GSEB HSC MATHS

Question Papers (All Sets)

Time: 3 hours Q. Paper set I MATHS – I (050)(E) MARKS – 75

XII – Science ①

Secondary School

Q-1(A) (1) Obtain the formula for the area of the ΔABC, where $A(x_1, y_1), B(x_2, y_2) \& C(x_3, y_3) \in R^2$ (3)

(2) Find the area of the triangle with the vertices (5, 3), (4, 5) & (3, 1) by shifting the origin at (5, 3). (1)

Q-1(B) Calculate any two (4)

(1) Find the points which divide the line segment joining (0, 0) and (a, b) in to n equal parts

(2) Prove that the coordinates of all three vertices of an equilateral triangle cannot be rational numbers

(3) If (1, −1), (−4, 4) and (3, 6) are the vertices of rhombus, find its coordinates of fourth vertex.

Q-1(C) Calculate any two (4)

(1) Area of ΔABC is 4. coordinates of A and B are resp. A(2, 1) and B(4, 3). Find the coordinates of C if it lies on line $3x - y - 1 = 0$.

(2)

(2) A line passes through $(\sqrt{3}, -1)$ and the length of the segment perpendicular to it from the origin is $\sqrt{2}$. Find the eqn of the line.

(3) Find the eqn of the lines passing through $(4, 5)$ and parallel to and perpendicular to $2x + y - 1 = 0$

Q-1(D) Obtain p-α form of a line (3)

Q-2(A) (1) what do you mean by con-current lines? Obtain the necessary and sufficient condition for three lines in R^2 to lie concurrent? (3)

(2) The cartesian eqn of $\overleftrightarrow{AB}$ is $4x - 3y + 10 = 0$. If one parametric eqn is $x = 3t - 1, t \in R$ then obtain the second parametric eqn. (1)

Q-2(B) (1) obtain the general form of the eqn of a circle in R^2. obtain the condition for this eqn to represent a circle and find its centre and radius (2)

(2) Find the measure of angle between the lines $6x^2 - xy - y^2 = 0$ (1)

(3)

(3) Find the combined eq^n of lines through the origin which are perpendicular to lines $ax^2+2hxy+by^2=0$

Q.2(C) (i) Show that points of intersection of the lines representation by $2x^2-5xy+2y^2+7x-5y+3=0$ with the axes lie on a circle, find the eq^n of this circle (3)

OR

(i) Find the eq^n of the circle which is orthogonal to the circles $x^2+y^2-6x+1=0$ and $x^2+y^2-4y+1=0$ and whose centre lies on the line $3x+4y+6=0$ (2)

(ii) Find the set of intersection of the circle $x^2+y^2=25$ & the line $x+y-7=0$ (1)

Q.2(D) Prove that the eq^n of the lines through the origin which make an angle of measure of α and $x+y=0$ is $x^2+2xy\sec 2\alpha+y^2=0$ $\left(0<\alpha<\frac{\pi}{4}\right)$ (3)

OR

In ΔABC, A is $(4,-3)$ and two of the medians lie along the lines $2x+y+1=0$ and $x+5y-1=0$. Find the coordinates of B and C

(4)

Q-3(A) (1) Define the conic section and hence write the condition that conic section becomes a parabola. only write the eqn of focus and directrix (2)

(2) If a focal-chord of the parabola $y^2=4ax$ forms an angle of measure θ with the positive direction of the X-axis, then show that its length is $4a\,\mathrm{cosec}^2\theta$ (2)

OR

Find the set of points P so that

(1) the sum of the slopes of tangents drawn to the parabola from P is a constant k.

(2) The product of slopes of the tangents drawn to the parabola from P is a constant k.

Q-3(B) (1) If P is on the ellipse and S and S' are the focii then prove that $SP+S'P=2a$ (2)

(2) If the line containing the chord joining α and β passes through the focus $(ae,0)$ then prove that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{e-1}{e+1}$

OR

(5)

points P and Q are corresponding points of ellipse and auxillary circle resp. If the line parallel to $\overleftrightarrow{OQ}$ and passing through the point P intersects the axes in E and F resp, then P.T PE = b and PF = a

Q-3(C) (1) Explain the auxillary circle and the eccentric angle of the hyperbola (2)

(2) For a point on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ prove that $SP \cdot S'P = CP^2 - a^2 + b^2$ (2)

Q-3(D) (1) Find the standard eq^n of hyperbola whose focus is (3, 0) and b = 2 (1)

(2) Find the length of the chords of the circle $x^2+y^2+2gx+2fy+c=0$ cut on the axes. (2)

Q-4(A) Which curve is represented by the eq^n $x^2+2xy+y^2+\sqrt{2}x-\sqrt{2}y=0$?
Obtain the eq^n of the curve in the standard form. Find the coordinates of the foci, the eq^n of directrix, the length of axes and the eccentricity?

OR

6

(A) Determine the following curves by converting it in standard form :

(1) $x^2+y^2-4x-6y-2=0$

(2) $xy=16$

(B) (1) State the Schwartz inequality in R^3. From this, obtain the triangular inequality (2)

(2) Show that for any $a \in R$, the direction of $(2,3,5)$ and $(a, a+1, a+2)$ cannot be same or opposite.

(C) (1) By vector method, obtain position vector of incentre of triangle (2)

(2) If G is the centroid of ΔABC and P is any point in plane of this triangle, then P.T

$$\vec{PA}+\vec{PB}+\vec{PC}=3\vec{PG} \quad (2)$$

(D) (1) A river flows with a speed of 5 km/s. One desire to cross the river in direction $\perp$ar to the flow. Find in what direction he swim if his speed is 8 km/s (2)

(1) Find a unit vector in R^2 which is $\perp$r to $(1,3)$

(7)

Q-5 (A) (1) Prove that the necessary condition for two distinct lines $\bar{r} = \bar{a} + k\bar{l}$; $k \in R$ and $\bar{r} = \bar{b} + k\bar{m}$; $k \in R$ in R^3 to intersect each other is $(\bar{a} - \bar{b}) \cdot (\bar{l} \times \bar{m}) = 0$ (2)

(2) If the length of the ⊥r from the origin to the plane is p and the direction angles of ⊥r are α, β, γ then show that the eqn of the plane is $x\cos\alpha + y\cos\beta + z\cos\gamma = p$

OR (2)

Obtain vector eqn and cartesian eqn of the plane passing through two parallel lines.

