2.5 and AE=3, find EC



Q.2. If $PQ = 4cm, PT = 6cm, QS = 4cm$ and $TR = 6cm$, Prove that $ST \parallel QR$

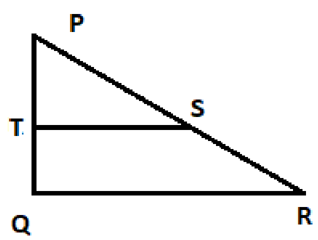


Q.3. ΔABC is isosceles with $AD = 5cm, BD = 4cm$. Find AE and EC when DE ∥ BC

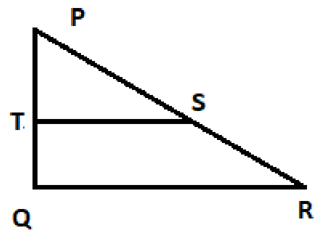


Q.4. Prove that the line segment from the common vertex of equal sides of a triangle divides the triangle into congruent triangles.

Q.5. In ΔPQR, ST ∥ QR, PT = TQ = 3cm, PS =6cm, Find TR.

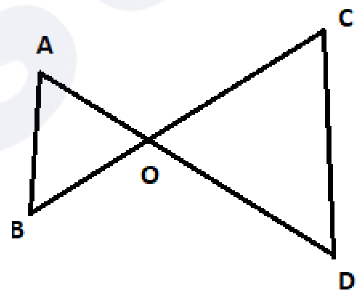


Q.6. In the given figure, if PT=TQ=4cm, PS = SR= 8cm, prove that TS ∥ QR



Q.7. The angles of ΔABC, ∠A=60, ∠B=90 and ∠C=30. In ΔPQR, ∠P=30^0, ∠Q=90^0, ∠R= 30^0. Show that: $\frac{AC}{RP} = \frac{BC}{PQ} = \frac{AB}{RQ}$

Q.8. In the given figure, if AB ∥ CD, prove that ΔABO ≃ ΔDOC

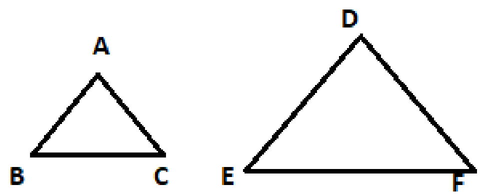


Q.9. In the figure given in Q.8, if ∠B=∠C, prove that ΔABO ≃ ΔDOC

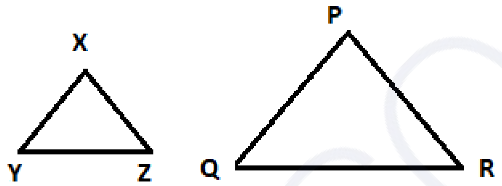
Q.10. Prove that the median in an isosceles triangle divide it into two similar triangles.

Q.11. If perpendicular in a triangle bisects the side of the triangle, prove that the triangle is isosceles.

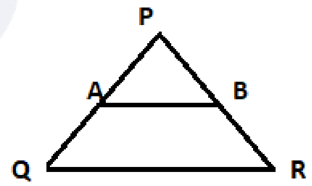
Q.12. In the given figure, ΔABC ≃ ΔDEF. If $ar\Delta ABC = 81cm^2$ and $ar\Delta DEF = 256cm^2$. Find DE when AB=9cm



Q.13. ΔXYZ and ΔPQR are two equilateral triangles. If $ar\Delta XYZ = 4\sqrt{3}cm^2$ and side of the ΔPQR is 6cm, find the side of ΔXYZ and $ar\Delta PQR$



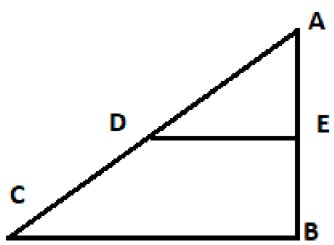
Q.14. In the given triangle AB ∥ QR. If $ar\Delta PAB = ar(ABRQ)$ and AP =5cm, find AQ



Q.15. The hypotonus and one of the other two sides of a right-angled triangle are 15cm and 9cm, find the third side.

Q.16. ΔABC is right-angled at ∠B. If DE ∥ BC, prove that:

$$DE^2 + AE^2 = BC^2 + AC^2$$

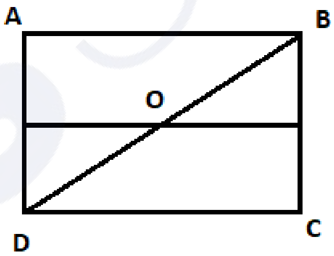


Q.17. Two trees (A) and (B) are of height 21m and 12m respectively. Shows that: $\frac{Shadow\ of\ tree\ (A)}{Shadow\ of\ tree\ (B)} = \frac{7}{4}$

Q.18. If the areas of two similar right-angled triangles are $81cm^2$ and $225cm^2$, show that ratio of their hypotonus is 3:5

Long Answer Type Questions

Q.1. ABCD is a rectangle. If EF ∥ DC, prove that EO=OF and DO=OB



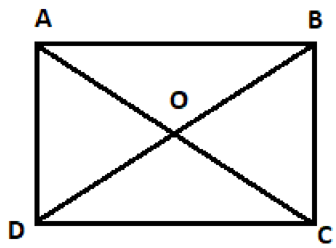
Q.2. Prove that the diagonal of a parallelogram divides it into two triangles which are similar as well congruent.

Q.3. Prove that a square is divided into congruent as well similar triangles by any of the diagonal.

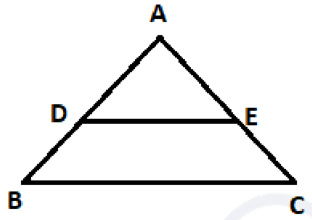
Q.4. ABCD is a rectangle. AC and BD are the diagonals which intersect each other at point O. Prove that:

I) $\Delta AOB \simeq \Delta DOC$

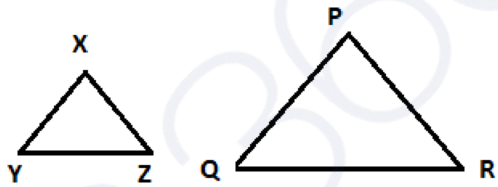
II) $\Delta AOD \simeq \Delta BOC$



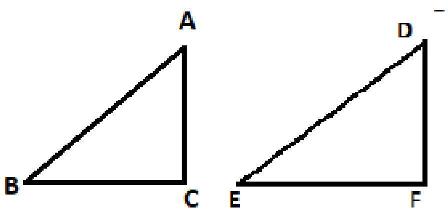
Q.5. In the given figure, DE ∥ BC, find ∠ADE, ∠AED and ∠ A if ∠B=60 and ∠C=65



Q.6. The area of $\Delta XYZ = 25cm^2$ and $\Delta PQR = 36cm^2$. If XY=5cm, YZ=10cm and XZ=2.5cm, find PQ, QR and PR

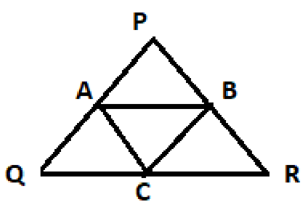


Q.7. $\Delta ABC \sim \Delta DEF$, ΔABC and ΔDEF are right angled triangles right angled at ∠C and ∠F respectively. If $ar\Delta ABC = ar\Delta DEF$ and BC = 3cm, AC=4cm, find EF, DF, DE



Q.8. If the area of two similar triangles is equal, prove that they are congruent.

Q.9. A, B, C are the mid points of the sides PQ, QR and PR of ΔPQR respectively. Prove the $ar\Delta ABC = \frac{1}{4} ar\Delta PQR$

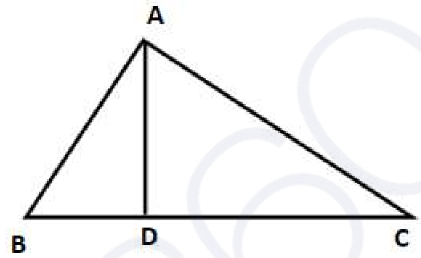


Q.10. Which one of the following triplets as the sides of triangle represent it as right-angled triangle?

(a) 5cm, 6cm, 7cm (b) 6cm, 8cm, 10cm

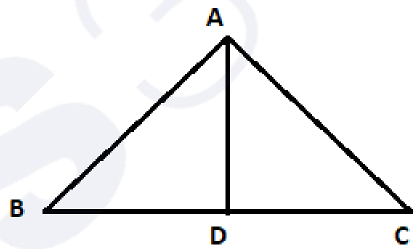
Q.11. ΔABC is right-angled at ∠A. If AD ⊥ BC, prove that

$AD^2 = BD.DC$



Q.12. In the given figure, ΔABC is right-angled at ∠A. If AD ⊥ BC, if BD=DC, prove that

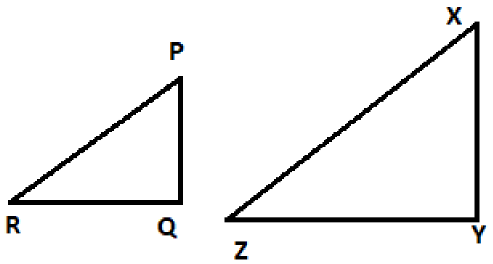
$AD = DC$



Q.13. A 10m ladder is placed 6m away from the foot of a tree. If the top of the ladder touches at the one-third of the height of the tree. Find the height of the tree.

Q.14. ΔPQR and ΔXYZ are two right-angled similar triangles at ∠Q and ∠Y respectively. If the ratio of the areas of ΔPQR to ΔXYZ is $\frac{1}{2}$, Prove that

$2PR.RQ = XZ.YZ$



Q.15. A 20m high tree makes 15m shadow on the ground at a particular time. If another tree makes 12m shadow on the ground at the same time, find the height of the tree.

Answers

Objective:

Q1	(c)
Q2	(c)
Q3	(d)
Q4	(d)
Q5	(c)
Q6	Similar
Q7	Similar
Q8	True
Q9	false
Q10	True
Q11	Similar and right-angled
Q12	(b)
Q13	(c)
Q14	false
Q15	False
Q16	False
Q17	True
Q18	False
Q19	False
Q20	Similar
Q21	Similar

Very Short Answer Type

Q3	5cm
Q4	Yes ST ∥ QR
Q6	Yes, Superimpose each other
Q7	Yes Perpendicular to each
Q11	a) Congruent
	b) Congruent
	c) Not similar
	d) May or may not be similar
Q15	Yes

Short Answer Type

Q1	EC=3.7cm
Q3	AE=5cm, EC=4cm
Q5	TR=6cm
Q12	DE =14cm
Q13	4cm and $9\sqrt{3}cm^2$
Q14	AQ= $5(\sqrt{2}-1)cm$
Q15	12cm

Long Answer Type

Q5	∠AED= 60
	∠AFE = 65
	∠A=55
Q6	PQ=6cm, QR = 12cm, PR = 3cm
Q7.	EF = 3cm. DF =4cm, DE = 5cm
Q10	(b)
Q13	24cm
Q15	16cm

CORORDINATE GEOMETRY

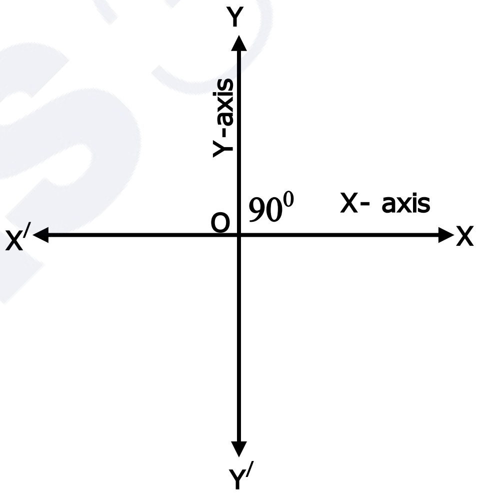
CLASS – 10TH **CHAPTER – 7**

7. 1 Introduction:

Coordinate Geometry is that branch of geometry which defines the position of a point in a plane by a pair of algebraic numbers. It is also called ALGEBRAIC GEOMETRY** OR **ANALYTICAL GEOMETRY.

7. 2 Rectangular Axis and Origin:

Let $X^/OX$ and $Y^/OY$ be two perpendicular straight lines intersecting at the point "O". Then



i. $X^/OX$ is called the *axis of "x"* or *the "x-axis"*.

ii. $Y^/OY$ is called the *axis of "y"* or the *"y – axis"*.

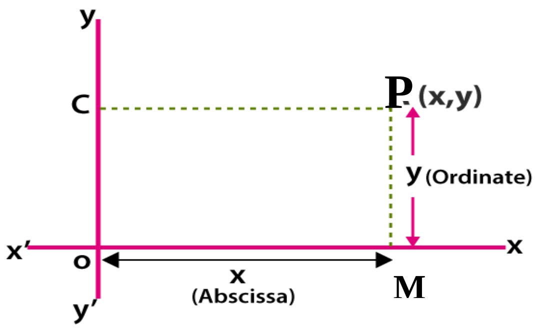
iii. Both $X^/OX$ and $Y^/OY$ taken together, in this very order, are called the *"Rectangular axes"* or the *"axes of Co-ordinates"* or *the "Coordinate axes"* or simply the *"axes"*.

(**Note – 1 :** *They are called Rectangular axes because the angle between them is a Right angle.)*

iv. Their point of intersection "O" is called the origin.

7. 3 Cartesian Co-ordinates of a point:

Let $X^/OX$ and $Y^/OY$ be two perpendicular lines intersecting at the point "O". Let "P" be any point in the plane of the axes. From "P", draw PM perpendicular $X^/OX$, then



a) "OM" is called the *"x-ordinate"* or "*abscissa of P"* and is denoted by "x"

b) "MP" is called the *"y-coordinate"* or "*Ordinate of "P"* and is *denoted by "y".*

c) The numbers "x" and "y" are called the *Cartesian rectangular Coordinate* or simply the "*Coordinate of P"*, represented by *P(x,y).*

*(**Note – 2 :** In this symbolic representation i.e. P(x,y), the "Abscissa is always written first and separated from the Ordinate (at second place) by a comma.*

Remember:

1. Abscissa is the perpendiculars distanc from "y – axis".
2. Ordinate is the distance of a point from "x axis".
3. Abscissa is +ve to the right to the "y-axis" and –ve to the left of "y-axis".
4. Ordinate is +ve above "x-axis" and –ve below "x-axis".
5. Abscissa of any point on "y – axis" is zero
6. Ordinate of any point on "x-axis" is zero.
7. Corodinates of the origin are (0,0).

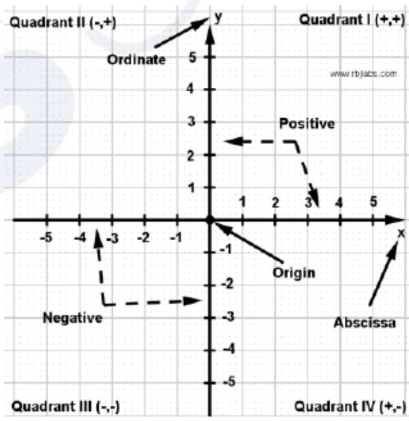
***Note – 3:** The two axis divide the plane into four regions called the Quadrants. The signs of the Co-ordiantes in different Quardants are:*

a) (+,+) ⟶ both abscissa and Ordinate are +ve in first quadrant.

b) (-, +) ⟶ in second quadrant.

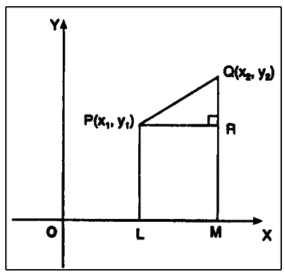
c) (-, -) ⟶ in third quadrant.

d) (+, -) ⟶ in fourth quadrant.



7. 4 Distance between two Points: *(Distance Formula)*

Let P(x_1, y_2) and Q (x_1, y_2) be the given points. Draw PL and QM perpendicular OX and PR perpendicular QM. Then



PR = **LM** = **OM – OL** = $x_2 - x_1$

RQ = **MQ – MR** = **MQ – LP** = $y_2 - y_1$

In right angled Δ PRQ

BY Pythagoras Theorem

$PQ^2 = PR^2 + RQ^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$$

Corollary: Distance of a point (x, y) from the origin (0, 0) = $\sqrt{x^2 + y^2}$

Note – 4: *When three points are given and it is required to prove that they form:*

a) *an Isosceless triangle, show that two of its sides are equal*
b) *an equilateral triangle, show that its three sides are equal.*
c) *a right angled triangle, show that the square of one side is equal to the sum of the square of other two sides.*
d) *They are collinear, show that the sum of the distances between two points is equal to the distances between the first point and the third point.*

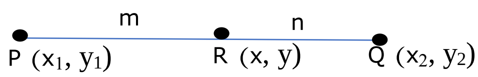
Note – 5: *When four points are given and it is required to prove that they form a;*

a) *Square, show that all sides are equal and diagonals are equal.*
b) *Rhombus, show that all sides are equal and diagonals are unequal.*
c) *Rectangle, show that the opposite sides are equal and diagonals are also equal.*
d) *Parallelogram, show that the opposite sides are equal.*

7. 5 Section Formula:

Let R(x,y) be the point which divides the joining of P(x_1, y_1) and Q = (x_2, y_2) in the ratio m:n internally. The "R" lies between "P" and "Q" and have Coordinates as

$$\mathbf{R(x,y) = R}\left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right]$$



i.e. *x-Coordinate of R is* $\frac{mx_2 + nx_1}{m+n}$ *and*

y-coordiante of R is $\frac{my_2 + ny_1}{m+n}$

This is known as ***Section Formula.***

Note: If we have to find the ratio in which "R" divides "PQ", it is convenient to take the ratio K:1 instead of m:n

Corollary: Mid Point Formula: If "R" is the mid – point of "PQ", then "m = n" i.e. the ratio is 1:1.

Therefore, Co-ordinates of the mid-point of the line – segement joining P(x_1, y_1) and Q (x, y_2) are

$$R \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) \quad \text{i.e}$$

$$R\left[\frac{\textit{Sum of absissae}}{2}, \frac{\textit{Sum of Ordinates}}{2}\right]$$

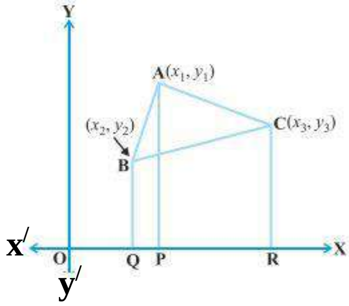
Remember:

1. The median is a line joining a vertex of the triangle to the middle point of the opposite side.
2. The point of intersection of the medians of a triangle is called the "*Centroid*" of the triangle.
3. The Centroid of a triangle divides each median in the ratio 2:1, 2 always being on the sides of the vertex.
4. The Co-ordiantes of the Centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are:

$$\left\{ \frac{\mathbf{x_1+x_2+x_3}}{\mathbf{3}} \quad , \quad \frac{\mathbf{y_1+y_2+y_3}}{\mathbf{3}} \right\}$$

7. 6 Area of a Triangle:

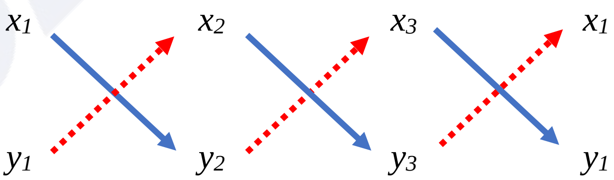
If A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are the vertices of a triangle ABC, then the area of triangle is given by

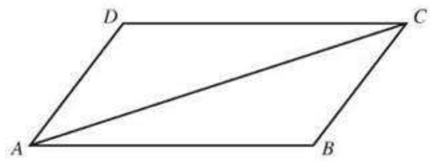


$$\text{Area of } \Delta \text{ ABC} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

Note – 6:

1. *If during calculations, area comes out to be negative, then we take it absolute value.*
2. *If we have to find some conditions, where area of a triangle is given, then we take both signs under consideration.*
3. *We can find area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) by using following diagram.*





We multiply the terms connected by arrows, for bold arrow, we put plus sign and for dotted arrow, we put negative sign.

Area of Triangle = $\frac{1}{2}$ **{ $x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3$ }**

4. *If we have to find the area of a quadrilateral ABCD, we can draw one diagonal and divide the quadrilaterals into two triangles and then apply formula for finding area of a triangle.*

Area (Quad. ABCD) = ar $(\Delta\, \boldsymbol{ABC})$ + ***ar*** $(\Delta\, \boldsymbol{BCD})$

7. 7 Condition for Collinearity:

A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are collinear (i.e. the three points lie on same straight line). If the area of triangle formed by these vertices is zero.

i.e. Area of Triangle = 0, if A, B, C are collinear.

1 MARK QUESTIONS

Choose the correct Answer

Q1 The distance of the point P (2, 3) from the x-axis is

a) 2 b) 3 c) 1 d) 5

Q.2 The distance between the points A (0,6) and B (0, -2) is

a) 6 b) 8 c) 4 d) 2

Q.3 The distance of the point P (-6, 8) from the origin is

a) 8 b) $2\sqrt{7}$ c) 10 d) 6

Q.4 The distance between the points (0, 5) and (-5, 0) is

a) 5 b) $5\sqrt{2}$ c) $2\sqrt{5}$ d) 10

Q.5 AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5,0). The length of its diagonal is

a) 5 b) 3 c) $3\sqrt{4}$ d) 4

Q.6 The perimeter of a triangle with vertices (0,4), (0,0) and (3,0) is

a) 5 b) 12 c) 11 d) $7\sqrt{5}$

Q.7 The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is 28 (T/F)

Q.8 The points (-4, 0), (4, 0), (0, 3) are vertices of an Isosceles Triangle. (T/F)

Q.9 If the distance between the point (2,-2) and (-1, x) is 5, one of the value of x is 1 (T/F)

Q.10 The mid points of the line segment joining the points A (- 2, 8) and B (-6, -4) is (- 4, 2) (T/F)

Q.11 The points A (9, 0), B (9, 6), C (-9, 6) and D (-9, 0) are the vertices of a Rhombus (T/F)

Q.12 The point which divides the line segment joining the points (7,-6) and (3, 4) in the ratio 1:2 internally lies in the

a) 1 - Quadrant b) II – Quadrant c) III – Quadrant d) IV – Quadrant.

Q.13 The point which lies on the perpendicular bisector of the line segment joining the point A (-2,-5) and B (2, 5) is

a) (0, 0) b) (0, 2) c) (2, 0) d) (-2, 0)

Q.14 The fourth vertex D of a parallelogram ABCD whose three vertices are A (-2, 3), B (6, 7) and C (8, 3) is

a) (-2, 3) b) (0,-1) c) (-1, 0) d) (1, 0)

Q.15 If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then

a) AP = $\frac{1}{3}$ AB b) AP = PB c) PB = $\frac{1}{3}$ AB d) AP = $\frac{1}{2}$ AB

Q.16 If P ($\frac{a}{3}$, 4) is the midpoint of the line segment joining the point Q (-6, 5) and R (-2, 3), then the value of "a" is

a) -4 b) -12 c) 12 d) -6

Q.17 Line intersects the y-axis and x-axis at the points P and Q respectively, if (2,-5) is the midpoint of PQ, then the coordinates P and Q are respectively

a) (0,-5) and (2, 0) b) (0, 10) and (-4, 0) c) (0, 4) & (-10, 0) d) (0,10) and 4,0)

Q.18 The area of a triangle with vertices (a, b + c), (b, c + a) and (a, b + c) is

a) (a+ b + c) ^2 b) 0 c) (a + b+ c) d) ab

Q.19 If the distance between the points (4,p) and (1,0) is 5, then the value of p is

a) 4 only b) $\pm$ 4 c) -4 only d) 0

Q.20 If the points A (1, 2), O (0,0) and C (a, b) are collinear, then

a) a = b b) a = 2b c) 2a = b d) a = -b

Q.21 If P (1, 2), Q (4, 6), R (5, 7) and S (a, b) are the vertices of a parallelogram PQRS then a =……………., b =…………….

Q.22 There are / is ……………. number of points on x-axis which are at a distance of 2 units from (2,4)

Q.23 The distance of the points (h, k) from x-axis is …………….

Q.24 ……………. is The area of the triangle with vertices at the points (a, b+ c) (b, c+ a) and (c, a+ b)

Q.25 The distance of A (5, -12) from the origin is …………

Answers

Q1.	(b)	Q.2	(b)	Q.3	(c)	Q.4	(b)	Q.5	(c)	Q6	(b)
Q.7	(F)	Q.8	(T)	Q.9	(F)	Q.10	(T)	Q.11	(F)	Q.12	(d)
Q.13	(a)	Q.14	(b)	Q.15	(d)	Q.16	(b)	Q.17	(d)	Q.18	(b)
Q.19	(b)	Q.20	(c)	Q.21	a=2, b=3	Q.22	(0)	Q.23	ǀKǀ		
Q.24	0	Q.25	(13)								

Short Answer Type Questions

Q.1 If A (6, -1), B(1,2) and C (K,3) are three points such that AB = BC. Find the value of K.

Q.2 The distance between the points P(a Sin θ, a cos θ) and Q (a cos θ,- a sin θ).

Q.3 Check whether the points (1,5), (2, 3) and (-2, -11) are collinear or not.

Q.4 Find a relation between "x" and "y", if the points (x, y), (1, 2) and (7, 0) are collinear.

Q.5 Find the point on x – axis which is equidistance from (2, -5) and (9, 1).

Q.6 Find the point on y – axis, each of which is at a distance of 13 units from the point (-5, 7).

Q.7 Show the point on A(1,2), B(5,4), C (3,8), D (-1, 6) are vertices of a square.

Q.8 If two vertices of an equilateral triangle are (0, 0) and (3,0), find the third vertex.

Q.9 Find the coordinates of the mid point of the line segement joining the points A (3,0) and B (5,4).

Q.10 The mid point of the line segment joining A(2a, 4) and B (-2, 3b) is M (1,2a +1). Find the vlaues of a and b.

Q.11 In what ratio does the points P (2,5) divide the line segment joining A (8, 2) and B (-6, 9).

Q.12 Find the coordinates of the point of trisection of the line segement joining the points (4, -1) and (-2, -3).

Q.13 Find the lengths of the medians of the triangle where vertices are A (1,-1), B(0,4) and C (-5, 3).

Q.14 AB is a diameter of a circle with centre C (-1, 6). If the coordinates of A are (-7, 3). Find the coordinates of B.

Q.15 Find the coordinates of the point P which is three – fourth of the way from A (3, 1) to B (-2, 5).

Q.16 Find th centroid of a triangle ABC whose vertices are A(-1,0), B(5,2) and C (8,2)

Q.17 Determine if the points (1,5), (2,3) and (-2, -11) are collinear using area of triangle.

Q.18 (Ref. using area of triangle) find the value of K if the points (8,1), (K, -4) and C (2, -5) are collinear.

Q.19 If A (2, 1), B (6,0), C(5, -2) and D (-3, -1) are the vertices of a quadrilateral. Find the area of quaderilatral ABCD.

Q.20 Find the value of K for which the area formed by the triangle with vertices A (K,0), B (4,0) and C (0, 2) is 4 square units.

ANSWER

Q.1	$\sqrt{33}+1$	Q.2	$a\sqrt{2}$	Q.3	(Not) area of triangle ≠0
Q.4	(x+3y – 7 = 0)	Q.5	(2, 0)	Q.6	(0, 19) or (0,-5)
Q.8	$(\frac{3}{2}, \frac{3\sqrt{3}}{2})$ or $(\frac{3}{2}, -\frac{3\sqrt{3}}{2})$	Q.9	(-1,2)	Q.10	a = 2, b = 2
Q.11	(3:4)	Q.12	(2, -5/3) and (0,-7/3)	Q.13	$\frac{\sqrt{130}}{2}$, $\sqrt{3}$
Q.14	(5,9)	Q.15	(6/7, 19/7)	Q.16	(4,0) Q.17 No
Q.18	(K=3)	Q.19	+ 15 square unit	Q.20	K = (0, 8)

Long Answer Type

Q.1 Prove that the points A (a, a), B (-a, - a) and C ($-\sqrt{3}$ a, $\sqrt{3}$ a) are the vertices of equilateral triangle. Calculate the area of this triangle.

Q.2 If the distance of P(x, y) from A (5, 1) and B (-1, 5) are equal. Prove that 3x = 3y

Q.3 If A, B and P are the points (-4, 3), (0, -2) and (α, β) respectively and P is equidistance from A and B. Show that $8\alpha \neq 10\beta + 21 = 0$

Q.4 If P (a, -11), Q (5, b), R (2, 15) and S (1, 1) are the vertices of a parallelogram PQRS. Find the value of "a and b".

Q.5 In what ratio is the line segment joining A (6, 3) and B (-2, -5) is divided by the x-axis. Also find the coordinates of the point of intersection of AB and the x – axis.

Q.6 The point P (-4, 1) divide the line segment joining the points A (2,-2) and B is the ratio 3: 5. Find the point B.

Q.7 Show that the points (3,-2), (5, 2) and (8, 8) are collinear by using Section formula.

Q.8 A (3, 2) and B (-2, 1) are two vertices of a triangle ABC whose centroid is G (5/3, – 1/3). Find the coordinates of the third vertex "C".

Q.9 Calculate the ratio in which the line joining the points A (6, 5) and B (4, -3) is divided by the line y = 2. Also find the coordinates of the point of intersection.

Q.10 Find the area of the triangle formed by joining the mid points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

ANSWERS

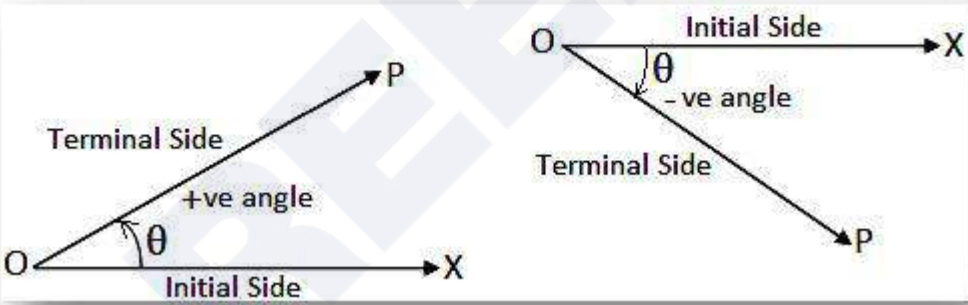
Q.1	$2\sqrt{3}\, a^2$	Q.4	(a=4, b =3)	Q.5	{(3:5, (3, 0)}	Q.6	(-14, 6)
Q.8	(4, - 4)	Q.9	{3:5 , (21/4, 0)}	Q.10	(1 Sq unit, 1:4)		

INTRODUCTION TO TRIGONOMETRY

Trigonometry originated as part of the study of triangle. The word ***Trigonometry*** is derived from the Greek Word "***Tri (means three) Gon (meaning sides)*** and ***metron (means measure)".*** In fact trigonometry means the measurement of three concerned figures, and the first definitions were in form of triangles. However, ***trigonometric functions*** can also be defined using the unit circle, ***a definition that makes them periodic or repeating.*** Many naturally occurring processes are also periodic, days and nights, seasons, water level in a tidal basin, the blood pressure in a heart, an alternating current and the position of air molecules, transmitting a musical note, all fluctuate regularly. Such a Phenomenon can be presented by Trigonometric functions.

The Sine and Cosine functions are commonly used to model periodic function phenomenon, such as sound and light waves. The position and velocity of harmonic oscillators, sunlight intensity and day length and average. Temperature variations throughout the year.

ANGLE: Angle is the figure obtained by the rotation of a given ray about its end point from its initial position to the terminal position.



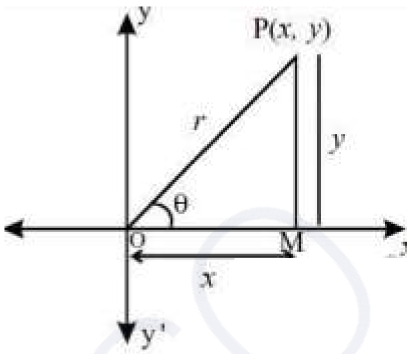
The measure of an angle is the amount of rotation from its initial position to the terminal position. If the *ray rotates in anticlockwise sense, the angle formed is taken positive.* If the *ray rotates in clockwise sense, the angle formed is taken negative.*

Remarks:

"OP" and "OX" are called arms of angle <POX and point "O" is called vertex of the angle.

Trigonometric Ratio (T – Ratio) of an acute angle of a Right Triangle:

In "XOY" – plane, let a revolving line "OP" starting from "OX", traces angle XOP= θ. From "P (x, y)" draw "PM perpendicular to "OX".



In right angled triangle OMP, OM = "x" (adjacent side), PM = "y" (opposite side): OP = "r" (hypotenuse).

1. Sin θ = $\frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{y}{x}$
2. Cos θ = $\frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{x}{r}$
3. Tan θ = $\frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{y}{x}$
4. Cosec θ = $\frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{r}{y}$
5. Sec θ = $\frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{r}{x}$
6. Cot θ = $\frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{x}{y}$

Reciprocal Relations:

1	sinA.cosecA = 1	$\sin A = \frac{1}{\text{cosec}A}$	$\text{cosec}A = \frac{1}{\sin A}$
2	tanA.cotA = 1	$\tan A = \frac{1}{\cot A}$	$\cot A = \frac{1}{\tan A}$
3	cosA.secA = 1	$\cos A = \frac{1}{\sec A}$	$\sec A = \frac{1}{\cos A}$

Quotient Relations

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ *and* $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Remark 1 : Sin θ is read as the ***"Sine of angle θ"*** and it should never be interpreted as the product of ***"Sin" and "θ".***

Remark 2 : **Notation:** ***(Sin*** θ)^2 is written as ***Sin***^{2} θ ***(read "Sin square θ").*** Similarly ***(Sin θ)***^{n} is *written as* ***Sin***^{n} θ (read ***Sin nth*** and ***power "θ"), "n"*** being ***positive integer.***

Note : **(Sin** θ)^2 should not be written as ***Sin*** θ^2 or as ***Sin***^{2} θ^2

Remark 3 : Trigonometric ratios depend only on the value of θ and are independent of the lengths of the sides of the right angle triangle.

Trigonometric Ratios of Complementary Angles:

- **Sin (90 – θ) = Cos θ** **Cos (90 – θ) = Sin θ**
- **Tan (90 – θ) = Cot θ** **Cot (90 – θ) = Tan θ**
- **Sec ((90 – θ) = Cosec θ** **Cosec (90 – θ) = Sec θ**

TRIGONOMETRIC RATIOS FOR ANGLE OF MEASURE 0^0, 30^0, 45^0, 60^0, AND 90^0 IN TABULAR FORM

$\theta =$	0^0	30^0	45^0	60^0	90^0
$\sin\theta =$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta =$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta =$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined
$\text{cosec } \theta =$	Undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\ \theta =$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
$\cot\ \theta =$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Identities

An equation involving ***trigonometric ratios of an angle*** is said to be a ***trigonometric identity,*** if it is ***satisfied*** for all ***values of θ*** for which the given ***trigonometric ratios*** are ***defined***.

Identity – 1 $\quad \sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Identity – 2 $\quad \sec^2\theta - \tan^2\theta = 1$

$\tan^2\theta = \sec^2\theta - 1$

Identity – 3 $\quad \mathrm{cosec}^2\theta = 1 + \cot^2\theta$

$\cot^2\theta = \mathrm{cosec}^2\theta - 1$

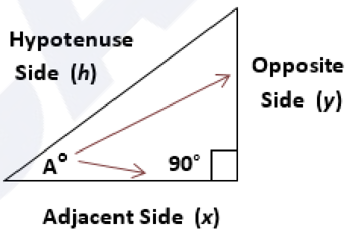
Remark – 1 $\quad \dfrac{\sin^2\theta}{1-\cos\theta} = 1 + \cos\theta$

Remark – 2 $\quad \dfrac{\cos^2\theta}{1-\sin\theta} = 1 + \sin\theta$

Remark – 3 $\quad \sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$

Remark – 4 $\quad \mathrm{cosec}\,\theta - \cot\theta = \dfrac{1}{\mathrm{cosec}\theta + \cot\theta}$

SOME TIPS

Right Triangle	SOH-CAH-TOA Method	Coordinate System Method
	SOH: $\text{sine}(A) = \sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\sin(A) = \frac{y}{h}$
	CAH: $\text{cosine}(A) = \cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\cos(A) = \frac{x}{h}$
	TOA: $\text{tangent}(A) = \tan(A) = \frac{\text{Opposite}}{\text{Adjacent}}$	$\tan(A) = \frac{y}{x}$
	$\text{cosecant}(A) = \csc(A) = \frac{1}{\sin(A)} = \frac{\text{Hypotenuse}}{\text{Opposite}}$	$\csc(A) = \frac{1}{\sin(A)} = \frac{h}{y}$
	$\text{secant}(A) = \sec(A) = \frac{1}{\cos(A)} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$	$\sec(A) = \frac{1}{\cos(A)} = \frac{h}{x}$
	$\text{cotangent}(A) = \cot(A) = \frac{1}{\tan(A)} = \frac{\text{Adjacent}}{\text{Opposite}}$	$\cot(A) = \frac{1}{\tan(A)} = \frac{x}{y}$

Some Trigonometric functions in terms of the other five

in terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Note: ***Csc*** θ ***is same as Cosec*** θ

Additional Formula

- ✓ ***Sin (A + B) = Sin A Cos B + Cos A Sin B***
- ✓ ***Sin (A – B) = Sin A Cos B – Cos A Sin B***
- ✓ ***Cos (A + B) = Cos A Cos B – Sin A Sin B***
- ✓ ***Cos (A – B) = Cos A Cos B + Sin A Sin B***
- ✓ ***Sin 2 A = 2 Sin A Cos A***
- ✓ ***Tan 2 A*** = $\frac{2 \tan A}{1 - \tan^2 A}$

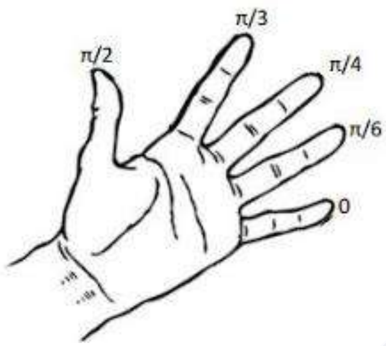
TRIGONOMETRIC HAND TRICK

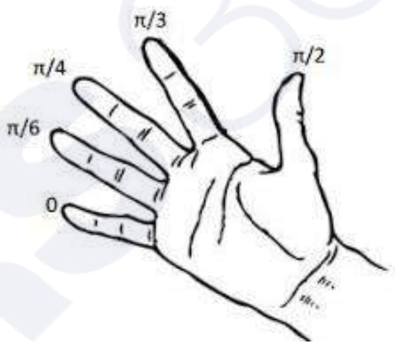
This is an easy way to remember the values of common values of trigonometric functions in the first quadrant. It's a lengthy explanation, but once you know this by heart, you can use this trick for all four quadrants. All you need is your non-dominant hand.

Step – 1 : Hold out your non-dominant hand.

Step – 2 : Assign" the following values to your fingers.

If your non-dominant hand is your left hand	**If your non-dominant hand is your right hand.**





Step – 3 : Find a trig problem. e.g. Cos $(\frac{\pi}{6})$

Step – 4 : Hold down the finger assigned for that angle.
For example: Hold down your ring finger for π/6

Step – 5 : Know the following Formulas

$$\sin\theta = \frac{\sqrt{bottom\ fingers}}{2} \qquad \cos\theta = \frac{\sqrt{top\ fingers}}{2} \qquad \tan\theta = \frac{\sqrt{bottom\ fingers}}{\sqrt{top\ fingers}}$$

"Bottom fingers" refer to how many fingers are ***"below"*** the finger you've held down. ***"Top fingers"*** refer to how many fingers ***"above"*** the finger you've held down. Your thumb counts.

Step – 6 : Calculate the values for your trig expression using the appropriate formula.

For example: When you hold down your ring finger, there is *1* finger below your ring finder (your pinkie), and there are *3* fingers above your ring finger (your thumb, your index finger, and your middle finger). Therefore, Cos (π/6) = $\sqrt{3}$ /2 If you need Sin (π/6) = $\sqrt{1}/2$ = ½

1 Mark Questions

Q.1 Define Identity.

Q.2 Sin θ = $\frac{3}{4}$ for any value of θ (T/F)

Q.3 What is the value of (1 – Cos^2θ) $Cosec^2$ θ

Q.4 The value of $\frac{2 \text{ Tan } 30^0}{1 - \text{Tan}^2 30^0}$ is

a) Sin 60^0 b) Cos 60^0 c) Tan 60^0 d) Sin 30^0

Q.5 In Triangle PQR right angled at Q. PQ+QR=25 cm PQ=5cm then the value of Sin P is

a) $\frac{7}{25}$ b) $\frac{24}{25}$ c) $\frac{12}{13}$ d) None of the these

Q.6 What is the Maximum value of $\frac{1}{\operatorname{cosec}\theta}$

Q.7 The value of $Cos1^0$ Cos 2^0 $Cos3^0$– Cos 180^0

a) 1 b) 0 c)- 1 d) None of these

Q.8 If $Cosec^2$θ (1+Cos θ) (1- Cos θ) = K then value of K is

a) -1 b) 2 c) 1 d) -1

Q.9 If x tan 45^0 Cos 60^0 = Sin 60^0 Cot 60^0 then x is equal to

a) 1 b) √3 c) ½ d) 1/ √2

Q.10 ……… is The value of θ for which √3 Sin θ = Cos θ

Q.11 If tan A = ¾ and A+B =90^0 the value of Cot B is

a) ¾ b) 5/4 c) 3/5 d) ¾

Q12 If tan θ = ¾ then Cos^2 θ – Sin^2θ =

a) 7/25 b) 1 c) -7/25 d) 4/25

Q13 If A, B and C are interior angles of Tringale ABC then Sin ($\frac{B+C}{2}$) =

a) Sin $\frac{A}{2}$ b) Cos $\frac{A}{2}$ c) Sin $\frac{A}{2}$ b) Cos $\frac{A}{2}$

Q14 The value of Sin A Cos $(90^0 - A)$ + Cos A Sin $(90^0 - A)$ is

a) -1 b) 0 c) 2 d) 4

Q.15 Define Angle.