(B) (1) Obtain the necessary and sufficient conditions for the eqn $x^2+y^2+z^2+2ux+2vy+2wz+d = 0$ represent a sphere (1)

(2) Find the intersection of the lines

$$\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-1}{1} \quad \&$$

$$\frac{x}{2} = \frac{y+1}{0} = \frac{z+3}{3} \qquad (3)$$

OR

Find the eqn of the line ⊥r to $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passing through $(3, -1, 11)$

(8)

(C) (1) Find the coordinates of a point equidistant from $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and $(0, 0, 0)$ (2)

(2) verify that the eq^n

$$3x^2+3y^2+3z^2-3x-2y-6z-66=0$$

represents the sphere, or not. If it represents the sphere then find the centre and radius of sphere. (2)

(D) Obtain the eq^n of the plane containing $\bar{r}=(1,1,1)+k(2,1,2)$, $k \in R$ and passing through $(1,-1,2)$

OR

(D) Find the length, the foot and the eq^n of $\perp$r from $(2,-1,2)$ to the plane $2x-3y+4z-44=0$

Solution of paper set I MATHS - I

PAPER ÷ 4 (1)

Q I (A) (1) Theo. Text page - 9
Ch - I

Ans (2) Shifting the origin at (5, 3),
new co-ord of (5, 3) are
$(5-5, 3-3) = (0, 0)$

New co-ord of (4, 5) are
$(4-5, 5-3) = (-1, 2)$

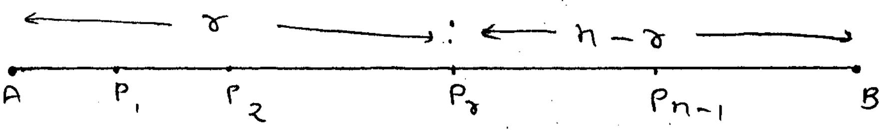
New co-ordinates of (3, 1) are
$(3-5, 1-3) = (-2, -2)$

$$\therefore D = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ -2 & -2 \end{vmatrix} = 2+4 = 6$$

$\therefore$ The area of the triangle
$= \frac{1}{2}|D| = \frac{1}{2}|6| = 3$ units

Q I (B)

(1) Let $P_1, P_2, \cdots, P_r, \cdots P_{n-1}$ be the points which divide $\overline{AB}$ in n equal parts.



Let $P_r = P_r(x_r, y_r)$, then P_r divides $\overline{AB}$ in ratio $r : n-r$ from A (see figure) $r = 1, 2, \cdots, (n-1)$

$$x_r = \frac{\frac{r}{n-r} \cdot a + 0}{\frac{r}{n-r} + 1} \qquad y_r = \frac{\frac{r}{n-r} \cdot b + 0}{\frac{r}{n-r} + 1} \quad ②$$

$$x_r = \frac{ar}{r+n-r} \qquad y_r = \frac{br}{r+n-r}$$

$$\therefore x_r = \frac{ar}{n}, \qquad y_r = \frac{br}{n}$$

$\therefore$ The required points of division are

$P_r(x_r, y_r) = P_r\left(\frac{ra}{n}, \frac{rb}{n}\right) \quad r = 1, 2, \ldots (n-1)$.

(2) Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of an equilateral triangle and if possible let all $x_i, y_i \in Q$, for $i = 1, 2, 3$.

$\therefore AB = BC = AC = a$ and

$x_1, y_1, x_2, y_2, x_3, y_3 \in Q \quad \therefore a^2 \in Q$

$(\because a^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 \in Q)$

$\therefore$ Area of $\triangle ABC = \Delta = \frac{1}{2}|D|$

where $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Since all $x_i, y_i \in Q$, $D \in Q$

$\therefore \Delta \in Q$

But $\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} \cdot a \cdot a \sin 60$

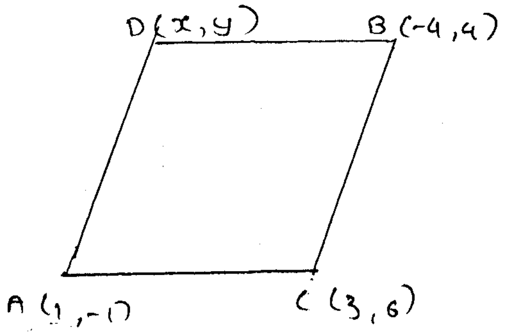
(In equilateral triangle $m\angle A = 60$)

$= \frac{\sqrt{3}}{4} a^2 \notin Q \quad (\because a^2 \in Q, \sqrt{3} \notin Q)$

Thus $\Delta \in Q$ and $\Delta \notin Q$ which are contradictory statements. 3

$\therefore$ All the co-ordinates of all the vertices of an equilateral ΔABC can not be rational.

(3)



Let $A(1,-1)$, $B(-4,4)$, $C(3,6)$ then

$AB^2 = (1+4)^2 + (-1-4)^2$

$= 25 + 25 = 50$

$BC^2 = (-4-3)^2 + (4-6)^2$

$= 49 + 4 = 53$

$AC^2 = (1-3)^2 + (-1-6)^2$

$= 4 + 49 = 53$

Since $AC = BC$, construct parallelogram $ABCD$. This will be rhombus due to $AC = BC$. Let $D(x, y)$.

Now mid-point of $\overline{CD}$ = mid point of $\overline{AB}$

$$\therefore \left(\frac{x+3}{2}, \frac{y+6}{2}\right) = \left(\frac{1-4}{2}, \frac{-1+4}{2}\right)$$

$\therefore x = -6$, $y = -3$

$\therefore D(x, y) = D(-6, -3)$ is the fourth vertex of the rhombus

Q1 (c) ④

(1) The co-ordinates of any point C on line $3x-y-1=0$ can be taken as $(x, 3x-1)$ $(\because y = 3x-1)$

$\therefore$ Area of $\Delta ABC = 4$

$\therefore \frac{1}{2}|D| = 4 \quad \therefore D = \pm 8$

where $D = \begin{vmatrix} x & 3x-1 & 1 \\ 2 & 1 & 1 \\ 4 & 3 & 1 \end{vmatrix} = 4x$

$\therefore 4x = \pm 8 \quad \therefore x = \pm 2$

For $x = 2$, point $C(x, 3x-1) = C(2, 5)$

For $x = -2$, point $C(x, 3x-1) = C(-2, -7)$

(2) Length of the perpendicular from origin on the line is given $\sqrt{2}$.

$\therefore$ The equation of line is p-α form is

$$x\cos\alpha + y\sin\alpha = p = \sqrt{2}$$

The line passes through $(\sqrt{3}, -1)$

$\therefore \sqrt{3}\cos\alpha - \sin\alpha = \sqrt{2}$

$\therefore \frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha = \frac{1}{\sqrt{2}}$

$\therefore \cos\alpha\cos\frac{\pi}{6} - \sin\alpha\sin\frac{\pi}{6} = \cos\frac{\pi}{4}$

$\therefore \cos\left(\alpha + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)$

$\therefore \alpha + \frac{\pi}{6} = \pm\frac{\pi}{4}$

$\therefore \alpha = \frac{\pi}{4} - \frac{\pi}{6}$ or $\alpha = -\frac{\pi}{4} - \frac{\pi}{6}$

$\therefore \alpha = \frac{\pi}{12}$ or $\alpha = -\frac{5\pi}{12}$ $(\because -\pi < \alpha \leq \pi)$ ⑤

$\therefore \cos\alpha = \cos\frac{\pi}{12}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore \sin\alpha = \sin\frac{\pi}{12}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos\alpha = \cos\left(-\frac{5\pi}{12}\right)$

$= \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin\alpha = \sin\left(-\frac{5\pi}{12}\right)$

$= -\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore$ Required equations are

(1) $\frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = \sqrt{2}$

$\therefore (\sqrt{3}+1)x + (\sqrt{3}-1)y = 4$

(2) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}x - \frac{(\sqrt{3}+1)}{2\sqrt{2}}y = \sqrt{2}$

$\therefore (\sqrt{3}-1)x - (\sqrt{3}+1)y = 4$

(3) The equations of lines parallel to and perpendicular to $2x+y=1$ are respectively $2x+y=k$ and $x-2y=k'$ both these lines are passing through $(4,5)$.