ANSWERS

Q.1	Definition	Q.2	(F)	Q.3	(1)	Q.4	(c)	Q.5	(c)
Q.6	(1)	Q.7	(b)	Q.8	(c)	Q.9	(a)	Q.10	($30^{0)}$
Q.11	(d)	Q.12	(a)	Q.13	(b)	Q.14	(d)	Q.15	Definition

2 Mark Questions

Q.1 Evaluate:

$$\frac{2 \text{ Tan } 53^0}{\text{Cot } 37^0} - \frac{\text{Cot } 80^0}{\text{Tan } 10^0}$$

Q.2 Prove that:

$$(1 - \text{Cos}^2\theta) \text{ Cosec}^2\theta = 1$$

Q.3 Prove that:

$$\text{Cosec } \theta \sqrt{1 - \text{Cos}^2\theta} = 1$$

Q.4 Find the value of x, if

$$\text{Tan } 3x = \text{Sin } 45^0 \text{ Cos } 45^0 + \text{Sin } 30^0$$

Q.5 Solve the equation when $0^0 < \theta < 90^0$

$$3 \text{ Tan}^2\theta - 1 = 0$$

Q.6 Evaluate:

$$\text{Cos}^2 13^0 - \text{Sin}^2 77^0$$

Q.7 Evaluate:

$$\text{Sin } 60^0 \text{ Cos } 30^0 + \text{Cos } 60^0 \text{ Sin } 30^0$$

(Use Sin A Cos B + Cos A Sin B = Sin (A + B)

Q.8 If triangle ABC is a right angled at "C", then what is

Cos (A + B) + Sin (A + B) equal to

Hint *(A + B + C = 180^0 ⟹ A + B = 180^0 – C ⟹ A + B = $180^0 - 90^0 = 90^0$)*
i:e A + B = 90^0

Q.9 Evaluate:

$$\text{Tan}10^0 \text{ Tan } 15^0 \text{ Tan } 75^0 \text{ Tan } 80^0$$

Q.10 If A = 45^0, verify that

Sin 2 A = 2 Sin A Cos A

ANSWER

Q.1	(1)	Q.4	(15^0)	Q.5	($\theta = 30^0$)	Q.6	(0)
Q.7	(1)	Q.8	(1)	Q9	(1)		

3 Mark Questions

Q.1 If Cos A = $\frac{7}{25}$, find the value of Tan A + Cot A

Q.2 If θ is acute angle and Sin θ = Cos θ. Find the value of $2Tan^2\theta + Sin^2\theta - 1$.
(Hint: Sin θ = Cos θ ⟹ Sin θ / Cos θ = 1 ⟹ Tan θ = Tan 45^0, θ = 45^0)

Q.3 Given that Sin (A + B) = Sin A Cos B + Cos A Sin B. Find the value of 75^0
(Hint: Put A = 45^0, B = 30^0)

Q.4 Given that Sin 2 A = 2 Sin A Cos A. Find the value of Sin 120^0
(Hint: Put A = 60^0)

Q.5 Evaluate $4 (Sin^4 60^0 + Cos^4 30^0) – 3 (Tan^2 60 – Tan^2 45^0) + 5 Cos^2 45^0$

Q.6 Prove that Tan 1^0 Tan 2^0 Tan 3^0…….. Tan 89^0 = 1

Q.7 If Sin (θ + 36^0) = Cos θ, where θ +36^0 is acute angle. Find θ
(Hint: Use Cos θ = Sin (90 – θ))

Q.8 If Tan θ + Cot θ = 2. Find value of $Tan^2\theta + Cot^2\theta$. *(Hint: Squaring both sides*

Q.9 **Prove that:**

$$\frac{\sin\theta}{1-\cos\theta} = \operatorname{cosec}\theta + \cot\theta$$

Hint L.H.S = $\frac{sin\theta}{1 - cos\theta} = \frac{sin\theta}{1 - cos\theta} \times (\frac{1 + cos\theta}{1 + cos\theta})$

Q.10 Express Sin 85^0 + Cosec 85^0 in terms of Trigonometric ratios of angles between 0^0 and 45^0. *(Hint: Use Sin (90 – θ) = Cos θ and Cosec (90 – θ) = Sec θ)*

ANSWERS

Q.1	$\frac{625}{168}$	Q.2	$\frac{3}{2}$	Q.3	$\frac{1}{\sqrt{2}}(\sqrt{3}+1)$	Q.4	$\frac{\sqrt{3}}{2}$	Q.5	1
Q.7	27^0	Q.8	2	Q.10	Cos 5^0 + Sec 5^0				

4 Mark Question

Q.1 In triangle ABC right angled at "C", if Tan A $=\frac{1}{\sqrt{3}}$. Find the value of Sin A + Cos B + Cos A Sin B

***(Alternate method: Hint**: A + B +C = 180^0, A+B= 180^0– C = $180^0 - 90^0 = 90^0$)*
Sin A Cos B + Cos A Sin B = Sin (A + B) = Sin 90^0 = 1

Q.2 If Sin B = $\frac{1}{2}$, Show that 3 Cos B – 4 Cos^3 B = 0

Q.3 If Tan θ = $\frac{20}{21}$ Show that

$$\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{3}{7}$$

Q4 If x = a Sec θ + b Tan θ , y = a tan θ + b Sec θ . Prove that $x^2 - y^2 = a^2 - b^2$

Q.5 Evaluate

$$\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$$

Q.6 Prove the identity

$$\frac{\sin\theta}{(\cot\theta + \cos ec\,\theta)} = 2 + \frac{\sin\theta}{(\cot\theta - \cos ec\,\theta)}$$

Hint $$\frac{\sin\theta}{(\cot\theta + \cos ec\,\theta)} - \frac{\sin\theta}{(\cot\theta - \cos ec\,\theta)} = 2$$

Q.7 If sin θ + cos θ = P and Sec θ + Cosec θ = q. Show that q (P^2-1) = 2P

Q.8 If Sin θ + Sin^2 θ = 1 prove that Cos^2 θ + $\cos^4$ θ = 1
(Hint: Sin θ =1 – Sin^2 θ = Cos^2 = Sin θ = Cos^2 θ)
Now Cos^2 θ + Cos^4 θ = Cos^2 θ + $(\text{Cos}^2\ \theta)^2$ = Cos^2 θ + Sin^2 θ =1

Q.9 If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$. Prove that $(\frac{x}{a})^{2/3} + (\frac{y}{b})^{2/3} = 1$

Q.10 Prove that

$(1 - \sin \theta + \cos \theta)^2 = 2(1 + \cos \theta)(1 - \sin \theta)$

(Hint $(1 - \sin \theta + \cos \theta)^2$ ***use $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$***

$= 1 + (-\sin \theta)^2 + \cos^2 \theta - 2\sin\theta - 2\sin\theta\cos\theta + 2\cos\theta$

$= 2 - 2\sin\theta + 2\cos\theta(1 - \sin\theta)$

$= 2(1 - \sin\theta) + 2\cos\theta(1 - \sin\theta) = 2(1 - \sin\theta)(1 + \cos\theta)$

Q.11 **Evaluate**

$$\frac{\sin^2 20^0 + \cos^2 70^0}{\cos^2 20^0 + \cos^2 70^0} + \frac{\sin(90 - \theta)\sin\theta}{\tan\theta} + \frac{\cos(90 - \theta)\cos\theta}{\cot\theta}$$

Q.12 **Evaluate**

$$\frac{2}{3}(\cos^4 30^0 - \sin^4 45^0) - 3(\sin^2 60 - \sec^2 45^0) + \frac{1}{4}\text{ Cost}^2 30^0$$

ANSWERS

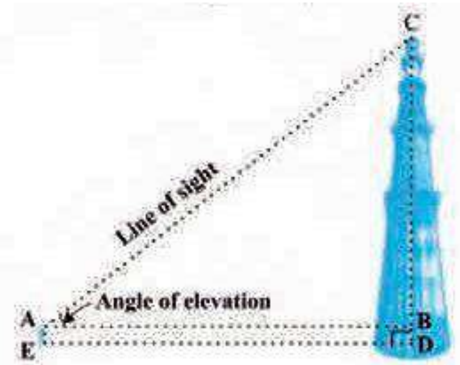
Q.5 (2) Q11 (2) Q.12 $\frac{113}{24}$

SOME APPLICATIONS TO TRIGONOMETRY

IMPORTANT FORMULAS AND CONCEPTS

Angle of Elevation:

In the below figure, the line AC is drawn from the eye of the student to the top of the minar is called the line of sight. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal is called the angle of elevation of the top of the minar from the eye of the student. Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.



Angle of Depression:

In the below figure, the girl sitting on the balcony is looking down at a flower pot placed on a stair of the temple. In this case, the line of sight is below the horizontal level. The angle so formed by the line of sight with horizontal is called the angle of depression. Thus the angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when point is below the horizontal level, i.e. the case when we lower our head to look at the point being viewed.

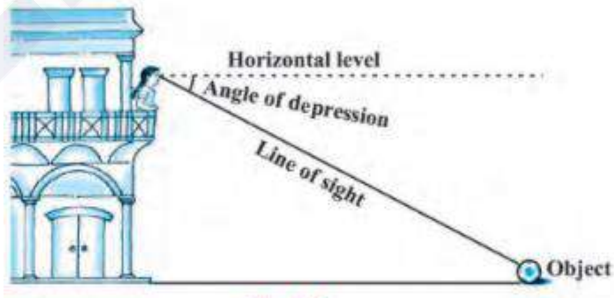
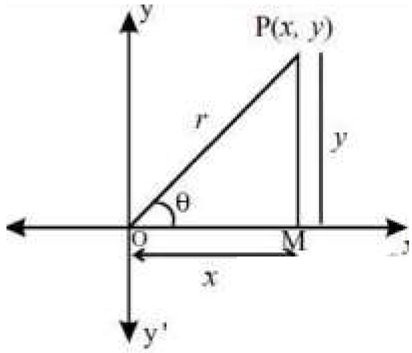


Fig. 9.3

Trigonometric Ratio (T – Ratio) of an acute angle of a Right Triangle:

In “XOY” – plane, let a revolving line “OP” starting from “OX”, traces angle XOP= θ. From “P (x, y)” draw “PM perpendicular () to “OX”.

In right angled triangle OMP, OM = “x” (adjacent side), PM = “y” (opposite side): OP = “r” (hypotenuse).



1. Sin θ = $\frac{\text{Opposite side}}{\text{Hypotenuse}}$ = $\frac{y}{r}$

2. Cos θ = $\frac{\text{Adjacent side}}{\text{Hypotenuse}}$ = $\frac{x}{r}$

3. Tan θ = $\frac{\text{Opposite side}}{\text{Adjacent side}}$ = $\frac{y}{x}$

4. Cosec θ = $\frac{\text{Hypotenuse}}{\text{Opposite side}}$ = $\frac{r}{y}$

5. Sec θ = $\frac{\text{Hypotenuse}}{\text{Adjacent side}}$ = $\frac{r}{x}$

6. Cot θ = $\frac{\text{Adjacent side}}{\text{Opposite side}}$ = $\frac{x}{y}$

Reciprocal Relations:

1	$\sin A = \frac{1}{\operatorname{cosec} A}$	$\operatorname{cosec} A = \frac{1}{\sin A}$
2	$\tan A = \frac{1}{\cot A}$	$\cot A = \frac{1}{\tan A}$
3	$\cos A = \frac{1}{\sec A}$	$\sec A = \frac{1}{\cos A}$

Quotient Relations

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ *and* $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Trigonometric Ratios of Complementary Angles:

- **Sin (90 – θ) = Cos θ** **Cos (90 – θ) = Sin θ**
- **Tan (90 – θ) = Cot θ** **Cot (90 – θ) = Tan θ**
- **Sec ((90 – θ) = Cosec θ** **Cosec (90 – θ) = Sec θ**

TRIGONOMETRIC RATIOS FOR ANGLE OF MEASURE 0^0, 30^0, 45^0, 60^0, AND 90^0 IN TABULAR FORM

$\theta =$	0^0	30^0	45^0	60^0	90^0
$\sin\theta =$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta =$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta =$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined
$\text{cosec}\ \theta =$	Undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta =$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
$\cot\theta =$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

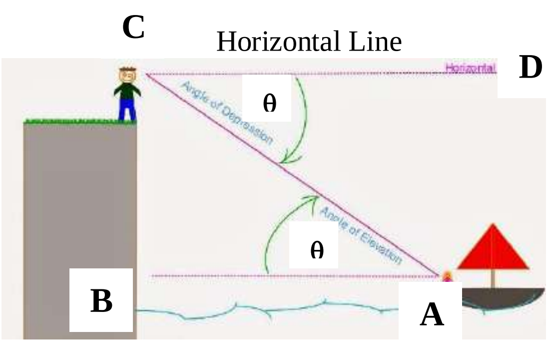
Remarks

In the figure above the angle of depression is angle ACD $= \theta$

Since DC ll AB and AC is transversal.

Hence angle ACD = angle CAB = θ

Thus angle CAB = θ is also be taken as angle of depression.



OBJECTIVE TYPE

(1 Marks Questions)

Q.1 **What is the angle of elevation of the sun when the shadow of a pole is $\sqrt{3}$ times the length of the pole?**

a) 30^0 b) 45^0 c) 60^0 d) None of these.

Q.2 **The shadow of a tower is 15 m, when the Sun's elevation is 30^0. What is the length of the shadow, when the Sun's elevation is 60^0?**

a) 3 m b) 4 m c) 5 m d) 6 m

Q.3 **What is the angle of elevation of the Sun, When the shadow of a pole of height "x" m is $\frac{x}{\sqrt{3}}$.**

a) 30^0 b) 45^0 c) 60^0 d) 75^0

Q.4 **A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts a shadow of 40 m long on the ground. The height of the tower is**

a) 60 m b) 65 m c) 70 m d) 72 m

Q.5 **The tops of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60^0 with the horizontal, then the length of the wire**

a) $8\sqrt{3}$ m b) 8m c) $6\sqrt{3}$ m d) 6 m

Q.6 **The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30^0. When it is 60^0. Then what is the height of the tower?**

a) 25 m b) $25\sqrt{3}$ c) $\frac{25}{\sqrt{3}}$ m d) 30 m

Q.7 **The angle of elevation of the tip of a tower from a point on the ground is 45^0. Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60^0. What is the height of the tower, to the nearest metre?**

a) 48 m b) 49 m c) 50 m d) 51 m

Q.8 **The angle of depression from the top of a light house of two boats are 45^0 and 30^0 towards the west. If the two boats are 5 m apart, then the height of the light house is**

a) $(2.5\sqrt{3} - 1)$ m b) $2.5(\sqrt{3} - 1)$ m c) $2.5\sqrt{3} + 1)$ m d) $2.5(\sqrt{3} + 1)$ m

Q.9 The angle of elevation of the top of an unfinished pillar at a point 150 m from the base is 30^0. If the angle of elevation at the same point is to be 45^0, then the pillar has to be raised to a height of how many meters?

a) 59.4 m b) 61.4 c) 62.4 d) 63.4 m

Q.10 From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower are observed to be 30^0 and 45^0, respectively. What is the height of the tower?

a) 400 m b) $400\sqrt{3}$ m c) $400/\sqrt{3}$ m d) None of these

Q.11 On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30^0 to 45^0. The height of the chimney is

a) 120 m b) $60(\sqrt{3}m - 1)$ c) $60(\sqrt{3}m + 1$d) None of these

Q.12 A man standing at a point "P" is watching the top of elevation of 30^0. The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60^0. If the height of the tower is 30 m, then the distance he moves is

a) 20 m b) $20\sqrt{3}m$ c) 22 m d) $22\sqrt{3}m$

Q.13 The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building, if the height of the tower is 75 m and the angle of elevation of the top of the tower from the bottom of the building is 60^0 ?

a) 25 m b) 37.5 m c) 50 m d) 60 m

Q.14 The angles of elevation of the top of a tower from two points which are at distance of 10 m and 5 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is

a) 5 m b) 15 m c) $\sqrt{15}m$ d) $\sqrt{75}m$

Q.15 The angles of elevation of a top of an inaccessible tower from two points on the same straight line from the base of the tower are 30^0 and 60^0, respectively. If the points are separated at a distance of 100 m, then the height of the tower is close to

a) 86.6 m b) 84.6 m c) 82.6 m d) 80.6 m

Q.16 Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m. What is the distance between their tops?

a) 13 m b) 17 m c) 18 m d) 23 m

Directions: *(Q No. 17 – 20) read the following information carefully to answer the questions that follow.*

As seen from the top and bottom of a building of height "h m", the angles of elevation of the top of a tower of height $\frac{(3+\sqrt{3})h}{2}$ *m and "α" and "β", respectively.*

Q.17 If β = 30⁰, then what is the value of Tan α

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) None of these.

Q.18 If α = 30⁰, then what is the value of Tan β

a) 1 b) $\frac{1}{2}$ **c)** $\frac{1}{3}$ d) None of these

Q.19 If α = 30⁰ and h = 30 m, then what is the distance between the base of the building and the bae of the tower

a) $15 + 15\sqrt{3}m$ b) $30 + 15\sqrt{3}m$ c) $45 + 15\sqrt{3}m$ d) None of these

Q.20 If β = 300 and if q is the angle of depression of the foot of the tower as seen from the top of the building, then what is tan q equal to ?

a) $\frac{3-\sqrt{3}}{3\sqrt{3}}$ b) $\frac{3+\sqrt{3}}{3\sqrt{3}}$ c) $\frac{2-\sqrt{3}}{3\sqrt{3}}$ d) None of these

1 Mark Questions

Q.1 Define line of sight

Q.2 Define angle of Elevation

Q.3 Define angle of Depression

ANSWERS

Q.1	(a)	Q.2	(c)	Q.3	(c)	Q.4	(a)	Q.5	(a)
Q.6	(b)	Q.7	(c)	Q.8	(d)	Q.9	(d)	Q.10	(d)
Q.11	(c)	Q.12	(b)	Q.13	(c)	Q.14	(c)	Q.15	(a)
Q.16	(a)	Q.17	(b)	Q.18	(a)	Q.19	(c)	Q.20	(a)

2 Marks Questions

Q.1 The height of a tower is 10 m. What is the length of its shadow when sun's altitude is 45^0?

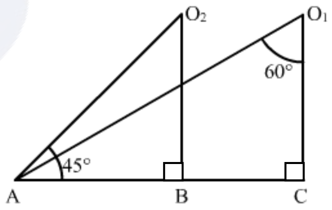
Q.2 If the ratio of the height of a Tower and the length of its shadow is $\sqrt{3}:1$. What is the angle of elevation of the sun?

Q.3 What is the angle of elevation of the sun when the length of the shadow of a vertical pole is equal to its height?

Q.4 A vertically straight tree 15 m high is broken by the wind in such a way that its top just touches the ground and makes an angle of 60^0 with the ground. At what height from the ground did the tree break?

Q.5 An observer 1.5 m tall is 28.5 m away from a tower. The angle of elevation of the top of a tower from her eyes is 45^0. What is the height of a tower?

Q.6 In the given figure what are the angles of depression from the observing position O_1 and O_2 of the object at "A"?



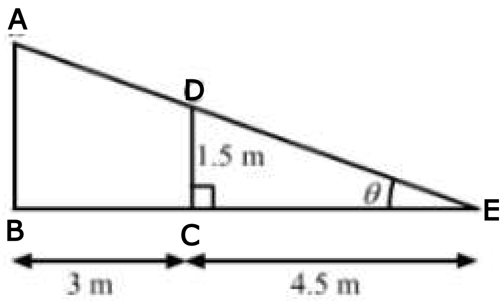
Q.7 A ladder makes an angle of 60^0 with the ground when placed against wall. If the foot of ladder is 2 m away from the wall, then length of the ladder is.

Q.8 The length of shadow of tower on the plane ground is $\sqrt{3}$ times the height of tower. The angle of elevation of sun is?

Q.9 The angle of depression of a car standing on the ground from the top of a 75 m tower is 30^0. Then the distance of car from the base of the tower is?

Q.10 A kite is flying at a height of 60 m above the ground. The inclination of the string with the ground is 60^0. Find the length of string, assuming that there is no slack in the string.

Q.11 A tower is 50 m high. Its shadow is "x" meters shorter, when the sun's altitude is 45^0 than when it is 30^0. Find value of "x".



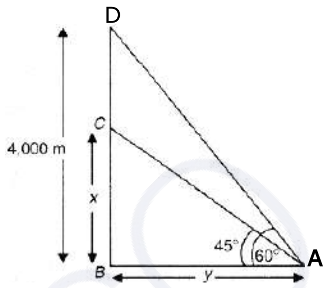
Q.12 If a 1.5 m tall girl stands at a distance of 3 m from a lamppost and costs shadow of length 4.5 m on the ground. Find the length /height of the lamppost. *(Hint: In Δ DCE, Tan θ =* $\frac{1.5}{4.5} = \frac{1}{3}$ *and in Δ ABE Tan θ =* $\frac{AB}{7.5}$*, AB =* $\frac{7.5}{3}$ *=2.5 m)*

ANSWERS

Q.1	(10 m)	Q.2	(60^0)	Q.3	(45^0)	Q.4	(6.9 m)
Q.5	(30 m)	Q.6	(30^0, 45^0)	Q.7	(2√3)	Q.8	(30^0)
Q.9	75√3	Q.10	(40√3	Q.11	(50√3 -1)		

3 Mark Questions

Q.1 An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Find the vertical distance between the aeroplane at that instant.



Q.2 A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top of tower is 60^0. What is the height of the tower?

Q.3 A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 60^0 with the level of ground. Determine the height of the wall.

Q.4 The length of a string between a kite and a point on the ground is 90 m. If the string makes an angle θ with the ground level such that Tan θ = $\frac{15}{8}$. How high is the kite?

Q.5 A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff. At a point on the plane 70 meters away from the tower, an observer notices that the angles of elevation of the top and bottom of the flag staff are respectively 60^0 and 45^0. Fin the height of flag staff and that of tower.

Q.6 The shadow of a tower when the angle of elevation of the sun is 45^0 is found to be 10 m longer than when it was 60^0. Find the height of tower.

Q.7 The angles of elevation of the top of a rock from the top and foot of a 100 m high tower are respectively 30^0 and 45^0. Find the height of the rock.

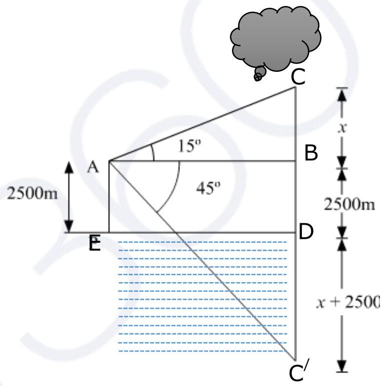
Q.8 As observed from the top of a 15 m tall light house, the angles of depression of two ships approaching it are 30^0 and 45^0. If one ship is directly behind the other. Find the distance between the two ships.

Q.9 The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45^0 and 30^0 respectively. Find the height of the light house.

Q.10 The length of a shadow of a tower standing on level plane is found to be 2 x meters longer, when the suns altitude is 30^0 than when it was 45^0. Prove that the height of tower is x ($\sqrt{3}$ + 1).

Q.11 The angle of elevation of a tower from a point on the same level as the foot of the tower is 30^0. On advancing 150 m towards the foot of the tower, the angle of the elevation of the tower becomes 60^0. Show that the height of the tower is 129.9 m (use $\sqrt{3}$ = 1.732).

Q.12 The angle of elevation of a stationary cloud from a point 2500 m above the lake is 15^0 and the angle of depression of its reflection in the lake is 45^0. What is the height of the cloud above the lake level? (Use Tan 15^0 = 0.268).



(Hint let " C/" be the reflection of cloud "C" in the lake. Then "C/D"= 2500 m+ x, so that BC/ = 5000 +x.

In triangle ABC/ Tan 45^0 = $\frac{BC'}{AB}$

- ➢ *AB = 5000 +x*

Take Δ ABC, Tan 15^0 =$\frac{BC}{AB}$ = $\frac{x}{5000+x}$ and solve it.

ANSWERS

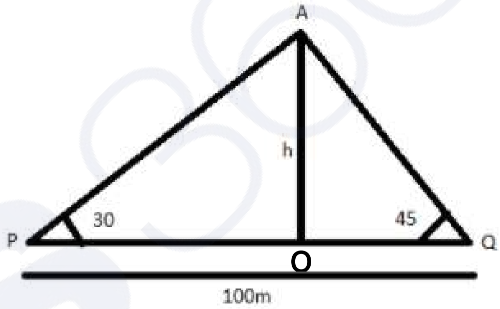
Q.1	$\frac{4000(\sqrt{3}-1)}{\sqrt{3}}$	Q.2	$20\sqrt{3}$	Q.3	$2\sqrt{3}$	Q4	79.41
Q.5	51.24 m, 70 m	Q.6	23.66 m	Q.7	236.5 m	Q.8	109.5 m
Q.9	273.2 m	Q.12	2500 $\sqrt{3}$ m				

4 MARK QUESTIONS

Q.1 The horizontal distance” between two towers is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30^0. If the height of the second tower is 60 m. Find the height of the first tower.

Q.2 The angle of elevation of a cloud from a point 60 m above the lake is 30^0 and angle of depression of the reflection of cloud in the lake is 60^0. Find the height of the cloud.

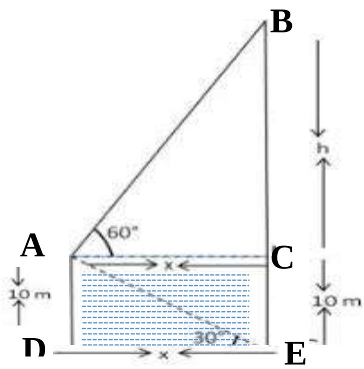
Q.3 There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. “P” and Q” are points directly opposite to each other in the two banks and in a line with a tree. If the angles of elevation of the top of tree from “P” and “Q” are respectively 30^0 and 45^0. Find the height of the tree.



(Hint: Tan $30^0 = \frac{OA}{OP}$, Tan $45^0 = \frac{OA}{OQ}$, OP + OQ= $(\sqrt{3}+1)h$, h =$\frac{100}{\sqrt{3}+1}$)

Q.4 From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30^0 and 60^0. Find the height of the tower.

Q.5 A man standing on the deck of a ship which is 10 m above water level. He observe the angle of depression of the base of the hill as 30^0. Calculate the distance of the hill from the ship and the height of the hill.



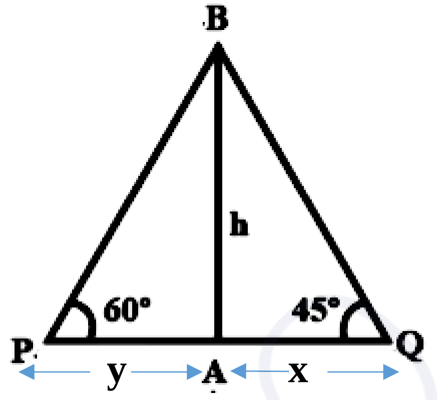
(Hint: Tan 60^0 =$\frac{\sqrt{h}}{x}$, h = $\sqrt{3}$, Tan 30^0=$\frac{10}{x}$, x = $10\sqrt{3}$, h= $\sqrt{3}(10\sqrt{3}$= 30).

Q.6 As observed from the top of a light house 100 m above the sea level, the angle of depression of a ship, sailing directly towards it changes from 30^0 to 45^0. Determine the distance travelled by ship.

Q.7 An aeroplane at a altitude of 200 m observes the angles of depression of opposite points on the two banks of a river to be 45^0 and 60^0. Find the width of the river.

Q.8 The shadow of a vertical tower on the level ground increases by 10 m, when the altitude of the sun changes from angle of elevation 45^0 to 30^0. Find the height of the tower.

Q.9 A fire in a building "B" is reported on telephone to two fire stations "P" and "Q" 20 Km apart from each other on a straight road. "P" observes that the fire is at an angle of 60^0 to the road and "Q" observes that it is at an angle of 45^0 to the road. Which station should send its team and how much will this team have to travel.



(Hint: AP <AQ, station "P" must send the team. Tan $60^0 = \frac{h}{y}$, $h = \sqrt{3}y$, *Tan* $45^0 = \frac{h}{r}$, $h = r$, $x + y = 20$ *proceed)*

Q.10 From a window 15 m high above the ground in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 30^0 and 45^0 respectively. Show that the height of opposite house is 23.66 m (take $\sqrt{3} = 1.732$).

ANSWER

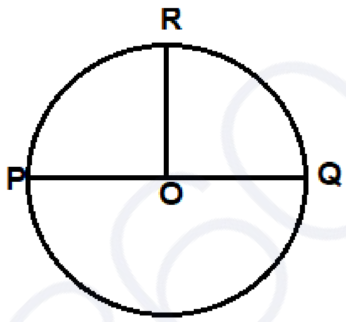
Q.1	140.83 mtrs.	Q.2	120 m	Q3	$\frac{100}{\sqrt{3}+1}$	Q4	40 m
Q.5	$10\sqrt{3}$, 40	Q.6	73.2 m	Q7	$200\,(\frac{\sqrt{3}+1}{\sqrt{3}}$	Q.8	$5(\sqrt{3}+1$
Q.9	7.32 Km	Q.10	23.66 m				

Class 10^{th} Mathematics

Circles

Circle:

A circular path equidistant from a fixed point (Centre) is called circle.



Diameter:

A diameter is a line segment from one point to another point on circumference which passes through the centre of the circle. In the given circle PQ is diameter

Radius:

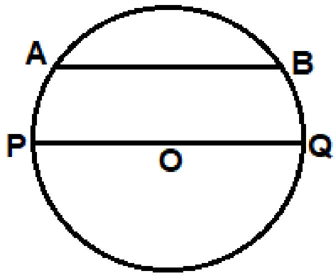
Radius is a line segment from centre to any point of the boundary of the circle. Radius is half of the diameter. In the given circle, OR is the radius.

Circumference:

The length of the boundary of the circle is called circumference of circle. It is calculated by using the formula $2\pi r$ or πd, where $'r'$ is the radius and $'d'$ the diameter of circle.

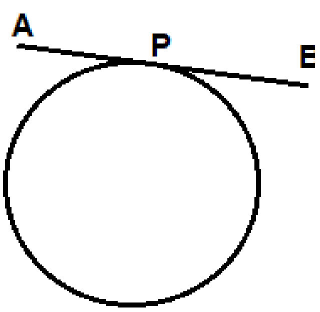
Chord:

A chord in a circle is a line segment which has the end points on the boundary of the circle. In the given circle, both AB and PQ are the chords. PQ is also the diameter and the chord.



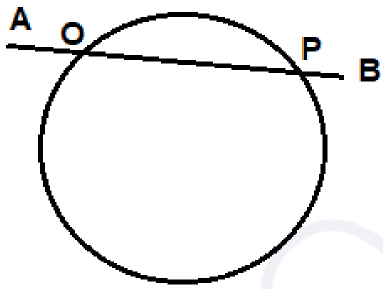
Tangent:

A tangent is a line segment which touches the circle only at one point. In the given figure, AE is a tangent which touches the circle at point P



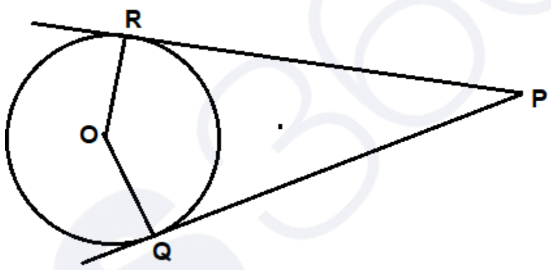
Secant:

A secant is a line segment which touches the circle at two different points. In the given circle, AB is a secant which the touches the circle at point O and B



Pair of tangents:

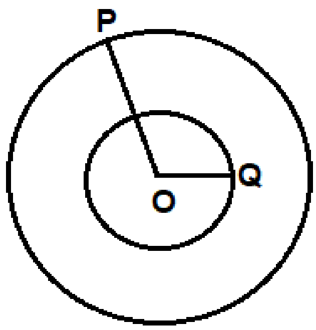
From an external point, two tangents are drawn to a circle. In this figure, from external point P, two tangents PR and PQ are drawn. These tangents touch the circle at R and Q respectively. Also, $PR = PQ$.



At any point on the circumference of the circle, we can draw only one tangent. In this way we can as many as tangents to a circle.

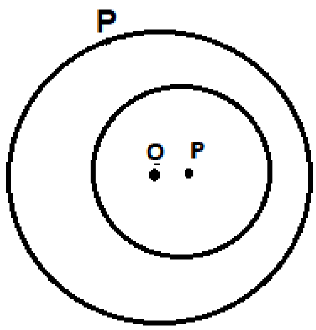
Concentric Circles with same centre:

Two or more circles are said to be concentric with common centre if they have common centre and of different radii. (See given figure) In this figure, there are two concentric circles with same centre O and having radii as OQ and OP



Concentric Circles with different centres:

Two or more circles are said to be concentric with different centres if they lie in one another and their centres are different. (See given figure) In this figure, there are two concentric circles centre O and P. The bigger circle has the centre O and the smaller circle P



Questions for Practice

Objective type

Q.No.1. A line segment which touches the circle at two different points and passes out from the circle is called:

a) Chord b) Secant

c) Tangent d) Radius

Q.No.2. A line segment which touches the circle only at one point is called:

a) Chord b) Secant

c) Tangent d) Diameter

Q.No.3. How many tangents can be drawn from an external point to a circle?

a) 1 b) 2

c) Infinite c) 0

Q.No.4. How many tangents can be drawn from a point on the boundary of a circle?

a) only 1 b) Infinitely many

c) More than one d) 0

Q.No.5. $ABCD$ is a cyclic quadrilateral with $\angle A = 80^0$ and $\angle B = 90^0$. The measure of $\angle C$ and $\angle D$ is respectively;

a) $90^0, 90^0$ b) $100^0, 80^0$

c) $100^0, 90^0$ d) $90^0, 100^0$

Q.No.6. If two circles touch each other at one point only, the number of common tangents to the circles are;

a) 2 b) 1

c) Infinite d) None

Q.No.7. A circle can have .... parallel tangles.

a) 2 b) 4

c) 0 d) Infinite

Q.No.8. Two circles which do not touch each other will have .... tangents in common.

a) 0 b)1

c) 4 d) Infinite

Q.No.9. A tangent touches the circle at .... points

a) 1 b) 2

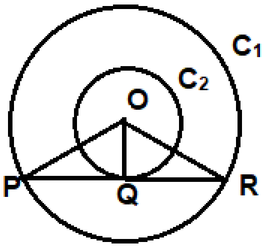
c) Infinite points d) 4

Q.No.10. How many tangents can be drawn from a point which is inside the circle?

a) Infinite b) Only one

c) No tangent d) More than one

Q.No.11. In the given figure, $C1$ and $C2$ are concentric circles with common centre O. If PQ the chord of circle $C1$ is tangent to circle $C2$, which one of the following may not be true?



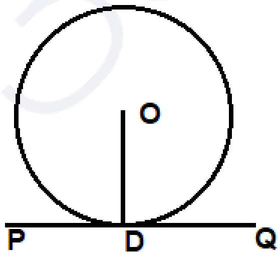
a) $OP = OR$

b) $PQ = QR$

c) $OR = QR$

d) $\triangle OPQ$ is right angled triangle

Q.No.12. In the given circle, PQ is a tangent and OD the radius. The measure of ∠PDO is equal to.....



a) 180^0 b) 90^0

c) 360^0 d) 100^0

Q.No.13. Tangent and the radius which meets the circle at the contact point of tangent are .... to each other.

a) Perpendicular b) parallel

c) Coincident d) All of these

Q.No.14. Two tangents to a circle are parallel to each other. The line segment joining the point of contacts of the tangents is as ....

a) radius b) Diameter

c) Any chord d) Secant

Q.No.15. The longest chord in the circle is ....

Q.No.16. A chord which passed through the centre of the circle is called ....

Q.No.17. The line joining the point of contacts of two parallel tangents of a circle must pass through ....

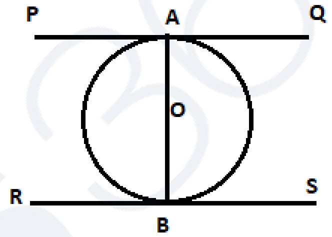
Q.No.18. We can have only three concentric circles (true/False)

Very Short Answer Type Questions

Q.No.1 Draw a rough sketch of a circle and a pair of tangents to it from any external point.

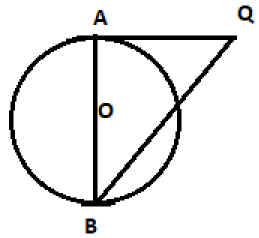
Q.No.2. Draw a circle with centre P. Draw any two tangents which are parallel to each other.

Q.No.3. In the given figure, PQ and RS are two tangents parallel to each other. Show that AB is diameter

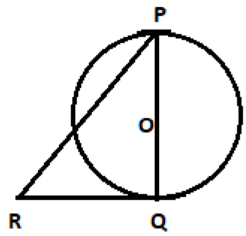


Q.No.4. Draw a rough sketch of any four parallel pairs of tangents.

Q.No.5. In the given figure, AQ is a tangent and AB the diameter of the circle with centre O. Show that ABQ is a right-angled triangle.



Q.No.6. In the given figure, $PQ = RQ$. If diameter $PQ = 12cm$, Find PR.



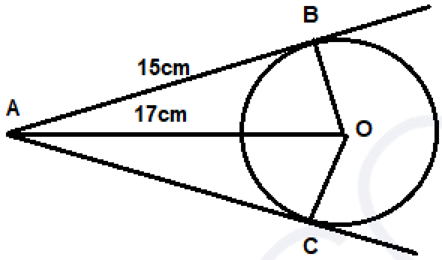
Q.No.7. Draw two concentric circles with common centre. Draw a line segment which is tangent to one circle and chord to another circle.

Q.No.8. Draw three concentric circles. Draw a line segment which chord in one circle, secant in another circle and tangent in third circle.

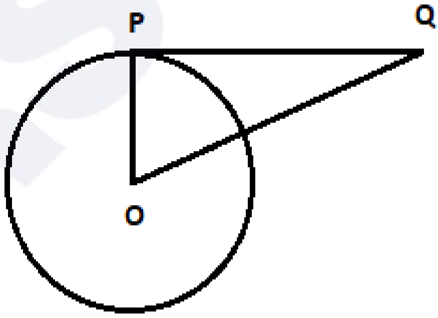
Q.No.9. Is it possible to have two circles with two points in common? If yes, draw at least one.

Short Answer Type Questions

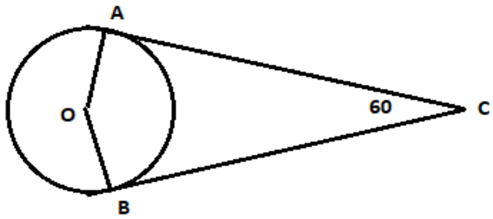
Q.No.1. AB and AC are two tangents which meets the circle at B and C respectively. If $OA = 17cm$ and $AB = 15cm$, find the radius of the circle.



Q.No.2. In the given figure, PQ is the tangent to the circle with centre O. If $OP = PQ$, show that $OQ = \sqrt{2}OP$



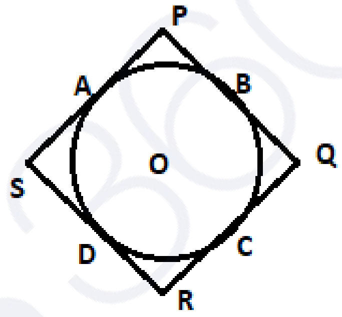
Q.No.3. In the given figure, AC and BC touches the circle at two points A and B respectively. If $\angle ABC = 60$ and $\angle AOB = 120$, prove that AC and BC are two tangents.



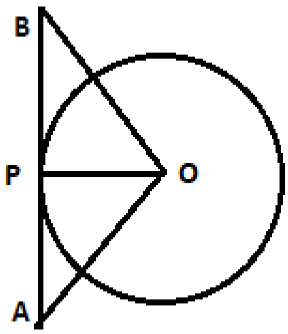
Q.No.4. Draw a circle with centre P. Draw two tangents to it from the external point D which meet the circle at points B and C. If the radius of the circle is $7cm$ and the distance between the external point D to the centre of the circle is $10cm$, find the length of each tangent

Q.No.5. A tangent from P is $16cm$ to the circle with radius $12cm$. Find the distance between the centre of the circle and the point P.

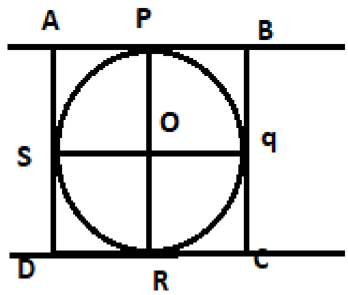
Q.No.6. In the given figure, PS, SR, RQ and QP are the tangents to the circle. If $PQRS$ is a parallelogram and $AP = BQ = SD$, prove that $PQRS$ is a rhombus.



Q.No.7. In the given figure, AB is the tangent to the circle with centre O. If $BP = AP$, show that $BO = OA$



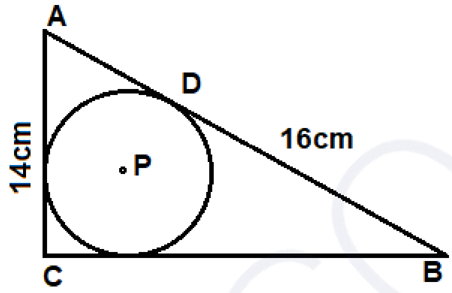
Q.No.8. In the given figure, AB and DC are two tangents which meets the opposite ends of the diameter PR. If $PB = RC$, prove that $PBCR$ is a rectangle.



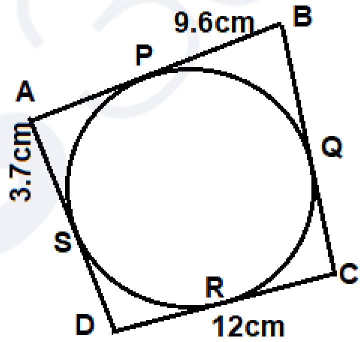
Q.No.9. Prove that the tangents drawn from external point to the circle are equal.

Long Answer Type Questions

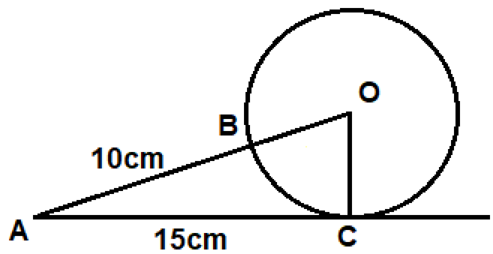
Q.No.1. $AB, BC \text{ and } AC$ are the tangents to the circle with centre P. If $AC = 14cm, DB = 16$ and $AB = 22cm$, find the perimeter of △ABC



Q.No.2. AB, BC, CD and DA are the tangents to the circle which touches the circle at P, Q, R and S respectively. If $PB = 9.6cm, DC = 12cm$ and $AS = 3.7cm$, find the perimeter of quadrilateral $ABCD$



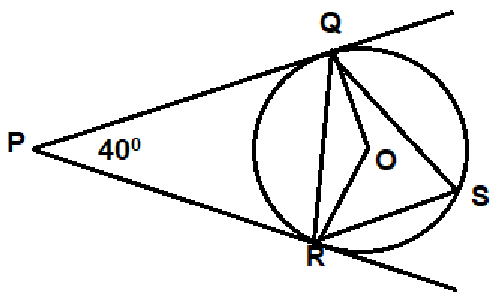
Q.No.3. In the given figure, $AC = 15cm$ is a tangent to circle which meets the circle at C. If $AB = 10cm$, find the radius of circle.