$\therefore 2(4)+5 = k$ | $\therefore 4-2(5) = k'$
$\therefore k = 13$ | $\therefore k' = -6$

$\therefore$ The required lines are respectively

$2x+y = 13$ and $x-2y+6 = 0$

Q.1.(D) Theory Text book page No. 41.

6

Q2 (A) (1) Ch-3 theory Page : 49-50

Ans : 2

(2) Substituting $x = 3t-1$ in $4x - 3y + 10 = 0$, we get

$4(3t-1) - 3y + 10 = 0$

$\therefore 3y = 12t + 6$

$\therefore y = 4t + 2$, $t \in R$ is the second parametric equation

Q2 (B) (1) Theorem circle Ch : 4

(2) Here $a = 6$, $h = \frac{1}{2}$, $b = -1$

$$\therefore \theta = \tan^{-1} \frac{2\sqrt{\frac{1}{4}+6}}{|6-1|}$$

$$= \tan^{-1} 1 = \frac{\pi}{4}$$

(3) $ax^2 + 2hxy + by^2 = 0$ is the combined equation of a pair of lines.

$$\therefore b\left(\frac{y}{x}\right)^2 + 2h\frac{y}{x} + a = 0$$

Put $\frac{y}{x} = m$, then m is the slope of the lines represented by equation (1)

Let m_1 and m_2 be the roots of $bm^2 + 2hm + a = 0$, then m_1 and m_2 are also the slopes of two seperate lines represented by (1)

$\therefore m_1 + m_2 = \frac{-2h}{b}$ and $m_1 \cdot m_2 = \frac{a}{b}$

(7)

The lines whose slopes are $m_1' = -\frac{1}{m_1}$ and $m_2' = -\frac{1}{m_2}$ are perpendicular to the lines represented by (1)

Now, $m_1' + m_2' = -\frac{1}{m_1} - \frac{1}{m_2} = -\left(\frac{m_1 + m_2}{m_1 m_2}\right)$

$$= -\frac{-2h/b}{a/b} = \frac{2h}{a}$$

$\therefore m_1' \cdot m_2' = \left(-\frac{1}{m_1}\right)\left(-\frac{1}{m_2}\right) = \frac{1}{m_1 m_2} = \frac{b}{a}$

$\therefore$ The combined equation of lines with slopes m_1' and m_2' is

$$y^2 - (m_1' + m_2')\,xy + m_1' \cdot m_2' \, x^2 = 0$$

$$\therefore y^2 - \frac{2h}{a} xy + \frac{b}{a} x^2 = 0$$

$$\therefore ay^2 - 2hxy + bx^2 = 0$$

$$\therefore bx^2 - 2hxy + ay^2 = 0$$

$\therefore$ This is the required equation.

Q 2 (c)

(i) Put $y = 0$ to find the intersection of pair of lines with x-axis,

$$2x^2 + 7x + 3 = 0$$

$$\therefore (x+3)(2x+1) = 0$$

$$\therefore x = -3, \quad x = \frac{-1}{2}$$

$\therefore$ The pair of lines intersect x-axis at $A(-3, 0)$, $B\left(-\frac{1}{2}, 0\right)$

Similarly putting $x = 0$, we get (8)

$2y^2 - 5y + 3 = 0$

$\therefore (2y-3)(y-1) = 0$

$\therefore y = \frac{3}{2}$, $y = 1$

$\therefore$ The pair of lines intersects Y-axis at $C(0,1)$ and $D(0, 3/2)$

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ --- (1) be the circle passing through $A(-3, 0)$, $B(-\frac{1}{2}, 0)$, $C(0, 1)$

$\therefore$ The co-ordinates of these points satisfy equation of that circle

$\therefore 9 - 6g + c = 0$ --- (2)

$\frac{1}{4} - g + c = 0$ --- (3)

$1 + 2f + c = 0$ --- (4)

Subtracting (3) from (2),

$-5g + \frac{35}{4} = 0$

$\Rightarrow g = 7/4$

Using (3), $\frac{1}{4} - \frac{7}{4} + c = 0$

$\Rightarrow c = 3/2$

Using (4), $1 + 2f + \frac{3}{2} = 0$

$\Rightarrow f = -5/4$

$\therefore$ The required circle is

$x^2 + y^2 + \frac{7}{2}x - \frac{5}{2}y + \frac{3}{2} = 0$

$\therefore 2x^2+2y^2+7x-5y+3=0$ ----- (5) ⑨

For $D(0, 3/2)$,

$0+2\times\frac{9}{4}+0-5\times\frac{3}{2}+3$

$=\frac{9}{2}-\frac{15}{2}+3=0$

$\therefore D$ is also a point on (5)

$\therefore A, B, C, D$ are on a circle whose eq^n is given by (5)

OR

(1) Let $x^2+y^2+2gx+2fy+c=0$ be the required circle.

The centre $c(-g,-f)$ of this circle is on line $3x+4y+6=0$

$\therefore -3g-4f+6=0$ ---- (1)

The required circle is orthogonal to the given circles

$\therefore$ using $2g_1g_2+2f_1f_2=c_1+c_2$, we get

$2g(-3)+2f(0)=c+1$ ---- (2)

$2g(0)+2f(-2)=c+1$ ---- (3)

Solving (1), (2), (3) for g, f, c, we get

$g=\frac{2}{3}$, $f=1$, $c=-5$

$\therefore$ The required circle is

$x^2+y^2+2gx+2fy+c=0$

ie $x^2+y^2+\frac{4}{3}x+2y-5=0$

ie $3x^2+3y^2+4x+6y-15=0$

(10)

(2) Solving $x+y=7$ and

$x^2+y^2=25$, $y=7-x$

$\therefore x^2+(7-x)^2=25$

$\therefore x^2+x^2-14x+49=25$

$\therefore 2x^2-14x+24=0$

$\therefore x^2-7x+12=0$

$\therefore (x-4)(x-3)=0$

$\therefore x=4$ or $x=3$

$\therefore x=4 \Rightarrow y=7-x=7-4=3$

$\therefore x=3 \Rightarrow y=7-x=7-3=4$

$\therefore$ The points of intersection are

$A(4,3)$, $B(3,4)$.

$\therefore$ Intersection set : $\{(4,3),(3,4)\}$

Q2 (0)

Slope of $x+y=0$ is -1.