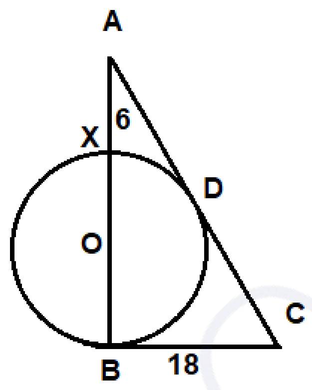
Q.No.4. In the given figure, $PQ$and PQ are two tangents to the circle with centre O. If $\angle QPR = 40$, find;

a) $\angle ROQ$ b) $\angle ORP$

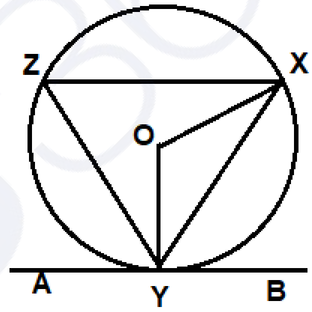
c) $\angle OQR$ d) $\angle RSQ$



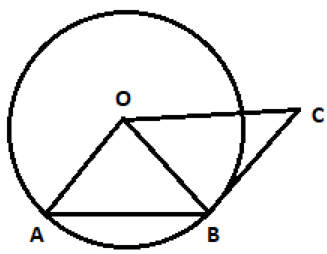
Q.No.5. BC and AC are two tangents which touches the circle at B and D respectively. If $AX = 6, AD = 12cm$ and $BC = 18,$ find the radius of the circle.



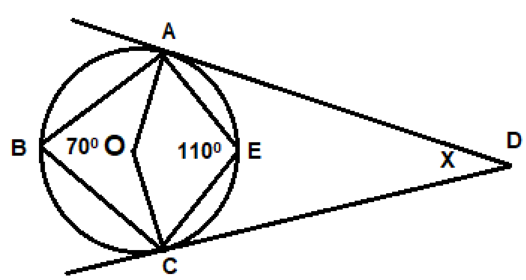
Q.No.6. A tangent AB touches the circle at Y. If XY is a chord such that $\angle XYB = 65^0$. Find $\angle XOY$



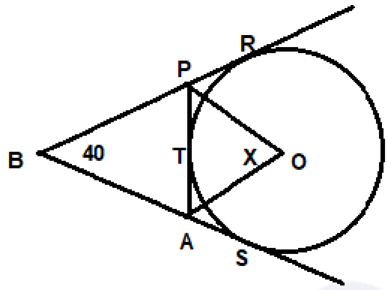
Q.No.7. In the given circle of radius $6cm$, Chord AB is equal to its radius. Tangent BC is equal to the chord AB. Find the length of OC



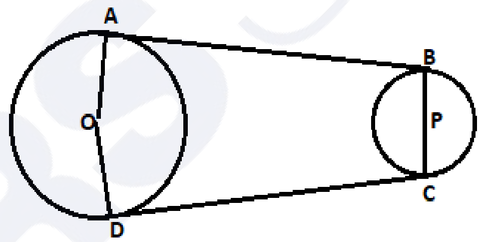
Q.No.8. AD and DC are two tangents from external point D which meets the circle at point A and C respectively. If $\angle ABC = 70, \angle AEC = 110$, find $\angle ADC$ and Obtuse $\angle AOC$.



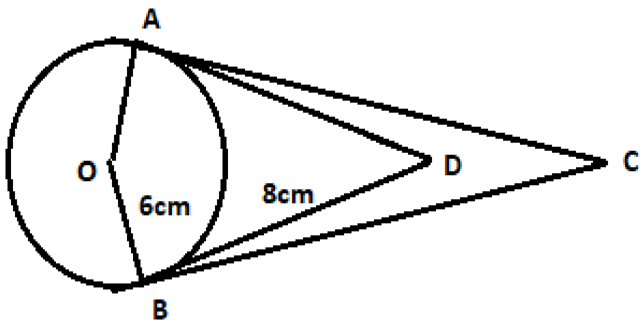
Q.No.9. In the given figure, BR and BS are two tangents which meets the circle at points R and S respectively. If $\angle PBA = 40$, $PO = AO$ and PA the tangent which meets the circle at point T, find $\angle POA$



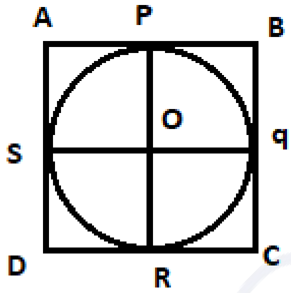
Q.No.10. In the given figure, AB and OD are the radii of circle with centre O and PB & PC are the radii of circle with centre P. If $AO = 7cm$, $PB = 3.5cm$ and $AB = 12cm$, find the area of polygon $AODCPB$



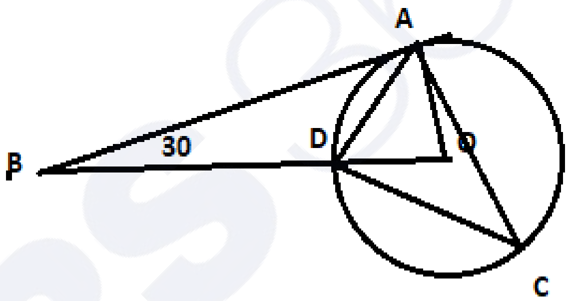
Q.No.11. In the given figure, AC and BC are two tangents from the point C. Also, AD and BD are two tangents from the point D. If the radius of the circle is $6\ cm, BD = 8cm$ and $DC = 5cm$, find the length of OC and BC



Q.No.12. In the given figure, AB, BC, CD, DA are the tangents to the circle. If $AB \parallel SQ \parallel DC$ and $AD \parallel PR \parallel BC$, show that $ABCD$ is a square.



Q.No.13. In the given figure, AB is a tangent to the circle. If $\angle ABD = 30$, find $\angle DCA$. Also show that $\angle ABO = 90 - 2\angle ACD$



Answers

Q1, b	Q2, c	Q3, b	Q4, a	Q5, c	Q6, b
Q7, d	Q8, a	Q9,a	Q10, c	Q11, c	Q12, b
Q13, a	Q14, b		Q15, Diameter	Q16, Diameter	

Q17, Centre Q18, False

Very Short Answer Type

Q6 $12\sqrt{2}$cm Q9, Yes

Short Answer Type

Q1, 8cm Q5, $\sqrt{51}$ Q5, 20cm

Long Answer type

Q1, 60cm Q2, 50.6cm Q3, $\frac{25}{4}\,cm$

Q4, ∠ROQ =140, ∠ORP =90, ∠OQR=20, ∠RSQ=70

Q5, 9cm Q6, 130, Q7, 10cm

Q8, ∠ADC=40, ∠AOC=140 Q9, 170

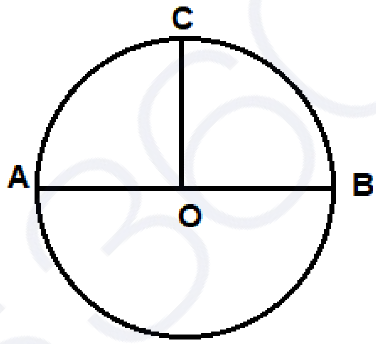
Q10, 136 cm^2, Q11, OC= 15cm, BC =$\sqrt{189}$

Class 10th Mathematics

Area Related to Circles

Parts of circle:

Observe the given circle carefully and remember the names of it different Parts



(a) AB is called diameter
A diameter is a line segment from one point on the circumference to another point on circumference which passes through the centre of the circle

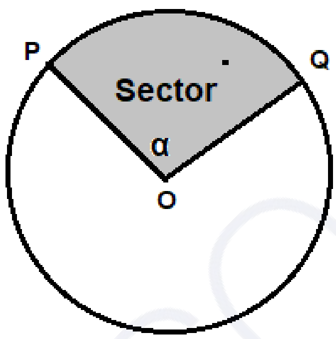
(b) OC is called radius
Radius is a line segment from centre to any point of the boundary of the circle. Radius is half of the diameter.

(c) Circumference of circle
The length of the boundary of the circle is called circumference of circle. It is calculated by using the formula $2\pi r$ or πd, where $'r'$ is the radius and $'d'$ the diameter of circle.

(d) Area of a circle:
Area of a circle is the measurement of the surface bounded by the circumference. It is calculated by using the formula πr^2. To know more about area formula for the circle, consult your textbook where it is clearly discussed.

Sector of a circle:

Sector is a portion of a circle between two radii and the circumference. The shaded portion in the given circle is a sector with angle **α** at centre. The remaining portion of the circle (Unshaded portion) is also a sector.



Area of the sector of a circle:

Since sector is part of a circle, therefore we can calculate the area of sector by using the formula $\frac{\theta}{360}\pi r^2$ where, θ is angle made b y the sector at the centre of the circle.

Why area of sector is $\frac{\theta}{360}\pi r^2$? Let us know about it

We know that:

Area of circle $= \pi r^2$

Let us divide the circle into 360 sectorial parts, the area of each of the sectorial part will be $\frac{\pi r^2}{360}$

The area of two sectorial parts is $2(\frac{\pi r^2}{360})$

Similarly, the area of $'n'$ sectorial parts of the circle is $n(\frac{\pi r^2}{360})$

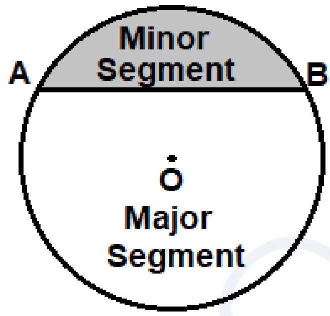
What is $'n'$? It is actually $n - degree$ angle out of 360^0

We replace the n by θ

This means the area of the sector will be calculated by $\frac{\theta}{360}\pi r^2$

Segment of a circle:

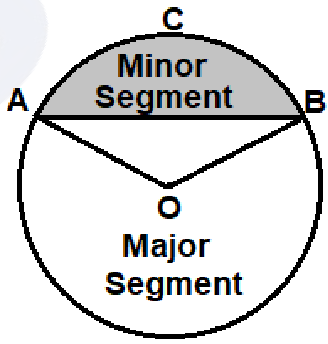
A segment of a circle is a portion in circle bounded by chord and the circumference of the circle. In the given figure, shaded portion is called Minor (small) segment while as unshaded portion is called major (bigger) segment.



Area of segment:

There is not a specific formula to calculate the area of a segment. But we can calculate the area of segment using concept of sector.

To calculate the area of minor segment in the circle, we need to calculate the area of sector ACBO and area of △AOB. Further, the area of the minor segment will be calculated as;



Area of minor segment = $Area\ of\ sector\ ACBO - Area\ of\ \triangle AOB$

Questions for Practice

Objective Type Questions

Q.No.1. The area of $\frac{1}{4}th$ part of a circle of radius 1m is …….

a) $\frac{\pi}{360}\,m^2$ b) $\frac{\pi}{1440}\,m^2$

c) $\frac{\pi}{4}\,m^2$ d) $\pi r^2\,m^2$

Q.No.2. If the area of a circle of radius $154cm^2$, the radius of the circle is:

a) $7cm$ b) $49cm$

c) $7\ cm^2$ d) 22

Q.No.3. A sector of a circle subtends at an angle of 90^0at the centre. If the radius of the circle is 7cm, the area of the circle is:

a) $38.5cm^2$ b) $14cm$

c) $154\ cm^2$ d) 22

Q.No.4. If a chord divides a circle into two segments of equal area, then the chord is:

a) Radius of circle b) Diameter of circle

c) Less than diameter of circle d) Greater than diameter of circle

Q.No.5. If the area of a circle of radius “r” is $154cm^2$. Then the diameter of the circle is;

a) 7cm b) 14cm

d) 21cm d) 22cm

Q.No.6. If the radius of circle (A) is twice the radius of circle (B), The ratio of their areas is;

a) 1:2 b) 2:3

c) 2:4 d) 4:1

Q.No.7. The radii of two circles are r and r^2. The ratio of their areas is;

a) 1:2 b) $1:r^2$

c) $1:r^3$ d) $1:r^3$

Q.No.8. If the area of circle (A) is half of the area of circle (B), then the ratio of their radii is respectively:

a) 1:2 b) 2:3

c) 3:1 d) $1:\sqrt{2}$

Q.No.9. If the area of a square is equal to the area of a circle with radius $\frac{5}{\sqrt{\pi}}$, then the side of the square is:

a) π b) 5

c) $\frac{1}{5}$ d) $\frac{\pi}{2}$

Q.No.10. Two circles of equal area will have:

a) Equal radii b) Equal Circumferences

c) Equal diameters d) All of these

Q.No.11. Two circles with radius $\frac{1}{\pi}$ and $\frac{2}{\pi}$ will differ in area by:

a) 1 sq unit b) $\frac{3}{\pi}$ sq unit

c) πunits d)3π units

Q.No.12. circle with radius $\frac{1}{\pi}$ has area;

a) $\frac{1}{\pi}$ b) π

c) 2π d) π^2

Q.No.13. Area of a sector with radius 1 unit and angle at centre 90^0, is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$

c) π d) 2π

Q.No.14. If area of circle (A) is less than the area of circle (B). Then the radius of circle (B) is .... the radius of circle (A)

(less than/ Greater than)

Q.No.15. Semicircle is not a sector (True/False)

Q.No.16. Sector with angle greater than 180^0 has more area than the semicircle of the circle. (True/False)

Q.No.17. Four equal sectors each of angle 90^0 of a circle makes the .... circle (Complete/ incomplete)

Q.No.18. If the circumference of two circles is same, then their radii are;

(Equal/ not equal)

Q.No.19. Which of the following has more area?

A circle of radius 7cm or a square of side 7cm

Q.No.20. Area of a sector of the circle is calculated by the formula ....

Q.No.21. A semicircle as a sector of a circle has .... angle at centre.

Very Short Answer type questions

Q.No.1. Find the area of a circle with radius 10.5cm.

Q.No.2. Find the area of a circle with radius x^2 units

Q.No.3. Find the area of a circle with radius $\frac{1}{\pi}$ units.

Q.No.4. Find the area of a circle with radius $\frac{1}{\pi^2}$ units.

Q.No.5. Find the area of a circle with radius $\sqrt{\pi}$ units.

Q.No.6. Find the area of a sector of a circle of radius 7cm having angle at centre equal to 90^0.

Q.No.7. Find the area of a sector of the circle of radius 10.5cm with angle at the centre 180^0.

Q.No.8. Find the area of semicircle of radius 21cm.

Q.No.9. Find the area of $\frac{1}{3}rd$ of a circle with radius 3.5cm

Q.No.10. Find the area of a sector of a circle with diameter 14cm and the angle at centre is 120^0.

Q.No.11. Two sectors in a circle are equal in area. Prove that their angles at the centre of the circle are equal.

Q.No.12. Area of a sector of a circle is $\frac{1}{4}th$ of the area of the circle. Prove that the angle of the sector at the centre of the circle is 90^0.

Q.No.13. Two circles C_1 and C_2 have the radii in the ratio 1:2, Find the ratio of their areas.

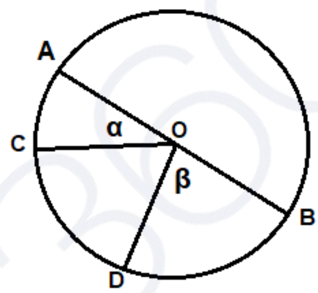
Q.No.14. Find the difference between the areas of two concentric circles of radii 5cm and 2cm.

Q.No.15. Show that the difference between the areas of two concentric circles with radii r and $3r$ is $1:9$

Short Answer type questions

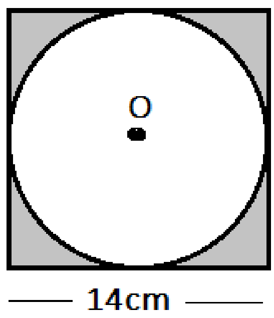
Q No.1. A sector of a circle subtends at an angle of 90^0at the centre. If the area of the sector is $\frac{77}{2}cm^2$, find the diameter of the circle.

Q.No.2. AB is a diameter of the circle. OC and OB are two radii of the circle. Sectors AOC and DOB have the angles at centre as α and β respectively. If the area of the sector AOC is half of the area of sector DOB, show that $2\alpha = \beta$



Q.No.3. If the circumference of a circle in meters is equal to its area in square meters, find the radius of the circle.

Q.No.4. Find then area of the shaded portion in the given square.



Q.No.5. A circle of radius $7cm$ has two equal sectors with sector angle 45^0 in it. Find the area of the reaming part of the circle.

Q.No.6. If the area of a circle with radius $'r'$ is 49π, find the radius of the circle.

Q.No.7. A circular mirror of diameter $21cm$ is one-side polished. If it costed ₹346.5 to polish the mirror, what is the rate of polishing per square centimetre?

Q.No.8. The ratio of the radii of three circles C_1, C_2, C_3 is 1:2:3. Find the ratio of their areas.

Q.No.9. If the sum of then areas of two concentric circles is 245π square units and the radius of one is twice the radius of another. Find their radii.

Q.No.10. If the rea of two circles are in the ratio 1:2. Show that their radii are in the ratio $1:\sqrt{2}$

Q.No.11. Circumference of a circle is 44cm. Find the area of the circle.

Q.No.12. If the ratio of the circumferences of two circles is 1:2. What is the ratio of their areas:

Q.No.13. The ratio of the areas of two circles is 2:3. What is the ratio of their circumferences?

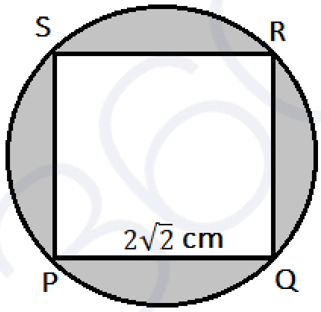
Q.No.14. The ratio of the areas of two circles is 1:9. What is the ratio of their radii?

Q.No.15. A circle is divided into 22 equal parts. If the area of each part is $7cm^2$, what is the radius of the circle?

Long Answer Type Questions

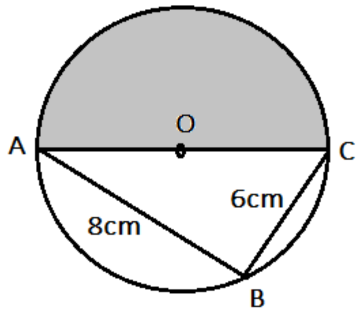
Q.No.1. A square shaped park of each side $10m$ has four circular flower beds in it. If the radius of each of each of the flower bed is $0.7m$, find the area of the remaining park.

Q.No.2. $PQRS$ is a square with side $2\sqrt{2}cm$. (See given figure) Find the area of shaded portion.



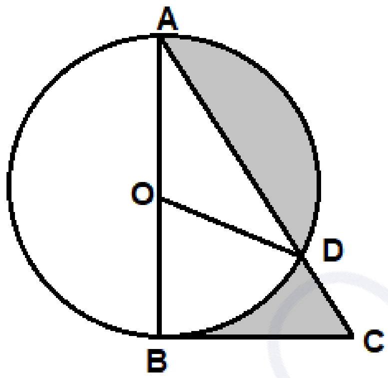
Q.No.3. If the area of square is equal to the area of a circle of radius $10.5cm$, find the perimeter of the square.

Q.No.4. In the given figure, $AB = 8cm, BC = 6cm$. If AC is the diameter of the circle, find the area of the shaded portion.

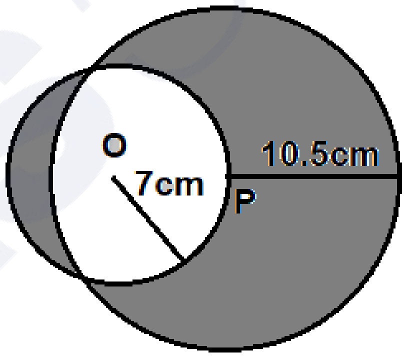


Q.No.5. Two circles of radius $'r'$ and $'2r'$ have two sectors of equal area. Find the relation between the angles of the sectors.

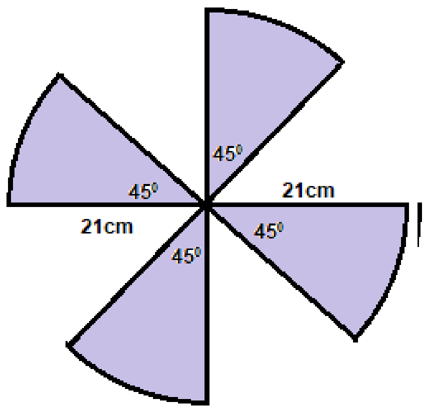
Q.No.6. In the given figure, ΔABC is right angled at $\angle B$. If the diameter AB of circle is $14cm$ and $BC = 12cm$, also ΔAOD is equilateral, find the area of the shaded portion.



Q.No.7. Given below are two circles with centre O and P having radius $7cm$ and $10.5cm$ respectively. (See given figure) If the centre of bigger circle lies on the boundary of smaller circle and the area of unshaded portion is 121cm, find the area of shaded portion.



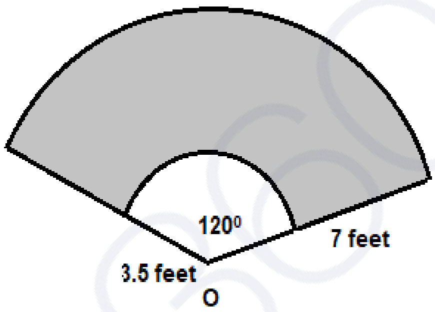
Q.No.8. A fan has four wings. (See given figure) Each wing is a sector of radius $21cm$ with angle at centre 45^0. Find the cost to paint the wings both sides @₹1000/m^2.



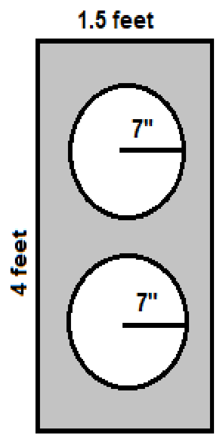
Q.No.9. If the area of a sector of a circle is $\frac{1}{3}rd$ of the area of the circle. What is the angle of sector?

Q.No.10. There are two circles, circle (A) of radius $21cm$ and circle (B) of radius $14cm$. Circle (A) has a sector whose area is equal to the area of a sector in circle (B). Find the ratio between the angles of the sectors when X and Y are the angles of circle (A) and (B) respectively.

Q.No.11. A signboard of a circle is a sectorial portion of a circle of radius $7\ feet$ without a small sectorial portion of radius $3.5\ feet$. (Coloured portion in the given figure) If the angle at the centre is 120^0, find the cost to design the signboard if the artist charges **₹300/** square feet.



Q.No.12. A window-pan of size $1.5\ feet \times 4\ feet$ has two circular mirrors in it. (See the given figure) If the radius of each mirror is 7 inches, find the area of wood. Also find the amount to paint it @ ₹30 per square feet.



Q.No.13. Find the radii of two concentric circles having difference in their areas 16π and the difference of their radii is 2.

Answers

Objective

Q1, c Q2, a Q3, a Q4, b Q5, b Q6, d

Q7, b Q8, d Q9, b Q10, d Q11, b Q12, a

Q13, b Q14, Greater than Q15, false Q16, False

Q17, Complete Q18, Equal Q19, Circle Q20, $\frac{\theta}{360}\pi r^2$

Q21, 180

Very Short Answer Type

Q1, 346.5cm^2 Q2, πx^4sq. units Q3, $\frac{1}{\pi}$ sq. units

Q4, $\frac{1}{\pi^3}$ sq. units Q5, 1 sq.unit Q6, 38.5cm^2 Q7, 173.25cm^2

Q8, 1386cm^2 Q9, 12.84cm^2 Q10, 205.34cm^2 Q13, 1:4

Q14, 21π cm^2

Short Answer type Questions'

Q1, 14cm Q3, 2 units Q4, 42cm^2 Q5, 115.5cm^2

Q6, 7cm Q7, Rs1/ Q8, 1:4:9 Q9, 7cm, 14cm

Q11, 7cm Q12, 1:4 Q13, 2:3 Q14, 1:3

Q15, 7cm

Long Answer Type Questions

Q1, 93.84cm^2 Q2, $\frac{32}{7}$cm^2 Q3, 74.46cm^2 (Apprx)

Q4, 39.29cm^2 Q5, $\theta_1 = 4\theta_2$ Q6, 15.9cm Q7, 258.5cm^2

Q8, Rs138.6/ Q9, 120^0, Q10, $4X = 9Y$ Q11, Rs11550/

Q12, 3.86 sft, Rs 115.83 Q13, 5cm, 3cm

SURFACE AREA AND VOLUMES

INTRODUCTION:

In our day-to-day life, we need a number of solids which are either a part of solid or a combination of these solids. We deal with cubes, cuboids, Cylinders, cones etc. In this chapter, we have to deal with surface area and volumes of various solids and also with the conversion of a solid into another.

Surface area refers to the area of the exposed surface of the three dimensional solid. There are two types of surface area.

1. Lateral Surface Area
2. Total Surface Area.

In general, lateral surface area does not include the base of the shape while the total surface is the area is the area of the entire object. Lateral surface area (lateral also means side), does not include the area of the top and bottom.

The volume of a solid refers to the three-dimensional space it occupies, often quantified numerically. One dimensional figure, namely lines and two-dimensional shapes, namely squares etc. are assigned zero volume in the three-dimensional space.

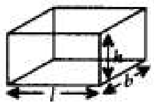
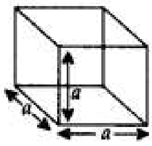
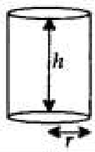
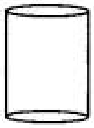
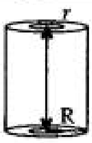
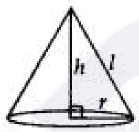
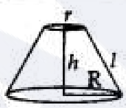
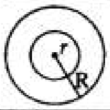
So, the surface area is the area that describes the material that will be used to cover a geometric solid. When we determine the surface areas of a geometric solid, we take the sum of the areas for each geometric form within the solid and the volume is a measure of how much a figure can hold. Surface area is measured in square units and volume is measured in cubic units.

Surface area is important to know situations where we want to wrap something, paint something and eventually while building things to get the best possible design. Finding the volume of an object can help us to determine the amount required to fill that object, like the amount of water needed to fill a bottle, an aquarium or tank.

For public works and industrial development activities, need of converting a solid into another solid of different shape or more than one solid of similar shape but with reduced size arises. When you convert a solid shape to another, its volume remains same, no matter how different the new shape is. In-fact, if one big cylindrical candle is melted to five small cylindrical candles, the sum of volume of the smallest candles is equal to volume of the bigger candle. Even if the conversion is to a different shape, the volume remains unchanged.

"The invention of surface area and volume is credited to Archimedes."

TABLE FOR SURFACE AREA AND VOLUME

Solid	Figures	Curved surface area (1)	Plane area (2)	Total area [1 + 2]	Volume	Remarks
Cuboid		Also known as lateral surface area $= 2(lh + bh)$	Area of: Top face $= lb$ Bottom face $= lb$ $\therefore lb + lb = 2lb$	$2(lb + bh + hl)$	$l.b.h$	l : length b : breadth h : height
Cube		Lateral surface area $= 4a^2$	Area of: Top face $= a^2$ Bottom face $= a^2$ $\therefore a^2 + a^2 = 2a^2$	$4a^2 + 2a^2 = 6a^2$	a^3	a : Side of cube
Right circular cylinder closed at top		Curved surface area $= 2\pi rh$	Area of: Top face $= \pi r^2$ Bottom face $= \pi r^2$ $\therefore \pi r^2 + \pi r^2 = 2\pi r^2$	$2\pi r^2 + 2\pi rh$ Or, $2\pi r(r + h)$	$\pi r^2 h$	r : radius h : height of cylinder
Right circular cylinder open at top		Curved surface area $= 2\pi rh$	Area of: Top face $= 0$ Bottom face $= \pi r^2$ $\therefore 0 + \pi r^2 = \pi r^2$	$2\pi rh + \pi r^2$ Or, $\pi r(2h + r)$	$\pi r^2 h$	r : radius h : height of cylinder
Hollow cylinder (Pipe)		$2\pi Rh$ • External surface area $= 2\pi Rh$ • Internal surface area $= 2\pi rh$	Area of: Top face $= \pi(R^2 - r^2)$ Bottom face $= \pi(R^2 - r^2)$	$2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$	$\pi R^2 h - \pi r^2 h$ (External Vol. - Internal Vol.)	R : Radius of outer base r : radius of inner base h = height
Cone		πrl	Area of: Bottom Face $= \pi r^2$	$\pi r^2 + \pi rl$ Or, $\pi r(r + l)$	$\frac{1}{3}\pi r^2 h$	h = height of cone r = radius of cone l = slant height $= \sqrt{h^2 + r^2}$
Frustum		$\pi l(R + r)$	Area of: Top Face $= \pi r^2$ Bottom Face $= \pi R^2$	$\pi r^2 + \pi R^2 + \pi l(R + r)$	$\frac{1}{3}\pi h \cdot (R^2 + r^2 + Rr)$	h = height of frustum r = radius of top face R = Radius of base l = slant height
Sphere		$4\pi r^2$	None	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r : radius of sphere
Hemisphere		$2\pi r^2$	πr^2	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r : radius of hemisphere
Spherical shell		$4\pi R^2$ (Outer) $4\pi r^2$ (Inner)	None	$4\pi R^2 + 4\pi r^2$	$\frac{4}{3}\pi \cdot (R^3 - r^3)$	R:Radius of outer shell r:Radius of inner shell

1 Mark Question

Q.1 If the radius and height of a cylinder are in the ratio 5:7 and its volume is 550 cm^3. Then its radius is equal to ($\pi = \frac{22}{7}$).

a) 6 cm b) 7 cm c) 5 cm d) 10 cm

Q.2 If the curved surface area of a solid right circular cylinder of height "h" and radius "r" is one-third of its total surface area, then

a) $h = \frac{1}{3} r$ b) $h = \frac{1}{2} r$ c) h = r d) h = 2r

Q.3 A hollow cylindrical pipe is 21 cm long. It its outer and inner diameters are 10 cm and 6 cm respectively, then the volume of the metal used in making the pipe is ($\pi = \frac{22}{7}$).

a) 1048 cm^3 b) 1056 cm^3 c) 1060 cm^3 d) 1064 cm^3

Q.4 **A cone is within the cylinder and cylinder is within a cube touch, by all vertical faces with same base and height, then the ratio of their volumes will be**

a) 14:11:13 b) 44:33:11 c) 56:36:22 d) None of these

Q.5 How many cubes each of edge 6cm can be cut from a cuboid of 42 cm x 36 cm x 24 cm:

a) 124 b) 142 c) 168 d) 186

Q.6 A medicine capsule in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. The capacity of the capsule is:

a) 0.36 cm^3 b) 0.35 cm^3 c) 0.34 cm^3 d) 0.33 cm^3

Q.7 A solid piece of iron in the cuboid form of dimensions 49 cm x 33 cm x 24 cm, in moulded to form a solid sphere. The radius of the sphere is

a) 21 cm b) 23 cm c) 25 cm d) 19 cm

Q.8 The radius and the height of a right circular cone are in the ratio 5:12. If its volume is 314 cm^3, the slant height and radius are:

a) 12, 5 cm b) 13,4 cm c) 1, 4 cm d) 13, 5 cm

Q.9 A mason constructs a wall of dimension 270 cm x 300 cm x 350 cm with the bricks each of size 22.5 cm x 11.25 cm x 8.75 cm and it is assumed that 1/8 space is covered by the mortar. Then the number of bricks used to construct the wall is:

a) 11100 b) 11200 c) 11000 d) 11300

Q.10 A cone is cut though a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called:

a) A frustum of a cone b) Cone c) Cylinder d) Sphere

Q.11 Total surface area of a lattu (top) as shown in figure is the sum of total surface area of hemisphere and the total surface of the cone (T/F)

Q.12 The total surface area of a hemisphere of radius is $2 r^2$ (T/F)

Q.13 Two solid metallic right circular cones have same height, the radii of their base are r_1 and r_2. The two cones are melted together and recast into a right circular cylinder of height "h" then the radius of the base of the cylinder is $\sqrt{\frac{1}{2}(r_1^2 + r_2^2)}$ (T/F)

Q.14 The lateral surface area of a cuboid of length "ℓ", breadth "b" and height "h" is $2(\ell + b) \times h$ (T/F)

Q.15 If the height of a cylinder doubles, then the volume of a cylinder increases by a factor of 8 (T/F)

Q.16 A cube with side length 10 cm would fit inside a sphere with diameter 10 cm (T/F)

Q.17 If the height of the cone is double and the base remains the same. The volume of the cone doubles (T/F)

Q.18 The total surface area is ………….. for a cube of edge length "a"

Q.19 ………….. is the surface area of a solid cylinder and radius 2 cm and height 10 cm in (mm^2)

Q.20 A cylindrical pencil sharpened at one end is the combination of ………….. and …………..

Q.21 During conversion of solid from one sphere to another, the volume of the new shape will …………..

Q.22 The surface area of a sphere is equal to the curved surface area………………

ANSWER

Q.1	(a)	Q.2	(c)	Q.3	(b)	Q.4	(b)	Q.5	(c)	Q.6	(a)
Q.7	(a)	Q.8	(d)	Q.9	(b)	Q.10	(a)	Q.11	(T)	Q.12	(F)
Q.13	(F)	Q.14	(T)	Q.15	(F)	Q.16	(F)	Q.17	(T)	Q.18	($6a^2$)

Q.19 (15072) Q.20 (Cone, cylinder) Q.21 Remain unaltered Q.22 Cylinder

2 MARKS QUESTIONS

Q.1 Three cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

Q.2 Define frustum of the cone.

Q.3 What is the capacity of a cylindrical vessel with a hemispherical portion raised upward to the bottom?

Q.4 A solid ball is exactly fitted inside the cubical box of side "a". What is the volume of the ball?

Q.5 A rectangular solid metallic cuboid 9 cm x 8 cm x 2 cm is melted and recast into solid cubes each of side 2 cm. How many solid cubes can be made?

Q.6 The radii of the ends of a frustrum of a cone 40 cm high are 20 cm and 11 cm. Find its slant height.

Q.7 Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Q.8 A cone, a hemisphere and the cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

Q.9 Match the Column

S.No	Type	Measurement
1	Volume of frustrum of cone	$\sqrt{h^2\ (r_1 - r_2)^2}$
2	Slant height of frustrum of a cone	$\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$
3	Curved surface area of frustrum of a cone	$\pi \ell(r_1+r_2)+\pi(r_1^2+r_2^2)$
4	Total surface area of a frustrum of a cone	$\pi \ell(r_1+r_2)$

Q.10 12 Spheres of same size are made from melting a solid cylinder of diameter 16 cm and 2 cm height. What is the diameter of each sphere?

Q.11 A cylinder has 5 cm height and 3 cm diameter. Find its total surface area.

Q.12 Find the curved surface area of a right circular cone whose slant height is 10 cm and the radius is 7 cm.

Q.13 Find the total surface area of a hemisphere of radius 21 cm.

Q.14 Give two point of difference between surface area and volume of a solid.

Q.15 Find the amount of water displaced by a solid spherical ball of diameter 28 cm.

ANSWER

Q.1	($350\ cm^2$)	Q.3	($\frac{\pi r^2}{3}(3h-2r)$)	Q.4	$\frac{\pi a^3}{6}$	Q.5	(18)
Q.6	(41 cm)	Q.7	(9 units)	Q.8	(1:2:3)	Q.10	(2 cm)
Q.11	($61.23\ cm^2$)	Q.12	($220\ cm^2$)	Q.13	($4158\ cm^2$)	Q.15	($11498\frac{2}{3}\ cm^3$)

3 Mark Questions

Q.1 The side of a solid metallic cube is 50 cm. The cube is melted and recast into 8000 equal solid cubical dice. Determine the side of the dice.

Q.2 A right circular cone of metal is 2.7 cm high and radius of base is 1.6 cm. It is melted and recast into a sphere. Find the radius of the sphere.

Q.3 The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross section. If the length of the wire is 36 m. Find the radius of the cross section.

Q.4 How many balls each of radius 0.5 cm can be made from a solid sphere of metal of radius 10 cm by melting the sphere.

Q.5 A copper rod of diameter 1 cm and length 8 cm is drawn into wire of length 32 m of uniform thickness (diameter). Find the thickness of the wire.

Q.6 A shot putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3. Find the mass of the shot putt.

Q.7 The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm. Find

(a) height of the cone (b) Slant height of the cone

Q.8 A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Q.9 The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

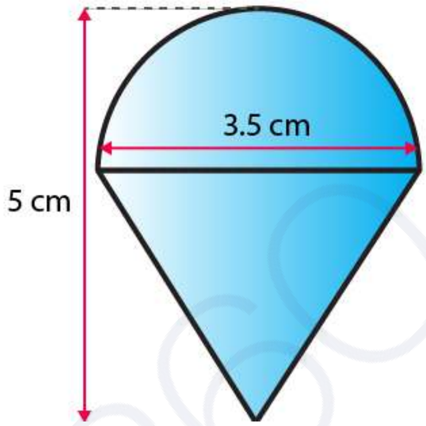
Q.10 The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of metal is 8.9 g per cm^3?

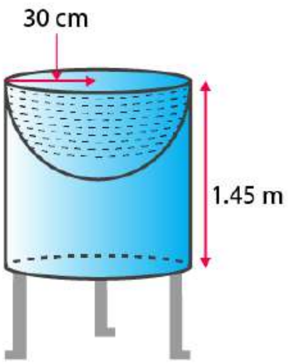
ANSWERS

Q.1	(2.5 cm)	Q.2	(1.2 cm)	Q.3	(0.1 cm)	Q.4	(8000)
Q.5	(0.05 cm)	Q.6	(3.85 Kg)	Q.7	(48 cm, 50 cm)	Q.8	(100π cm^3)
Q.9	($\frac{1}{4}$)	Q.10	345.39 g)				

4 Mark Questions

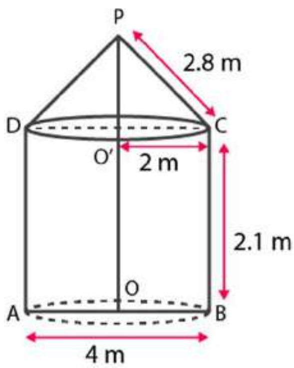
Q.1 Rameez got a playing top (Lattu) as his birthday gift which had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (take $\pi \frac{22}{7}$).



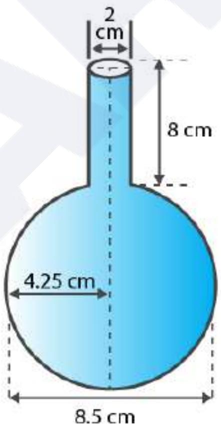


Q.2A person made a bird bath for his garden in the shape of a cylinder with a hemispherical depression at one end, as shown in the figure. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird bath. (take $\pi \frac{22}{7}$).

Q.3 A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m. Find the area of the canvas used for making the tent.



Q.4



A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter, the diameter of the spherical part is 8.5 cm. By measuring the amount of water, it holds. Its volume is 345 cm3 found by a child. Check whether she is correct ($\pi = 3.14$)

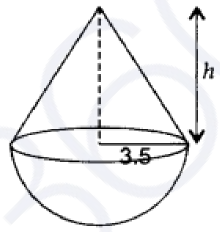
Q.5 A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A boy reshapes it in the form of a sphere. What is the radius of the sphere?

Q.6 Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Q.7 A canal is 300 cm wide, 120 cm deep. The water in the canal is flowing at a speed of 20 Km/h. How much area will it irrigate in 20 minutes, if 8 cm of standing water is desired.

Q.8 The radii of the circular ends of a bucket of height 15 cm are 14 cm and "r" cm (r < 14 cm). If the bucket has volume 5390 cm^3. Find "r". (Take $\pi \frac{22}{7}$).

Q.9 A solid wooden toy in the form of a hemisphere surmounted by a cone of same radius. The radius of the hemisphere is 3.5 cm and the total wood used in the toy is $\frac{1001}{6}$ cm^3. Find the height of the toy.



Q.10 A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the soup is filled to a height of 4 cm. How much soup is to be given to 250 patients?

Q.11 A building is in the form of a cylinder surrounded by a hemispherical vaulted dome and contains $41\frac{19}{21}$ cum of air. If the internal diameter of the building is equal to the total height cu.m above the floor. Find the height of the building.

Q.12 A heap of sand forms a cone whose diameter is 10.5 m and height 3 m. Find its volume. If it is to be protected by canvas from rain. Find the area of the canvas required.

ANSWERS

Q.1	(39.6 cm^2) approx.	Q.2	(3.3 m^2)	Q.3	(44 m^2)	Q.5	(6 cm)
Q.6	(r = 12 cm)	Q.7	(300000 m^2)	Q.8	(r = 7 cm)	Q.9(h= 9.5 cm)	
Q.10	(38500 cm^3)	Q.11	(4 cm)	Q.12	(86.625 m^3 & 99.825 m^2)		

CHAPTER - 14

STATISTICS

Introduction:

Statistics is the branch of mathematics which deals with the collection, analysis and interpretation of numerical data. In our day – to- day life, we come across a wide variety of information in the form of facts, numerical figures, tables, graphs etc. This information is provided by newspapers, televisions, magazines and other means of communications. You can relate to cricket batting or bowling averages, company profits, recorded city temperatures, expenditures in five year plan of the government, polling results etc. ***The facts or figures, which are numerical or otherwise collected with a definite purpose are called data.*** In "Latin" the singular form of "***data"*** is "***datum"***.

Nowadays, we are becoming more and more information oriented and we utilize data in every part of our life in one or the other form. Therefore, it is very important for us to know how useful information can be extracted from such data. This extraction of meaningful information is studied in Statistics a branch of Mathematics.

The word ***"Statistics"*** appears to have been derived from the Latin word ***"Status"*** meaning ***"a political state".*** In its origin Statistics was simply the collection of data on different aspects of people's life, useful to the State. But over a period of time, however, its scope broadened. ***Thus Statistics deals with the collection, organization, analysis and interpretation of data.***

Mean: The mean of a set of data values is the sum of all the data values divided by the number of data values.

$$\textbf{Mean} = \frac{\textbf{Sum of data values}}{\textbf{No. of data values}}$$

$$\bar{x} = \frac{\sum x}{n}$$

Arithmetic Mean:

$$\bar{x} = \frac{\textbf{Sum of terms}}{\textbf{Number of terms}} = \frac{x_1 + x_2 + x_3 \ldots\ldots\ldots + x_n}{n}$$

For two numbers "a" and "b"

$$\textbf{A.M.} = \frac{a + b}{2}$$

$\sum (x_i - \bar{x}) = 0$, where $\bar{x}$ is the Arithmetic mean.