Let the slope of line making angle of measure α with $x+y=0$ be m, then

$$\tan\alpha = \left|\frac{m-(-1)}{1+m(-1)}\right| = \left|\frac{m+1}{1-m}\right|$$

$$\therefore \frac{m+1}{1-m} = \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

or

$$\frac{m+1}{m-1} = \frac{\sin\alpha}{\cos\alpha}$$

$$\therefore \frac{m+1-(1-m)}{m+1+1-m} = \frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha} \quad \text{or}$$

$$\frac{m+1+m-1}{m+1-(m-1)} = \frac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}$$ (11)

$$\therefore m = \frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha} \quad \text{or} \quad m = \frac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}$$

These two values of m are the slopes of the required pair of lines through origin. Denote them by m_1 & m_2

$$\therefore m_1+m_2 = \frac{(\sin\alpha-\cos\alpha)^2+(\sin\alpha+\cos\alpha)^2}{\sin^2\alpha-\cos^2\alpha}$$

$$= -\frac{2}{\cos 2\alpha}$$

$\therefore m_1+m_2 = -2\sec 2\alpha$ and $m_1m_2 = 1$

$\therefore$ The equation of pair of lines is

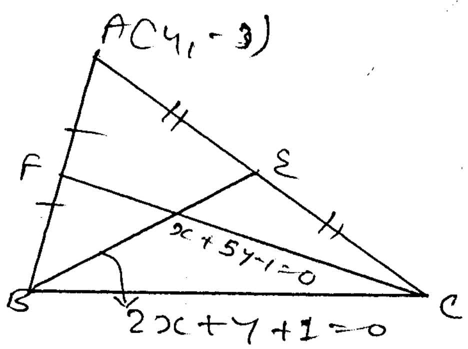
$$y^2-(m_1+m_2)xy+m_1m_2x^2 = 0$$

$$\therefore y^2-(-2\sec 2\alpha)xy+(1)x^2 = 0$$

$$\therefore y^2+2xy\sec 2\alpha+x^2 = 0$$

OR

(D)



Point A (4,−3) is not on the given lines containing medians ((4,−3), does not satisfy any of the equations $2x+y+1=0$ and $x+5y-1=0$)

$\therefore$ Take point A and the lines along the medians as shown in the figure

(12)

Let $B = B(a, b)$, then B is on $\overleftrightarrow{BE} : 2x + y + 1 = 0$

ie $2a + b + 1 = 0$ ------- (1)

mid-point of $\overline{AB}$ ie. F

$= F\left(\frac{a+4}{2}, \frac{b-3}{2}\right)$ is on $\overleftrightarrow{CF} : x + 5y - 1 = 0$

$\therefore \frac{a+4}{2} + \frac{5(b-3)}{2} - 1 = 0$

$\therefore a + 5b - 13 = 0$ ----- (2)

Solving (1) and (2), we get

$a = -2, \; b = 3$

$\therefore B(a, b) = B(-2, 3)$

Let $C = C(c, d)$, C is on $\overleftrightarrow{CF}$:

$x + 5y - 1 = 0$

$\therefore c + 5d - 1 = 0$ ----- (3)

The mid point of $\overline{AC}$:

ie $E = E\left(\frac{4+c}{2}, \frac{d-3}{2}\right)$,

E is on $\overleftrightarrow{BE} : 2x + y + 1 = 0$

$\therefore \frac{2(4+c)}{2} + \frac{d-3}{2} + 1 = 0$

$\therefore 2c + d + 7 = 0$ ---- (4)

Solving (3) and (4), we get

$d = 1, \; c = -4$

$\therefore C(c, d) = C(-4, 1)$

13

Q 3

Ans : 3 (A)

(1) Theorem Ch:5 - Parabola

(2) Let the end points of focal chord $\overline{PQ}$ be $P(t_1)$ and $Q(t_2)$ and let $\overleftrightarrow{PQ}$ form an angle of measure 60° with positive x - axis

$\therefore t_1 t_2 = -1$ and

$\tan\theta$ = slope of $\overleftrightarrow{PQ}$ $= \dfrac{2at_1 - 2at_2}{at_1^2 - at_2^2}$

$= \dfrac{2}{t_1 + t_2}$

$\therefore t_1 + t_2 = 2\cot\theta$ and $t_1 t_2 = -1$

$\therefore PQ^2 = (at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2$

$= a^2(t_1^2 - t_2^2)^2 + 4a^2(t_1 - t_2)^2$

$= a^2(t_1^2 - t_2^2)\cdot[(t_1 + t_2)^2 + 4]$

$= a^2[(t_1 + t_2)^2 - 4t_1t_2][(t_1 + t_2)^2 + 4]$

$= a^2[(t_1 + t_2)^2 + 4]^2 \quad (\because t_1 t_2 = -1)$

$= a^2(4\cot^2\theta + 4)^2 = a^2(4\operatorname{cosec}^2\theta)^2$

$PQ = 4a\operatorname{cosec}^2\theta$

OR

(14)

Let $P(x_1, y_1)$ be the point in the plane of the parabola $y^2 = 4ax$

Let m be the slope of tangent which passes through $P(x_1, y_1)$

$\therefore y = mx + \frac{a}{m}$ is the equation of such tangent

$\therefore y_1 = mx_1 + \frac{a}{m}$

$\therefore m$ satisfies $x_1 m^2 - y_1 m + a = 0$ ---- (1)

$\therefore$ If $\Delta = y_1^2 - 4ax_1 > 0$, then there are two roots m_1 and m_2 of equation (1)

$\therefore m_1 + m_2 = \frac{y_1}{x_1}$, $m_1 m_2 = \frac{a}{x_1}$

If sum of the slopes of tangent through $P(x_1, y_1)$ is constant, then

$\frac{y_1}{x_1} = m_1 + m_2 = k$ (Constant)

$\therefore (x_1, y_1)$ satisfies $y = kx$ which is the required equation of the set of points in (1)

If the product of the slopes of tangent is constant, then

$\frac{a}{x_1} = m_1 \cdot m_2 = k$

$\therefore kx_1 = a$

$\therefore (x_1, y_1)$ satisfies $kx = a$, which is the eq^n of set of points in (2)

(15)

Q 3 (B)

(1) Theorem Ch : 6 Ellipse

(2) $P(\alpha)$ and $Q(\beta)$ are the points on ellipse such that $\overline{PQ}$ passes through $S(ae, 0)$

The equation of $\overleftrightarrow{PQ}$ is

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$$

The line passes through $S(ae, 0)$

$$\frac{ae}{a}\cos\frac{\alpha+\beta}{2} + 0 = \cos\frac{\alpha-\beta}{2}$$

$$\therefore e = \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$$

$$\therefore \frac{e-1}{e+1} = \frac{\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}}$$

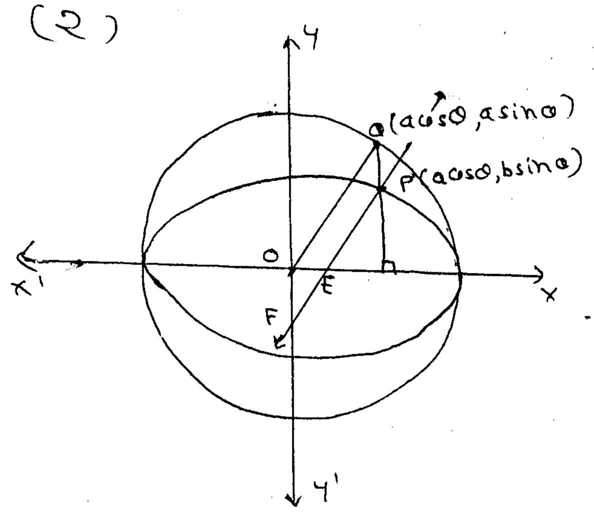
$$= \frac{2\sin\alpha/2 \, \sin\beta/2}{2\cos\alpha/2 \, \cos\beta/2}$$

$$= \tan\alpha/2 \, \tan\beta/2$$

OR

(16)