If $\bar{x}_1$ **and** $\bar{x}_2$ are the respective Arithmetic means of two different sets. If data having **"a_1"** and **"a_2"** elements respectively. Then the mean of the total set is

$$\bar{x} = \frac{a_1\bar{x}_1 + a_2\bar{x}_2}{a_1 + a_2} = (a_1 a_2 \ldots\, a_n)^{1/n}$$

Geometric Mean:

For two numbers "a" and "b"

$$\text{G.M.} = \sqrt{ab} = (ab)^{1/2}$$

Harmonic Mean:

$$\text{H.M} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}}$$

For two numbers "a" and "b"

$$\text{H.M} = \frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a + b}$$

Note AM ⩾GM ⩾HM

Mean of Ungrouped Data:

The information collected systematically regarding a population or a sample survey is called ***ungrouped data.***

$$\textbf{Mean} = \frac{\text{Sum of Observations}}{\text{Number of observations}}$$

$$\bar{x} = \frac{x_1 + x_2 + x_3 \ldots\ldots\ldots + x_n}{n}$$

Mean of grouped Data:

If $x_1, x_2 \ldots\ldots\ldots x_n$ are **n**-observations with respective frequencies $f_1, f_2, f_3 \ldots\, f_n$. then the mean.

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Example: Mean of the data shown below will be

Marks (f_i)	20	30	35	40
Number of students	10	6	6	2

Solution:

$$\bar{x} = \frac{20 \times 10 + 30 \times 6 + 35 \times 6 + 40 \times 2}{10 + 6 + 6 + 2}$$

$$= \frac{200 + 180 + 210 + 80}{24} = 27.91$$

Assumed mean method of calculating mean

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = a + \bar{d}$$

$$\bar{d} = \bar{x} - a = \bar{x} - a = \frac{\Sigma(f_i x_i)}{\Sigma f_i} = \frac{\Sigma(f_i x_i)}{\Sigma f_i} - \frac{\Sigma f_i a}{\Sigma f_i}$$

$$= \frac{\Sigma f_i\ (x_i - a)}{\Sigma f_i} = \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Example: The table below shows the number of people within different age group who visited the mall on week end.

Age group (Class interval)	**10 – 25**	**25 – 40**	**40 – 55**	**55 – 70**	**70– 85**	**85 - 100**
Number of people	3	11	10	8	6	2

Solution:

Class Interval	**Number of people (f_i)**	**Class mark (x_i)**	**$d_i = x_i - 47.5$**	**$f_i d_i$**
10 – 25	3	17.5	- 30	-90
25 – 40	11	32.5	- 15	-165
40 – 55	10	47.5	0	0

Class Interval	Number of people (f_i)	Class mark (x_i)	$d_i = x_i - 47.5$	$f_i d_i$
55 – 70	8	62.5	15	120
70 – 85	6	77.5	30	180
85 – 100	2	92.5	45	90
	$\Sigma f_i = 40$			$\Sigma f_i d_i$ 135

Thus $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 47.5 + \frac{135}{40} = 50.875$

Step deviation method:

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$u_i = \frac{x_i - a}{h}$$

Where **"a"** is the assumed mean and **"h"** is the class size.

Example:

Number of wickets	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 - 85
Number of bowlers	6	6	7	4	4	2	1

Solution:

Class Interval	Number of bowlers (f_i)	(x_i)	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{10}$	$f_i u_i$
15 – 25	6	20	-30	-3	-18
25 – 35	6	30	-20	-2	-12
35 – 45	7	40	-10	-1	-7

Class Interval	Number of bowlers (f_i)	(x_i)	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{10}$	$f_i u_i$
45 – 55	4	50	0	0	0
55 – 65	4	60	10	1	4
65 – 75	2	70	20	2	4
75 – 85	1	80	30	3	3
	Σf_i =30				=-26

$$\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) h$$

$$= \quad 50 + \left(\frac{-26}{30}\right) \text{ x } 10 \quad = \quad \mathbf{41.33}$$

Median: Arrange the numbers in ascending or descending order, and why they are arranged as such, median is

- ➢ The middle term when number of terms is odd.
- ➢ Then average of middle two terms when the number of terms is even.
- ➢ Divide the distribution in two equal parts

Median of grouped Data:

Data which have been arranged in groups or classes rather than showing all the original figures.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - c_f}{f}\right) \times h$$

Where l = lower limit of median class.

n = number of observations

cf = cumulative frequency of class preceding the median class

f = frequency of median class

h = class size

Note: *Find cumulative frequencies of all the classes and* $\frac{n}{2}$ *locate the class where cumulative frequency is greater than* $\frac{n}{2}$ *. That is called median class.*

Mode: The number which has the highest frequency in the mode.

Mode of grouped Data:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where

l	=	lower limit of median class.(Class with maximum frequency
h	=	size of class interval
$f0$	=	frequency of class preceding the modal class
$f1$	=	frequency of modal class
$f2$	=	frequency of succeeding to the modal class.

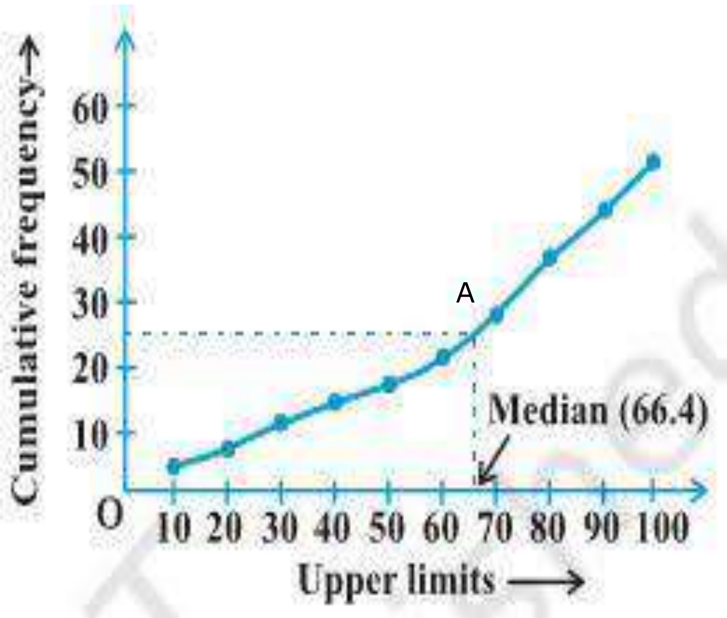
Note: *The empirical formula says Mode = 3 (median) – 2 (mean)*

There are three methods of drawing Ogive.

1. Less than Method:

*Steps involved in calculating median using less than **Ogive** approach:*

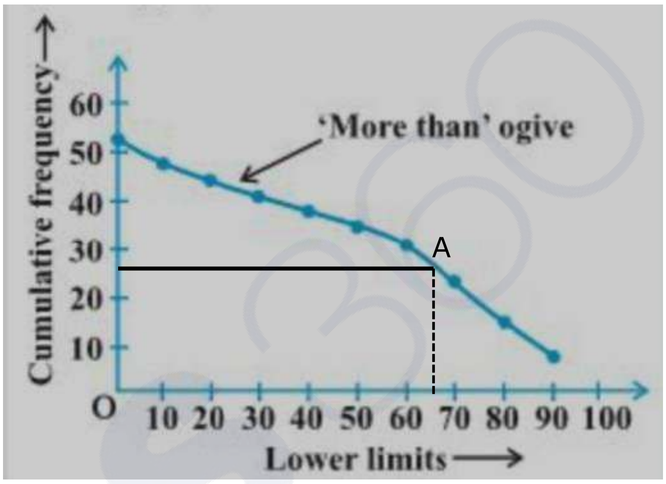
- Convert the series into a ***"less than"*** cumulative frequency distribution.
- Let ***"N"*** be the total number of students whose data is given. ***"N"*** will also be the cumulative frequency of the last interval. Find the $(\frac{N}{2})^{th}$ item and mark it on the Y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at the point **"A"**.
- From point ***"A"*** where the Ogive curve is cut, draw a perpendicular on the X-axis. The point at which it touches the X-axis will be median value of the series as shown in the graph:



2. More than Method:

*Steps involved in calculating median using more than **Ogive** approach:*

- Convert the series into a **“more than”** cumulative frequency distribution.
- Let ***“N”*** be the total number of students whose data is given. ***“N”*** will also be the cumulative frequency of the last interval. Find the $(\frac{N}{2})^{th}$ item and mark it on the Y-axis.

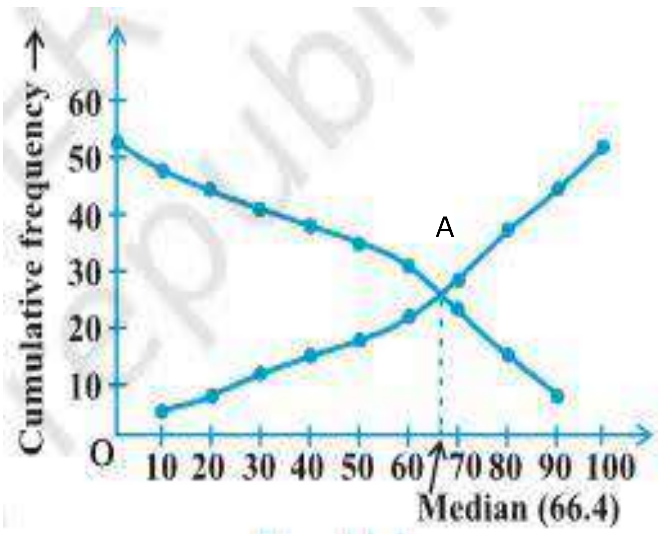


- Draw a perpendicular from that point to the right to cut the Ogive curve at the point **“A”.**
- From point ***“A”*** where the Ogive curve is cut, draw a perpendicular on the X-axis. The point at which it touches the X-axis will be median value of the series as shown in the graph:

3. Less than and More than Ogive Method:

*Another way of graphical determination of median is through simultaneous graphic presentation of both the **“less than”** and **“more than”** Ogives:*

- Mark the point ***“A”*** where the Ogive curves cut each other.
- Draw a perpendicular from ***“A”*** on the X-axis. The corresponding value on the X-axis would be the median value.
- The median of grouped data can be obtained graphically as the X-coordinate of the point of intersection of the two ogives of the data.



1 MARK QUESTIONS

Q.1 **The median for the data 2,4,6,8,10,12,14 is**

a) 6 b) 8 c) 9.5 d) 10

Q.2 **The mean weekly pay for ten persons equals to Rs. 100, if one of the person gets a taxi of Rs. 10 per week, what is the new mean weekly pay**

a) Rs. 99 b) Rs. 101 c) Rs. 200 d) Rs. 250

Q.3 **Each of the group formed from given data is called:**

a) frequency b) raw data c) median d) class-interval

Q.4 **Mean of the 5 items of a data is 10. If each term is multiplied by 4, then the new mean will be**

a) 40 b) 50 c) 30 d) 60

Q.5 **The Harmonic mean of 2, 4, 6 and 8 is**

a) 3 b) 3.5 c) 3.84 d) 9

Q.6 **If the AM as well as GM of two +ve numbers is 8, what is their H.M.**

a) 2 b) 4 c) 8 d) None of these

Q.7 **Geometric mean of 2, 6, 24 and 72 is**

a) 12 b) $13\sqrt{3}$ c) $8\sqrt{3}$ d) none of these

Q.8 **Given the mean (30), the mode (35), what is the median**

a) 30 b) 31.69 c) 40 d) 55.7

Q.9 **The range of the data 12, 16,17,23,29,13,2,5,19 is**

a) 23 b) 24 c) 27 d) 29

Q.10 **The mean of 80 items was 42. Later it was found that the two items were misread as 87 and 6 instead of 187 and 66. Which of the following will be the correct mean.**

a) 44 b) 45 c) 42.6 d) none of these

ANSWERS

Q.1	(b)	Q.2	(b)	Q3	(d)	Q.4	(a)	Q.5	(c)
Q.6	(c)	Q.7	(a)	Q.8	(b)	Q.9	(c)	Q.10	(a)

2 MARK QUESTIONS:

Q.1 Define mode.

Q.2 If the mean of the following data is 15

x:	5	10	15	20	25
f:	6	P	6	10	5

Find P.

Q.3 In a continuous frequency distribution, the median of the data is 21. If each observation is increased by 5, then find the new median.

Q.4 The mean of the following frequency table is 50, but the frequency f_1 and f_2 in class interval 20 – 40 and 60 – 80 respectively are not known. Find these frequencies, when sum of all the frequency is 120.

Class	Frequencies
0 – 20	17
20 – 40	f_1
40 – 60	32
60 – 80	f_2
80 – 100	19
Total	**120**

Q.5 Find the mode of the following frequency distribution.

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	12	35	45	25	13

Q.6 A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	Number of students
0 – 6	11
6 – 10	10
10 – 14	7
14 – 20	4
20 – 28	4
28 – 38	3
38 – 40	1

Q.7 Give three examples of collecting data from day to day life.

Q.8 Find the mode of the following items.

0, 5, 5, 1, 6, 4, 3, 0, 2, 5, 5, 6

Q.9 A student scored the following marks in 6 subject

30, 19, 25, 30, 27, 30.

Find his modal score

Q.10 Find mode using an empirical relation, when it is given that mean and median are 10.5 and 9.6 respectively.

Q.11 In a frequency distribution, if "a" assumed mean = 55. $\sum fi$ = 100, h = 10 and $\sum fiui$ = - 30, then find mean of the distribution.

ANSWER

Q.2	(8)	Q.3	(26)	Q.4	(1300 – 1450)	Q.5	(3)
Q.6	(5)	Q. 8	(5)	Q.9	(30) Q.10 (7.8)	Q.11	(52)

3 MARK QUESTIONS

Q.1 Find the value of "y" from the following observations, if they are already arranged in ascending order. The median is 63.

20, 24, 42, y, y+2, 73, 75, 80, 99

Q.2 While checking the value of 20 observations, it was noted that 125 was wrongly noted as 25 while calculating the mean and then mean was 60. Find the correct mean.

Q.3 The daily minimum steps climbed by a person during a week were as under:

Monday	35
Tuesday	30
Wednesday	27
Thursday	32
Friday	23
Saturday	28

Find the mean number of steps.

Q.4 From the following frequency distribution, find the median class.

Class Interval	Frequency
1000 – 1150	8
1150 – 1300	15
1300 – 1450	21
1450 – 1600	8

Q.5 Consider the following distribution, find the frequency of class 30 – 40.

Marks	No. of Students
0 or more	63
10 or more	58
20 or more	55
30 or more	51
40 or more	48
50 ore more	42

Q.6 Following table shows sale of shoes in a store during the month

Shoe Size	Pairs sold
3	4
4	18
5	25
6	12
7	5
8	1

Find the modal size of the shoes sold.

Q.7 The following table shows the ages of the patients admitted in a hospital during a year.

Age (in years)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode of the data.

Q.8 If the median of the distribution given below is 28.5. Find the values of "x" and "y".

Class Interval	Frequency
0 – 10	5
10 – 20	x
20 – 30	20
30 – 40	15
40 – 50	y
50 – 60	5
Total	**60**

Q.9 A company manufactures car batteries of a particular type. The lives of 40 such batteries were recorded as below:

Life of batteries (in years)	2 – 2.5	2.5 – 3.0	3.0 – 3.5	3.5 – 4.0	4.0 – 4.5	4.5 – 5.0
No. of batteries	2	6	14	11	4	3

Find the modal life of a battery in years.

Q.10 Consider the following distribution:

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 - 12	12 – 14
No. of Houses	1	.2	1	5	6	2	3

ANSWERS

Q.1	(61)	Q.2	(65)	Q.3	(29.17)	Q.4	(f_1=28, f_2=24)
Q.5	(33.33)	Q.6	(12.48 days)	Q.7	(36.8 years)	Q.8	(x=8, y=7)
Q.10	(8.1)						

4 MARK QUESTIONS

Q.1 The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of the number of boys to the number of girls who appeared in the examination.

Q.2 Some students of a class donated for the welfare of old age persons. The contributions are as follows:

Amount in Rs.	0 – 20	20 – 40	40 – 60	60 – 80	8 0– 100
No. of students	5	8	12	11	4

Find median and mode for their contribution.

Q.3 In a retail market, fruit vendors were selling mangoes in the packing boxes. These boxes contained varying number of mangoes. The following was the distribution.

No. of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
No. of boxes	15	110	135	115	25

Find the mean and median number of mangoes kept in a packing box.

Q.4 The length of 50 leaves of a plant are measured correct to the nearest millimeter and the data obtained is represented in the following table:

Length in (mm)	109 –117	118 – 126	127 – 135	136 – 144	145 – 153	154 – 162	163 - 171
No. of leaves	4	6	14	13	6	4	3

Find the mean length of the leaves.

Q.5 In a hospital, during the month of October, number of patients admitted for dengue and their ages are as follows:

Age in years	0 – 8	8 – 16	16 – 24	24 – 32	32 – 40	40 – 48	48 – 56	56 – 64	64 – 72
No. of patients	10	12	8	25	15	11	21	30	22

Find the mean and median age of patients

Q.6 Find the missing frequencies (f_1, f_2 and f_3) in the following frequency distribution when it is given that f_1, f_2 = 4:3 and mean = 50.

Class Interval	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	Total
Frequency	17	f_1	f_2	f_3	19	20

Q.7 A student noted the number of cars passing through a spot on a road for 100 periods, each of 3 minutes and summarized it in the table. Find the mode of the data.

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70– 80
Frequency	7	14	13	12	20	11	15	8

Q.8 Draw "less" than "Ogive" for the following frequency distribution.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70– 80	80– 90	90 - 100
No. of students	5	3	4	3	3	4	7	9	7	8

Q.9 Draw "more than" Ogive for the following distribution. Hence find median.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70– 80	80– 90	90 - 100
No. of students	5	3	4	3	3	4	7	9	7	8

Q.10 Draw "less than" Ogive and "more than" Ogive for the following distribution

Height	135 – 140	140 – 145	145 – 150	150 – 155	155 – 160	160 – 165
No of Plants	4	*7*	*18*	*11*	6	4

ANSWERS

Q.1 (3:2) Q.2 (Median = 51.66, Mode = 56) Q.3 (57.16)

Q.4 (137.30 mm) Q.5 (Mean = 41.92, Median = 45.09)

Q.6 (f_1=28, f_2 = 32, f_3 = 24) Q.7 (44.7) Q.9 (66)

PROBABILITY

Introduction:

In life we usually come across two types of experiments namely "*Deterministic Experiment and Random Experiment*".

Deterministic Experiment:

An experiment is said to ""*Deterministic*", it has unique outcome, when repeated under same conditions e.g.

1. Na + Cl $\longrightarrow$ Nacl
2. $2H_2$ + O_2 $\longrightarrow$ $2H_2O$
3. 7 + 5 $\longrightarrow$ 12
4. Every soul shall taste death.
5. Time and Tide wait for none.

Such experiments are full of certainty.

Radom Experiment

An experiment which has two or more than two outcomes is said to be *"Random" or "Probabilistic Experiment".* The outcomes of such experiment face uncertainty.

1. On a cloudy day our mothers provide us an umbrella, when we leave our homes, because it seems that it may rain.

2. Weather forecasting is another applicative example of Random experiment.

3. Tossing a coin.

4. Match between two teams.

5. Rolling a dice.

Probability is the measurement of certainty or uncertainty of outcomes in a Random experiment, where there is uncertainty, Probability steps in**:**

Sample Space: The set of all possible outcomes of a Random experiment is called sample space. It is usually denoted by "S".

1. **Rolling a die:**
   S = {1, 2, 3, 4, 5, 6}

2. **Rolling a pair of die:**

S	=	{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)}
		{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)}
		{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)}
		{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)}
		{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)}
		{(6,1), {6,2), (6,3), (6,4) (6,5) (6,6)}

3 **Tossing a Coin:**

S = {HT}

4 **Tossing two coins**

S = {(HH), (HT), (TH), (TT)}

5 **Tossing three coins**

S = {(***HHH, HHT, HTH, HTT,*** TTT, TTH, THT, THH}

(***Important Note :*** *Find these elements, then replace "H" by "T" and vice versa*)

6 **Tossing four coins**

S	**=**	***{ HHHH,***	***HHHT,***	***HHTH,***	***HTHH}***
		{ HHTT,	***HTTH,***	***HTHT,***	***HTTT}***
		{TTTT,	TTTH,	TTHT,	THTT}
		{TTHH,	THHT,	THTH	THHH}

(Write these elements, then replace "H" by "T" and vice versa)

Remarks:

1. In case of rolling "n" dies
   Total elements = 6^n
2. In case of Tossing "n" coins
   Total elements = 2^n

7 **Tossing a coin and die**

S = { H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

EVENT: Let "S" be a sample space associated with the Random experiment then any part (subset) of "S" is called "***Event".*** e.g. In case of rolling a dice

S = {1, 2, 3, 4, 5, 6}

Let “A” be the *event* getting *even number*. Then

$$A = \{2, 4, 6\}$$

Let “B” be the *event* getting *Prime number*. Then

$$B = \{2, 3, 5\}$$

(Compound even containing more than one element)

Let “C” be the *event* getting *even number. Then*

$$C = \{2\}$$

(Elementary event /simple event containing one element)

Let “D” be the event getting number less than “7”. Then

$$D = \{1, 2, 3, 4, 5, 6\}$$

(Certain / sure elements, containing all elements of sample space).

Impossible Event:

Let “E” be an event getting number less than “1”, *which is impossible, this is called Impossible or Null Event denoted by* $\emptyset$ *containing no element at all.*

Complimentary Event ($\bar{A}$):

Let “S” be a sample space, in case of match between two teams “E” and “F”, if “A” is the event winning match by team “E” then complement event of “A” denoted by $\bar{A}$ means losing match by team “E”. e.g.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

Then $\bar{A}$ is also called “***Negation of Event “A”*** or not ***“A”.*** Thus

$$\bar{A} = \{1, 3, 5\}$$

Remarks: $\bar{A}$ is also called *“negation of event” “A” or not “A”.*

Occurrence of an Event:

An event “A” associated to a “Random experiment” is said to occur if any one of the elementary events associated to the even “A” is an outcome. e.g.

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{2, 4, 6\}$$

Suppose that in Trial/Experiment, the outcome is “4”. We say event “A” has occurred.

In another trial, if outcome is “3”, then we say even “A” has not occurred.

CLASS – X

CHAPTER - 15

IMPORTANT FORMULAS & CONCEPTS

PROBABILITY:

Experimental or empirical probability "P(E)" of an even "E" is

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of Trials}}$$

The theoretical probability (also called classical probability of an event "A", written as "P(A)" is

$$P(A) = \frac{\text{Number of outcomes of favourable to A}}{\text{Number of all possible outcomes of the experiment}}$$

$$= \frac{n(A)}{n(S)}$$

Two or more events of an experiment, where occurrence of an event prevents of all other events are called "***Mutually Exclusive Events.***"

Compliment Events and Probability:

We denote the event *"not* E" by $\overline{E}$, so

$$P(E) + P(\text{not } E) = 1\ .$$

i.e. $P(E) + P\,\overline{E} = 1$, which gives us

$$P(\overline{E}) = 1 - P(E)$$

In general, it is true that for an even E

$$P(\overline{E}) = 1 - P(E)$$

- The probability of an event which is impossible to occur is 0. Such event is called ***impossible event, dentoted by P(∅).*** Thus

$$P(\emptyset = \frac{0}{n(s)} = 0 \longrightarrow \text{Least (minimum)}$$

- The probabilty of an event which is sure (or certain) to occur is "1". Such an event is called ***Sure Event*** or ***a Certain Event, denoted by P(S).*** Thus

$$\mathbf{P(S)} = \frac{n(S)}{n(S)} = \mathbf{1} \longrightarrow \text{Maximum}$$

- The probability of an event "E" is a number P(E) such that $\mathbf{0 \leq P(E) \leq 1}$
- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

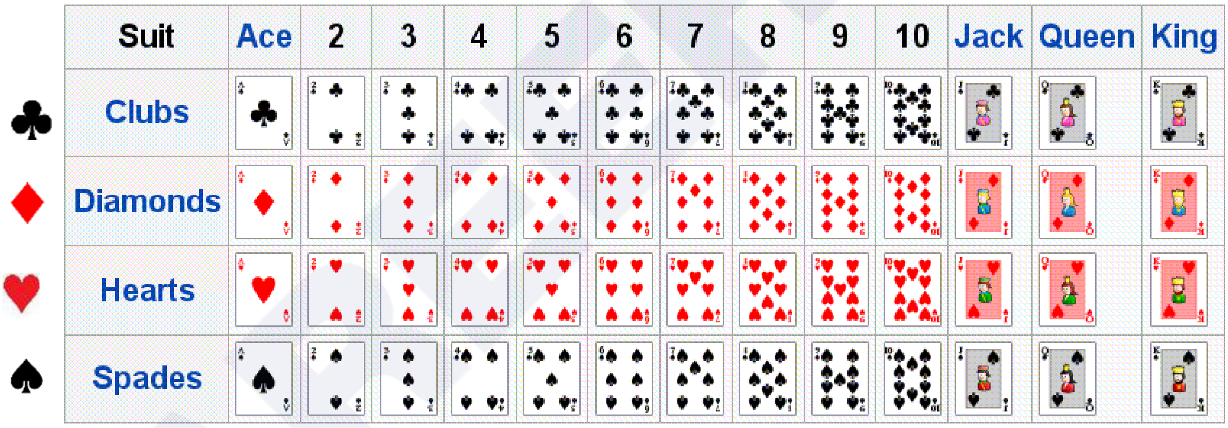
Deck of Cards and Probability

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are Black Spades (♠) Red Hearts (♥), Red Diamond (♦) and Black Club (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.

Kings, Queen and Jacks are called ***Face Cards.***

Example set of 52 poker playing cards



Suits	*Spade* ♠	*Club* ♣	*Diamond* ♦	*Heart* ♥
Cards	***13***	***13***	***13***	***13***
Colour	***Black (26 Cards)***		***Red (26 Cards)***	
Face Card (12)	***J, K, Q***	***J,K, Q***	***J,K,Q***	***A, J, K, Q (Total face Cards= 12)***
Court Cards	***A,J, K, Q***	***A, J,K, Q***	***A, J,K,Q***	***A, J, K, Q (Total face Cards= 16)***

Equally Likely Events: Two or more events are said to be equally likely, if each one of them has an equal chance of occurrence.

Mutually Exclusive Events: Two or more events are mutually exclusive, if the occurrence of each event prevents the every other event.

Complementary Events: Consider an event has few outcomes. Event of all other outcomes in the sample survey which are not in the favourable even is called complementary event.

Exhausive Events **:** All the events are exhausive events, if their union is the sample space.

Sure Events **:** The sample space of a random experiment is called ***sure or certain event*** as any one of its elements will surely occur in any trail of the experiment.

Impossible Event **:** An event which will occur on any account is called an ***impossible event.***

1 Mark Questions

Q.1 A coin is tossed 1000 times and 560 times a “head” occurs. The empirical probability of occurrence of the “head” in this case is

a) 0.5 b) 0.56 c) 0.44 d) 0.056

Q.2 Two coins are tossed 200 times and the following out comes are recorded

HH	HT/TH	TT
56	110	34

What is the empirical probability of occurrence of at least one “head” in the case

a) 0.33 b) 0.34 c) 0.66 d) 0.83

A die is thrown 200 times and the following outcome are noted, with their frequencies

Outcome	1	2	3	4	5	6
Frequency	56	22	30	42	32	18

Q.3 What is the empirical probability of getting a 1 in the above case.

a) 0.28 b) 0.22 c) 0.15 d) 0.21

Q.4 What is the empirical probability of getting a number less than 4?

a) 0.50 b) 0.54 c) 0.46 d) 0.52

Q.5 What is the empirical probability of getting a number greater than 4.

a) 0.32 b) 0.25 c) 0.18 d) 0.30

Q.6 On a particular day, the number of vehicles passing a crossing is given below:

Vehicle	Two wheeler	Three wheeler	Four wheeler
Frequency	52	71	77

What is the probability of a two wheeler passing the crossing on that day?

a) 0.26 b) 0.71 c) 0.385 d) 0.615

Q.7 The following table shows the blood- group of 100 students

Blood group	A^{+ve}	B^{-ve}	O^{+ve}	AB^{-ve}	B^{+ve}
Number of students	12	23	35	20	10

One student is taken a random. What is probability that his blood group is B^{+ve}

a) 0.12 b) 0.35 c) 0.20 d) 0.10

Q.8 In a bag, there are 100 bulbs, out of which 30 are bad ones. A bulb is taken out of the bag at random. The proability of the selected bulb to be good is

a) 0.50 b) 0.70 c) 0.30 d) None of these

Q.9 On a page of telephone directory having 250 telephone numbers, the frequency of the unit digit of those number is given below

0	1	2	3	4	5	6	7	8	9
18	22	32	28	40	30	30	22	18	10

A telephone number is selected from the page at random. What is the probability that its digit is

(a)2

a) 0.16 b) 0.128 c) 0.064 d) 0.04

(b) More than 6

a) 0.20 b) 0.25 c) 0.32 d) 0.16

(c) less than 2

a) 0.16 b) 0.18 c) 0.22 d) 0.32

Q.10 10 defective pens are accidentally mixed with 90 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is good one.

a) 0.10 b) 0.20 c) 0.90 d) 1.0

Q.11 Define Probability

Q.12 Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game.

Q.13 Probability of sure event is 1 *(T/F)*

ANSWERS

Q.1	(b)	Q.2	(d)	Q.3	(a)	Q.4	(b)	Q.5	(b)	Q.6	(a)
Q.7	(d)	Q.8	(b)	Q.9(a)	b	Q.9 (c) a		Q.9 (c) a		Q.10	(c)

Q.11 Definition Q.12 Because H and T have equal probabilities to occur

Q.13 (T)

2 Mark Questions

Q.1 An unbaised die is thrown. What is the probability of getting

i) An even number (ii) a multiple of 3
iii) An even number or multiple of 3 (iv) An odd number
v) A number between 2 and 6 (vi) A number less than 1.
vii) A number less than or equal to 6

Q.2 Two unbaised coins are tossed simultaneously. Find the probability of getting

i) Two heads ii) One Head iii) One Tail
iv) Atleast one head v) At most one head vi) No head.

Q.3 Three unbaised coins are tossed together. Find the probability of getting

i) All heads ii) Two Heads iii) One Head
iv) Atleast two heads v) At most two heads

Q.4 Two dice are thrown simultaneously. Find the probability of getting

i) An even number as Sum ii) The sum as prime number
iii) A total of at least 10
iv) A doublet and a doublet of odd number
(Hint A ={ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6))

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

v) A multiple of 2 on one die and a multiple of 3 on the other.
vi) A multiple of 3 as sum.

Q.5 There are six marbles in a box numbered 1 to 6. What is the probability of drawing a marble with prime number.

Q.6 The probability that it will rain tommorrow is 0.75. What is the probability it will not rain tommorrow?

Q.7 An Urn contain 10 read and 8 white balls. One ball is drawn at random. Find the probability that the ball drawn is white.

Q.8 A bag contains 3 red balls , 5 black balls and 4 white balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is

i) White ii) Red iii) Black iv) Not Red

Q.9 If probability of winning a game is 0.5. What is the probability of lossing it.

Q.10 In a lottery there are 10 prizes and 25 blanks. What is he probability of

i) getting a prize ii) not getting a prize

Q.11 What is the probability that a number selected from the numbers 1,2,3…. 20 is a mulitiple of 3.

ANSWERS

Q.1 i) $\frac{1}{2}$ ii) $\frac{1}{3}$ iii) $\frac{2}{3}$ iv) $\frac{1}{2}$ v) $\frac{1}{2}$ vi) 0 vii) 1

Q.2 i) $\frac{1}{4}$ ii) $\frac{1}{2}$ iii) $\frac{1}{2}$ iv) $\frac{3}{4}$ v) $\frac{3}{4}$ vi) $\frac{1}{4}$

Q.3 i) $\frac{1}{8}$ ii) $\frac{3}{8}$ iii) $\frac{3}{8}$ iv) $\frac{1}{2}$ v) $\frac{7}{8}$

Q.4 i) $\frac{1}{2}$ ii) $\frac{5}{12}$ iii) $\frac{1}{6}$ iv) $\frac{1}{6}$ v) $\frac{1}{12}$ vi) $\frac{11}{36}$ vii) $\frac{1}{3}$

Q.5 $\frac{1}{2}$ Q.6 0.25 Q.7 $\frac{4}{9}$ Q.8 i) $\frac{1}{3}$ ii) $\frac{1}{4}$ iii) $\frac{5}{12}$ iv) $\frac{3}{4}$

Q.9 0.95 Q.10 i) $\frac{2}{7}$ ii) $\frac{5}{7}$ Q.11 $\frac{3}{10}$

3 MARK QUESTIONS

Q.1 A lot consits of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy it if it defective. The shopkeeper draws one pen at random and gives it to her. What is a probability that

a) She will buy it   b) She will not buy it

Q.2 A jar contains 54 marbles each of which some are blue, some are green and some are white. The probability of selecting a green marble at random is 1/3 and the probability of selecting a blue marble at random is 4/9. How many white marbles does the jar contain. *(Hint: P(G) + P(B) + P (W)) = {1}*

Q.3 A letter is chosen at random from the letter of the word "ASSASSINATION". Find the probability that letter choosen is (i) a vowel (ii) an consonant (iii) A (iv) S (v) N.

Q.4 A letter is choosen at random from the letter of the word "INDEPENDENCE". Find the probability that the letter choosen is a (i) Vowel (ii) Consonant (iii) I (iv) N (v) D.

Q.5 A letter is choosen at random from the letter of the word "MATHEMATICS". Find the probability that the letter choosen is a (i) Vowel (ii) Consonant (iii) A (iv) T (v) M.

Q.6 A letter of English alphabet is choosen at random. Detrmine the probability that the letter is consonant.

Q.7 There are 1000 sealed envelopes in a box . 10 of them contain a cash prize of Rs. 100. 100 of them contain a cash prize of Rs. 50 each and 200 them contain a cash prize of Rs. 10 and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out. What is the probability that it contains no cash prize?

Q.8 Box "A" contains 25 slips of which 19 are marked Rs. 1 and other marked Rs. 5 each. Box "B" contains 50 slips of which 45 are marked Rs. 1, each and other are marked Rs. 13 each. Slips of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than Rs. 1?

Q.9 A carton of 24 bulbs contain 6 defective bulbs. One bulb is drawn at random. What is th probability that the bulb is not defective. If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest. What is the probability that the second is defective?

Q.10 A child's game has 8 triangles of which three are blue and rest are red and 10 squares of which six are blue and rest are red. One piece is lost at random. Find the probability it is a

(i) triangle (ii) square (iii) square of blue colur (iv) triangle of red colour

Q.11 In a game the entry fee is Rs. 5. The game consists of a tossing a coin three times. If one or two head show. Sweta gets her entry fee back. If she throws three heads, she receives double the entry fees. Otherwise, she will loss. For tossing a coin three times. Find the probability that she

(i) loses the entry fee (ii) gets double entry fee
(iii) Just gets her entry fee.

Q.12 A dice has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recored

(i) how many different scores are possible?
(ii) What is the probability of getting a total seven?
(Hint: Different Scores: 0,1,2,6,7,12.
Here "S" = (0,0) (0,1), (0,1) (0,1), (0,6), (0,6), (1,0), (1,1), (1,1), (1,1), ***(1,6), (1,6),*** *(1,0),.....* ***(1,6), (1,6),*** *(1,0)………..* ***(1,6), (1,6),*** *(6,0),* ***(6,1), (6,1), (6,1), (****6,6), (6,6), (6,0),* ***(6,1), (6,1), (6,1),*** *(6,6)(66)*

P (Total) = 12/36 = 1/3

Q.13 A lot consists of 48 mobile phones of which 42 are good, three have only minor defect and three have major defects. Varnika will buy a phone, if it is good, but the trader will only buy a mobile if it had no major defect. One phone is selected at random from the lot. What is the probability that it is

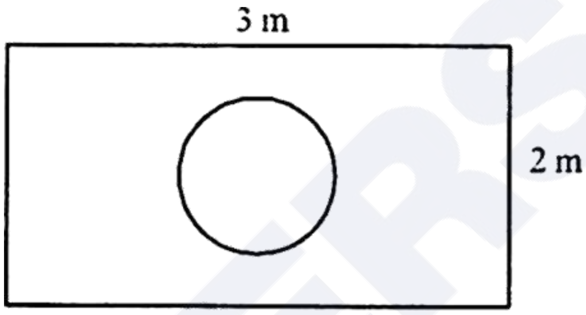
(i) a good phone (ii) a bad phone

Q.14 i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective.

ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective.

Q.15 A game of chance consits of spinnings an arrow which comes to rest poinitng at one of the numbers, 1, 2, 3, 4, 5, 6, 7, 8 (see fig) and these are equally likely outcomes. What is the probability that it will point at (i) 8 ? (ii) an odd number ? (iii) a number greater than 2? (iv) a number less than 9 ?.

Q.16 Suppose you drop a die at random on the rectangular region shown in above right sided figure. What is the probability that it will land inside the circle with diameter 1 m?



.17 A child has a die whose six faces show the letters as given below:

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A ? ii) D ?.

Q.18 A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanief wins if all the tosses give the same result i.e. three heads or three tails, and loses otherwise. Calculate the probability that Hanief will lose the game.

ANSWERS

Q.1	(i)	$\frac{31}{36}$	(ii)	$\frac{5}{36}$	Q.2	(i)	$\frac{2}{9}$			
Q.3	(i)	$\frac{6}{13}$	(ii)	$\frac{7}{13}$	(iii)	$\frac{3}{13}$	(iv)	$\frac{4}{13}$	(v)	$\frac{2}{13}$
Q.4	(i)	$\frac{5}{12}$	(ii)	$\frac{7}{12}$	(iii)	$\frac{1}{3}$	(iv)	$\frac{1}{4}$	(v)	$\frac{1}{6}$
Q.5	$\frac{4}{11}$		(ii)	$\frac{7}{11}$	(iii)	$\frac{2}{11}$	(iv)	$\frac{2}{11}$	(iii)	$\frac{2}{11}$
Q.6	$\frac{21}{26}$	Q.7	$\frac{69}{100} = 0.69$		Q.8	$\frac{11}{75}$	Q.9	(i)	$\frac{3}{4}$ (ii)	$\frac{5}{23}$
Q.10	(i)	$\frac{4}{9}$	(ii)	$\frac{5}{9}$	(iii)	$\frac{1}{3}$	(iv)	$\frac{5}{18}$		
Q.11	(i)	$\frac{1}{8}$	(ii)	$\frac{1}{8}$	(iii)	$\frac{3}{4}$	Q.12	(i)	six	(ii) $\frac{1}{3}$
Q.13	(i)	$\frac{14}{15}$	(ii)	$\frac{1}{15}$	Q.14	(i)	$\frac{1}{5}$	(ii)	$\frac{15}{19}$	
Q.15	(i)	$\frac{1}{8}$	(ii)	$\frac{1}{2}$	(iii)	$\frac{3}{4}$	Q.16	(i)	$\frac{\pi}{24} = \frac{11}{84}$	
Q.17	(i)	$\frac{1}{3}$	(ii)	$\frac{1}{6}$	Q.18	$\frac{3}{4}$				

4 Mark Questions

Q.1 Find the probability that a leap year selected at Random will contain 53 Sundays. *(Hint" In a leap year, we have 366 days or 52 weeks and 2 days. The remaining 2 days can be*

a) Sundays and Monday b) Monday and Tuesday
c) Tuesday and Wednesday d) Wednesday and Thursday
e) Thursday and Friday f) Friday and Saturday
g) Saturday and Sunday.

Hence required probability $= \frac{2}{7}$

Q.2 One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely drawn. Find the probability the card drawn is

a) An ace b) Red c) Either Red or King
d) Red and a King e) A face card f) A Red face card
g) 2 of Diamonds h) 10 of Black Suit.

Q.3 The King,Queen and Jack of Clubs are removed from a deck of 52 playing cards and then well shuffled. One card is selected from the remaining cards. Find the probability of:

a) a Heart b) a King c) a Club d) the 10 of hearts

Q.4 What is the probability that oridnary year has 53 Sundays?

Q.5 Red Queens and Black Jacks are removed from a pack of 52 playing cards. A card is drawn at Random from the remaining cards. Find the probabiity that card drawn is

a) a King b) of Red colour c) a Face card d) a Queen

Q.6 Cards drawn in a bag are numbered from 1 to 30. A card is drawn at random from this bag. Find the probability that the number of card is

a) not divisible by 3 b) a prime number greater than 7
c) not a perfect square number

Q.7 A dice is rolled twice. Find the probability that

a) 5 will not come up either time b) 5 will come up exactly one time

Q.8 A box contains cards numbered 3, 5, 7, 9, ………, 35, 37. A card is drawn at random from the box. Find the probability that the number on the drawn is a prime number.

Q.9 What is the probability that a number selected at random from the numbers 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, will be their average.

Q.10 If a number x is choosen at random from the numbers -2, -1, 0, 1, 2. What is the probability that $x^2 < 27$.

Q.11 A box contains 100 red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box. Find the probability that it will be

a) a blue card b) not a yellow card c) neither yellow nor blue card

ANSWERS

Q.1	$\frac{2}{7}$	Q.2	i)	$\frac{1}{13}$	ii)	$\frac{1}{2}$	iii)	$\frac{7}{13}$	iv)	$\frac{1}{26}$	v)	$\frac{3}{13}$	
vi)	$\frac{3}{26}$	vii)	$\frac{1}{52}$	viii)	$\frac{1}{26}$	Q.3	i)	$\frac{13}{49}$	ii)	$\frac{3}{49}$	iii)	$\frac{10}{49}$	
iv)	$\frac{1}{49}$	Q.4	i)	$\frac{1}{7}$	Q.5	i)	$\frac{1}{12}$	ii)	$\frac{1}{2}$	iii)	$\frac{1}{6}$	iv)	$\frac{1}{24}$
Q.6	i)	$\frac{2}{3}$	ii)	$\frac{1}{5}$	iii)	$\frac{5}{6}$	Q.7	i)	$\frac{25}{36}$	ii)	$\frac{5}{18}$		
Q.8	i)	$\frac{5}{9}$	Q.9	i)	$\frac{3}{10}$	Q.10	i)	$\frac{3}{5}$	Q11	i)	$\frac{1}{7}$		
ii)	$\frac{3}{7}$	iii)	$\frac{2}{7}$										