(2)



Let $P(a\cos\theta, b\sin\theta)$ $(-\pi < \theta \le \pi)$ be a point on ellipse $(\theta \neq 0)$

∴ The point corresponding to P on the auxiliary circle is $Q(a\cos\theta, a\sin\theta)$

∴ The slope of $\overleftrightarrow{OQ}$

$= \dfrac{a\sin\theta - 0}{a\cos\theta - 0} = \tan\theta$

∴ Equation of line parallel to $\overleftrightarrow{OQ}$ through P is

$y - b\sin\theta = \tan\theta(x - a\cos\theta)$

i.e. $y\cos\theta - b\sin\theta\cos\theta = x\sin\theta - a\sin\theta\cos\theta$

i.e. $x\sin\theta - y\cos\theta = (a-b)\sin\theta\cos\theta$ --- (1)

By substituting $y = 0$ in (1), we get the co-ordinates of E, the intersection of line (1) with x-axis and substituting $x = 0$ in (1), we get co-ord of points of intersection F of line (1) with y-axis

∴ $E = E((a-b)\cos\theta, 0)$, $F = F(0, -(a-b)\sin\theta)$

∴ $PE^2 = (a\cos\theta - (a-b)\cos\theta)^2 + (b\sin\theta - 0)^2$

$= b^2\cos^2\theta + b^2\sin^2\theta = b^2$ ∴ $PE = b$

$PF^2 = (a\cos\theta - 0)^2 + (b\sin\theta + (a-b)\cos\theta)^2$

$= a^2\cos^2\theta + a^2\sin^2\theta = a^2$ ∴ $PF = a$

(17)

Q-3(c)(1) theorem ch:7

(2) let $P(\theta)$ be a point on hyperbola
$P(\theta) = P(a\sec\theta, b\tan\theta)$, $C(0,0)$ is the centre of the hyperbola.

$\therefore CP^2 = a^2\sec^2\theta + b^2\tan^2\theta$ .... (1)

$S(ae, 0)$ and $S'(-ae, 0)$ are foci of the ellipse

$SP^2 = (a\sec\theta - ae)^2 + (b\tan\theta - 0)^2$

$= a^2(\sec^2\theta - 2e\sec\theta + e^2) + a^2(e^2-1)\tan^2\theta$

$= a^2\{\sec^2\theta - 2e\sec\theta + e^2 + e^2\tan^2\theta - \tan^2\theta\}$

$= a^2\{1 - 2e\sec\theta + e^2\sec^2\theta\}$ | $e > 1, |\sec\theta| \geq 1$

$= a^2(e\sec\theta - 1)^2$ | $\therefore |e\sec\theta| \geq 1$

$SP = |a(e\sec\theta - 1)|$

Similarly, $S'P = |a(e\sec\theta + 1)|$

$\therefore SP \cdot S'P = a^2(e^2\sec^2\theta - 1) = a^2\left\{\frac{a^2+b^2}{a^2}\sec^2\theta - 1\right\}$

$= \frac{a^2(a^2\sec^2\theta + b^2\sec^2\theta - a^2)}{a^2}$

$= a^2\sec^2\theta + b^2\sec^2\theta - a^2$

$= a^2\sec^2\theta + b^2(1+\tan^2\theta) - a^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + b^2 - a^2$

$= CP^2 + b^2 - a^2$

$= CP^2 - a^2 + b^2$ (using (1))

18

Q-3(D) (1)

Focus $S = (3,0)$, $b = 2$

$\therefore (ae, 0) = (3,0)$ $\therefore ae = 3$

$b^2 = a^2(e^2-1) = a^2e^2 - a^2 = (ae)^2 - a^2$

$\therefore 4 = 9 - a^2$ $\therefore a^2 = 9-4 = 5$

$\therefore$ Eq^n of curve : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore \frac{x^2}{5} - \frac{y^2}{4} = 1$

$4x^2 - 5y^2 = 20$

(2) centre $(-g,-f)$ $r = \sqrt{g^2+f^2-c}$

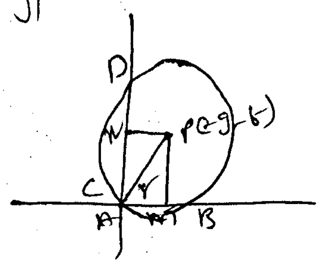
$P_1 = |-f|$ $P_2 = |-g|$

$AM^2 = r^2 - p^2$

$= g^2 - c$

$AM = \sqrt{g^2-c}$

$AB = 2\sqrt{g^2-c}$



CD can be obtained

(19)

Q-4(A)

comparing the equation with
$ax^2+2hxy+by^2+2gx+2fy+c=0$, $a=b=1$

Taking $\frac{\pi}{4}$ rotation : $x=\frac{x'-y'}{\sqrt{2}}$, $y=\frac{x'+y'}{\sqrt{2}}$

$\therefore$ Eqn in new system is

$$\frac{(x'-y')^2}{2}+\frac{2(x'-y')}{\sqrt{2}}\cdot\frac{(x'+y')}{\sqrt{2}}+\frac{(x'+y')^2}{2}+\sqrt{2}\left(\frac{x'-y'}{\sqrt{2}}\right)$$
$$-\sqrt{2}\left(\frac{x'+y'}{\sqrt{2}}\right)=0$$

$$\frac{4x'^2}{2}+x'-y'-x'-y'=0$$
$$x'^2=y'$$

$\therefore$ curve is a parabola

$\therefore$ Eccentricity is 1

comparing with $x^2=4ay$, $4a=1$ $\therefore a=\frac{1}{4}$
in (x',y') system,

Focus : $(0,a)=(0,\frac{1}{4})$; Directrix : $y'=-a=\frac{-1}{4}$
in (x',y') system,

focus : $x=\frac{x'-y'}{\sqrt{2}}=\frac{0-\frac{1}{4}}{\sqrt{2}}=\frac{-1}{4\sqrt{2}}$

$y=\frac{x'+y'}{\sqrt{2}}=\frac{0+\frac{1}{4}}{\sqrt{2}}=\frac{1}{4\sqrt{2}}$ $\therefore$ Focus $\left(\frac{-1}{4\sqrt{2}},\frac{1}{4\sqrt{2}}\right)$

(20)

Directrix : $y' = \frac{y-x}{\sqrt{2}} = \frac{-1}{4}$

$\therefore x - y = \frac{\sqrt{2}}{4}$

$\therefore x - y = \frac{1}{2\sqrt{2}}$.

OR

(A) (1) Given eq^n is $x^2+y^2-4x-6y-2=0$

$\therefore x^2-4x+4+y^2-6y+9-15=0$

$\therefore (x-2)^2+(y-3)^2=15$

Shift origin at $O'(2,3)$, so that $(x,y) = (X+2, Y+3)$

$\therefore$ the eq^n is $X^2+Y^2=15$ which is a circle.

(2) $xy=16$

$a=b=0$ $\therefore$ Taking $\frac{\pi}{4}$ Rotation axes,

$x = \frac{x'-y'}{\sqrt{2}}$, $y = \frac{x'+y'}{\sqrt{2}}$

$xy=16 \implies \frac{x'-y'}{\sqrt{2}} \cdot \frac{x'+y'}{\sqrt{2}} = 16$

$\therefore x'^2-y'^2=32$, comparing with $x^2-y^2=a^2$

$a^2=32$ $\therefore a=\sqrt{32}=4\sqrt{2}$

$\therefore$ curve is a rectangular hyperbola.

$\therefore e=\sqrt{2}$

(21)

Q.4(B) (1) theo. Ch : 9

(2) If the given directions are same or opposite then for some $k \in R - \{0\}$, $(2, 3, 5) = k(a, a+1, a+2)$

$\therefore ka = 2,\ k(a+1) = 3,\ k(a+2) = 5$

$\therefore ka = 2,\ ka + k = 3,\ ka + 2k = 5$

$\therefore 2 + k = 3\ ;\ 2 + 2k = 5$

$k = 1\ ;\ 2k = 3\ \therefore k = \frac{3}{2}$

Thus the eqn $ka = 2,\ k(a+1) = 3,\ k(a+2) = 5$ are not consistant.

$\therefore (2, 3, 5) \neq k(a, a+1, a+2)$ for any $k \in R - \{0\}$

$\therefore$ For any $a \in R$, the given directions cannot be same or opposite.

Q.4(C) (1) theo Ch-10.

(2) Let $P = P(\bar{o})$, $A = A(\bar{x})$, $B = B(\bar{y})$ and $C = C(\bar{z})$, where A, B, C are vertices of ΔABC and P is any point. The centroid of ΔABC is $G(\bar{g})$ where $\bar{g} = \frac{1}{3}(\bar{x} + \bar{y} + \bar{z})$

$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$

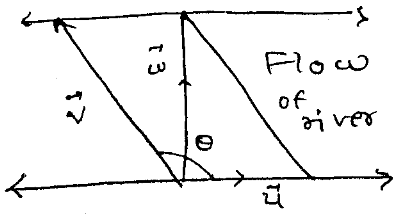
$= \bar{x} - \theta + \bar{y} - \theta + \bar{z} - \theta = \bar{x} + \bar{y} + \bar{z}$

$= \frac{3(\bar{x} + \bar{y} + \bar{z})}{3} = 3\bar{g} = 3\overrightarrow{PG}$

22

Q 4 (D)

(1)



Let $\hat{i}$ be the unit vector in the direction of flow of river and $\hat{j}$ be the unit vector $\perp^{lar}$ to the flow

Let the direction of swimmer make angle of measure θ with the direction of flow of river, so that resultant speed of swimmer be $\perp^{lar}$ to the flow

Speed of river $\vec{u} = 5\hat{i}$

Speed of swimmer is $\vec{v} = 8\cos\theta\hat{i} + 8\sin\theta\hat{j}$

The resultant speed of the swimmer is

$\vec{\omega} = \bar{u} + \bar{v} = 5\hat{i} + 8\cos\theta\hat{i} + 8\sin\theta\hat{j}$

$= (5 + 8\cos\theta)\hat{i} + 8\sin\theta\hat{j}$

Since $\vec{\omega} \perp \hat{i}$, $\vec{\omega}\cdot\hat{i} = 0$

$\therefore [(5 + 8\cos\theta)\hat{i} + (8\sin\theta)\hat{j}]\cdot\hat{i} = 0$

$\therefore 5 + 8\cos\theta = 0$

$\therefore \cos\theta = -5/8$

$\therefore \theta = \cos^{-1}\left(-\frac{5}{8}\right)$

$= \pi - \cos^{-1}\frac{5}{8}$

23

(1) Let $\bar{x} = (1, 3)$

Suppose $\bar{y} = (y_1, y_2) \in R^2$ such that

$\bar{y} \perp \bar{x}$ and $|\bar{y}| = 1$

$\therefore \bar{y} \cdot \bar{x} = 0 \Rightarrow y_1 + 3y_2 = 0$

$\Rightarrow y_1 = -3y_2 = k$ (say)

$\therefore y_1 = k$ and $y_2 = -k/3$

$|\bar{y}| = 1 \Rightarrow k^2 + \frac{k^2}{9} = 1$

$\Rightarrow k = \pm \frac{1}{\sqrt{10}}$

$\therefore \bar{y} = \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \right)$ or

$\bar{y} = \left(-\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right)$

24

Q 5 (A)

(1) Theo. Ch - 11

(2) Theory

OR

Theory

(B) (1)

(2) Let line $L_1 : \frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-1}{1} = k$

$(k \in R)$

$\therefore x-3 = k$, $y+2 = -k$, $z-1 = k$, $k \in R$

$\therefore$ Any point $P \in L_1$ can be expressed as

$P(x,y,z) = P(3+k, -k-2, 1+k)$, $k \in R$

Let line $L_2 : \frac{x}{2} = \frac{z+3}{3}$, $y+1 = 0$

If $L_1 \cap L_2 = \{P\}$, then P is on the line L_1 and also on L_2

$\therefore$ For some $k \in R$, $(3+k, -k-2, 1+k)$ should satisfy the eq^n of L_2

$\therefore \frac{3+k}{2} = \frac{1+k+3}{3}$ and $-k-2+1 = 0$

From $-k-2+1 = 0$, $k = -1$ which also satisfies

$\frac{3+k}{2} = \frac{1+k+3}{3}$

$\therefore$ The position vector (co-ordinates) of the point of intersection of L_1 and L_2 is

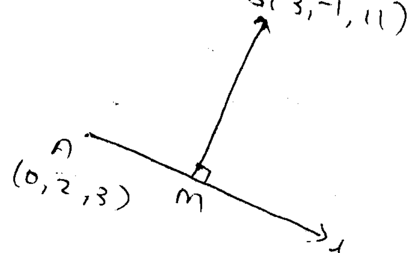
$(3+k, -k-2, 1+k) = (2, -1, 0)$

(For $k = -1$)

$\therefore L_1 \cap L_2 = \{P(2, -1, 0)\}$

OR

(2)



let $L : \frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = k$, say

$\therefore$ The direction of L is $\bar{l} = (2,3,4)$ and position vector of any point on l can be written as $(x, y, z) = (2k, 3k+2, 4k+3)$

$B(3,-1,11)$ is a point given outside the line. let m the foot of perpendicular from B on L. m is on L $\therefore$ For some $k \in R$, the position vector of m can be taken ~~(2k, 3k+3, 4k-8)~~ $(2k, 3k+2, 4k+3)$.

$\therefore \overrightarrow{Bm} = (2k, 3k+2, 4k+3) - (3, -1, 11)$

$= [(2k-3), (3k+3), (4k-8)]$

Since $\overrightarrow{Bm} \perp \bar{l}$, $\overrightarrow{Bm} \cdot \bar{l} = 0$

$\therefore 2(2k-3) + 3(3k+3) + 4(4k-8) = 0$

$29k - 29 = 0 \quad \therefore k = 1$

$\therefore$ The position vector of m is

$(2k, 3k+2, 4k+3) = (2, 5, 7) = \bar{c}$, say

(26)

$\therefore$ Eq^n of $\overrightarrow{BM}$ can be taken as $\bar{r} = \bar{b} + k(\bar{b} - \bar{c})$
where $B(\bar{b}) = B(3, -1, 11)$ & $M(\bar{c}) = M(2, 5, 7)$
$\therefore \bar{r} = (3, -1, 11) + k[(3, -1, 11) - (2, 5, 7)]$
$\bar{r} = (3, -1, 11) + k(1, -6, 4)$, $k \in R$

This is required line $\overrightarrow{BM}$ which passes through B & which is $\perp$ar to L.

Q.5(c) (i)

$A(\bar{x}) = A(a, 0, 0)$
$B(\bar{y}) = B(0, b, 0)$
$C(\bar{z}) = C(0, 0, c)$ &
$D(\bar{w}) = D(0, 0, 0)$
let $P(\bar{u}) = P(x, y, z)$
be at equal distance from
A, B, C, D.

$\therefore AP^2 = (x-a)^2 + y^2 + z^2$
$BP^2 = x^2 + (y-b)^2 + z^2$
$CP^2 = x^2 + y^2 + (z-c)^2$
$DP^2 = x^2 + y^2 + z^2$

Now $AP = BP = CP = DP$
$\therefore AP^2 = BP^2 = CP^2 = DP^2$
$\therefore AP^2 = BP^2 \Rightarrow (x-a)^2 + y^2 + z^2 = x^2 + y^2 + z^2$
$\Rightarrow x^2 - 2ax + a^2 + y^2 + z^2 - x^2 - y^2 - z^2 = 0$
$\Rightarrow a^2 - 2ax = 0 \therefore x = \frac{a^2}{2a} = \frac{a}{2}$

(27)

$BP^2 = DP^2 \Rightarrow -2by + b^2 = 0$

$\therefore y = \frac{b}{2}$

$CP^2 = DP^2 \Rightarrow -2cz + c^2 = 0$

$\therefore z = \frac{c}{2}$

$\therefore (x, y, z) = \left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

$\therefore P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from given four points

R)

(3) $3x^2 + 3y^2 + 3z^2 - 3x - 2y - 6z - 66 = 0$

$\therefore x^2 + y^2 + z^2 - x - \frac{2}{3}y - 2z - 22 = 0$

$\therefore u = -\frac{1}{2}, \; v = -\frac{1}{3}, \; w = -1, \; d = -22$

$\therefore r^2 = u^2 + v^2 + w^2 - d$

$= \frac{1}{4} + \frac{1}{9} + 1 + 22$

$= \frac{9 + 4 + 36 + 792}{36} = \frac{841}{36} > 0$

$\therefore$ Eq^n represents a sphere

$\therefore$ Centre $C = C(-u, -v, -w) = C\left(\frac{1}{2}, \frac{1}{3}, 1\right)$

Radius : $r = \sqrt{\frac{841}{36}} = \frac{29}{6}$

28

(4) $5x^2+5y^2+5z^2-5x-10y+15z+21=0$

$\therefore x^2+y^2+z^2-x-2y+3z+\frac{21}{5}=0$

$\therefore u=\frac{-1}{2}$, $v=-1$, $w=\frac{3}{2}$, $d=\frac{21}{5}$

$r^2 = u^2+v^2+w^2-d$

$= \frac{1}{4}+1+\frac{9}{4}-\frac{21}{5}$

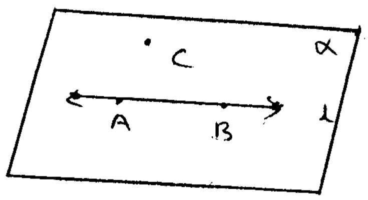
$= \frac{5+20+45-84}{20} = \frac{-19}{20} < 0$

$\therefore$ Eq^n does not represent a sphere in R^3.

Q5 (D)

(1) Let us select two different points on line

$\bar{r} = (1,1,1)+k(2,1,2)$, $k \in R$



For k=0 and 1, we get points A(1,1,1), B(3,2,3) on the given lines

Point C (1,-1,2) is not on given line

$\therefore$ The required plane passing through the given line and point C is same as the plane determined by non-collinear points A, B and C

(29)

$\therefore$ Its eq^n is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

ie $$\begin{vmatrix} x-1 & y-1 & z-1 \\ 3-1 & 2-1 & 3-1 \\ 1-1 & -1-1 & 2-1 \end{vmatrix} = 0$$

ie $$\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & 2 \\ 0 & -2 & 1 \end{vmatrix} = 0$$

ie $(x-1)(5) - (y-1)(2) + (z-1)(-4) = 0$

ie $5x - 2y - 4z - 5 + 2 + 4 = 0$

ie. $5x - 2y - 4z + 1 = 0$

OR

(ii) The normal to the plane $2x-3y+4z = 44$ is $(2,-3,4)$

$\therefore$ The direction of line $\perp^{lar}$ to the plane is $(2,-3,4)$ and that line passes through $A(2,-1,2)$

$\therefore$ The equation of this line is

$\bar{r} = \bar{a} + k\bar{l}$, $k \in R$

where $\bar{a} = (2,-1,2)$

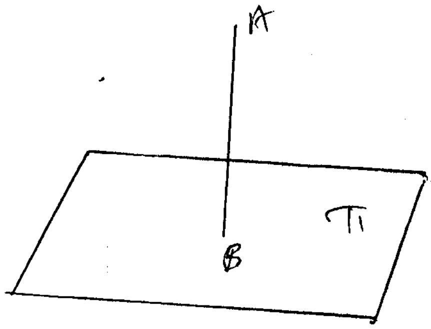
$\bar{l} = (2,-3,4)$

$\therefore \bar{r} = (2,-1,2) + k(2,-3,4)$, $k \in R$

$\therefore (x,y,z) = (2+2k, -1-3k, 2+4k)$

This is reqd eq^n of line (1)

30



The intersection of this line with the given plane is foot of ⊥lar from A on the plane

Let $P(x,y,z)$ be any general point on line (1), then for some $k \in R$, $P(x,y,z) = P(2+2k, -1-3k, 2+4k)$ must satisfy the eqⁿ of plane

$2x - 3y + 4z = 44$

$\therefore 2(2+2k) - 3(-1-3k) + 4(2+4k) = 44$

$\therefore 4 + 4k + 3 + 9k + 8 + 16k = 44$

$\therefore 29k = 29$

$\therefore k = 1$

$\therefore B = B(2+2k, -1-3k, 2+4k)$

$= B(4, -4, 6)$ (for $k = 1$)

$\therefore$ The position vector of foot of the normal is $(4, -4, 6)$

The length of the perpendicular

$AB = \sqrt{(2-4)^2 + (-1+4)^2 + (2-6)^2}$

$= \sqrt{29}$

Question paper set 2

Mathematics I

std. XII 050 (E)

Time : 3.00 Hours]

Instructions:

1. There are Five questions in this question paper. All are compulsory.

2. Figures to the right indicate the marks of the questions.

Q.1. (A) (1) Derive the co-ordinates of the point dividing $\overline{AB}$, from A in the ratio $\lambda:1$ if A is (x_1, y_1) and B is (x_2, y_2), where $\lambda \in R - \{-1\}$ (3)

(2) A, B, P are collinear and AP = 3AB. (1) Find the ratio in which P divides $\overline{AB}$ from A

(B) Attempt any two (4)

(1) For A (6,3), B (-3,5), C (4,-2) and P(x,y) show that

PBC

$$\frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{|x+y-2|}{7}$$

(2) If G is the centroid in ΔABC, prove that -

$$AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$$

(3) A(3,4), B(0,-5) and C(3,-1) are the vertices of ΔABC. Determine the length of the altitude from A on $\overleftrightarrow{BC}$.

(C) Attempt any two (4)

(1) Find the equations of the lines passing through (-2,3) and $\sqrt{3}x - 3y + 16 = 0$ forming an equilateral triangle with the line

(2) Find the equation of a line pass through $(2,6)$ if the length of the perpendicular segment to it from the origin is 2.

(3) An adjacent pair of vertices of a square is $(-1,3)$ and $(2,-1)$ Find the remaining vertices.

(D) Obtain the angle between two intersecting lines of slope m_1 and m_2

Q:2. (A) (1) Show that $l(a_1x+b_1y+c_1)+m(a_2x+b_2y+c_2)=0$ $(l^2+m^2\neq 0)$ represents a line through the intersecting point of the lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$

(2) If the length of the perpendicular segment from the origin is 10 and ~~$\alpha=\frac{5\pi}{6}$~~ $\alpha=\frac{5\pi}{6}$ find the equation of the line.

(B) (1) Find the equation of tangent and normal to a circle $x^2+y^2=a^2$ at the point (x_1,y_1).

(2) If the lines $a_1x+b_1y=1$, $a_2x+b_2y=1$ and $a_3x+b_3y=1$ are con-current, prove that the points (a_1,b_1), (a_2,b_2) and (a_3,b_3) are collinear.

(3) Obtain the perpendicular distance b/n the lines $2x+2y+5=0$ and $x+y-15=0$

3

Q:2 (C) (1) If the point $A(a,0)$, $A'(-a',0)$, $B(0,b)$, and $B'(0,-b')$ are on a circle then prove that $aa' = bb'$. Also find the equation of a circle (3)

OR

Find the equation of the circumscribed circle of the triangle formed by the three lines $x+y-6=0$, $2x+y-4=0$ and $x+2y-5=0$

(2) Obtain the equation of a circle (D), having centre $(-2,3)$ and touching Y axis. (3)

(D) Prove that $x^2-y^2-2xy\tan\theta+2ay\sec\theta-a^2=0$ represents a pair of lines and find their point of intersection.

OR

Obtain the equation a line passing through the point of intersection of the lines $3x+2y+4=0$ and $x-y-2=0$ and making a triangle of area 8 unit with the axes.

Q:3 (A) (1) Prove that the tangents at the end-points of a focal-chord intersects orthogonally at the directrix. (2)

(2) Find the co-ordinates of the focus, equation of the directrix and the length of the latus-rectum for the parabola $x^2 = -8y$ (2)

(4

OR

Find the equations of tangents to parabola at end of the latus rectum

Q: 3 (B)(1) Obtain the condition for the line $y = mx + c$, $c \neq 0$ to be a tangent of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ also obtain the co-ordinates of the point of contact.

(1) ecc.

(2)(i) Find the eccentric angle of the point $(-6, 4)$ of the ellipse $x^2 + 4y^2 = 100$ (2)

~~OR~~

(ii) Obtain the standard equation of the ellipse whose focii is $(\pm 2, 0)$ and eccentricity $\frac{1}{3}$ (1)

OR

(2) If the feet of the perpendicular drawn to the tangents at point P from focii S and S' are L and L' respectively, then show that $SL \cdot S'L' = b^2$ (2)

(C)(1) Define Rectangular hyperbola. Prove that its eccentricity is $\sqrt{2}$ ~~Obtain~~ Write its parametric eqn. (2)

(2) If (α, β) is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then show that its eccentricity is $\left(\frac{\alpha^2 + \beta^2 - a^2}{\alpha^2 - a^2}\right)^{\frac{1}{2}}$ (2)

5

Q:3 (D) (1) Show that the angle between two asymptotes of the hyperbola $x^2-2y^2=1$ is $\tan^{-1}(2\sqrt{2})$ (1)

(2) Prove that the circle $x^2+y^2-2x-2y+1=0$ touches on the X-axis. Also obtain the equation of a circle having radius 4 and which touches the above circle on X-axis. (2)

Q.4 (A) Which curve is represented by $x^2+xy+y^2+x-4y+1=0$. find the focii, directrix, eccentricity, length of the axes and the co-ordinates of the centre. (4)

OR

Which curves are represented by the following equations?

(1) $(x-1)(y+2)=2$ (2) $x^2+y^2+2xy+\sqrt{2}x-\sqrt{2}y=0$

(B) (1) Obtain the necessary and sufficient condition for two non-null vectors $\bar{x}, \bar{y}$ of R^3 to be collinear. (2)

(2) If θ is the measure of angle between unit vectors $\bar{a}, \bar{b}$ prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\bar{a}-\bar{b}|$ (2)

(C) (1) Obtain the formulae for the volume of a tetrahedron. (2)

(2) Show that ⊥r bisectors of side of any triangle are concurrent. (2)

D (1) A boat is speeding to the east with a speed of $10\sqrt{2}$ kms. A man on boat feels that the wind (2)

(6)

is blowing from the South East with speed of 5 kms. Find the true velocity and direction of wind.

(2) Prove that if $\bar{x}, \bar{y}$ are non-collinear vectors of R^3 then $\bar{x}, \bar{y}$ and $\bar{x} \times \bar{y}$ are non-coplanar. (1)

Q:5 (A) (1) Obtain the formulae for the shortest distance between two skew lines $\bar{r} = \bar{a} + k\bar{l}, k \in R$ and $\bar{r} = \bar{b} + k\bar{m}, k \in R$ (2)

(2) Obtain the formulae for the distance of a point and plane in R^3 (2)

OR

Derive the vector and cartesian equation of a plane passing through three distinct non-collinear points $A(\bar{a})$, $B(\bar{b})$ and $C(\bar{c})$

(B) (1) Write general equation of a sphere also state its centre and radius. (

(2) Show that angle between any two diagonals of a cube is $\cos^{-1}(\frac{1}{3})$ (

OR

Find the co-ordinates points on the line, $\bar{r} = (1,2,1) + k(-1,-2,1), k \in R$ distant $\sqrt{6}$ units from $(2,4,0)$

(7)

Q:5 (C) (1) In a parallelogram ABCD if $\overrightarrow{AC} = \bar{a}$ and $\overrightarrow{BD} = \bar{b}$ find the area, of the parallelogram. (2

(2) If a plane through (2,3,4) intersects the co-ordinate axes in A, B, C then find the equation of set of points which form the centre of sphere OABC. (2

(D) Obtain in cartesian form the equation of the plane through the lines $\frac{x-1}{3} = \frac{y+2}{5} = \frac{z-1}{1}$ and (3

$$\frac{2x-1}{6} = \frac{y+3}{5} = \frac{2z+1}{2}$$

OR

Find the equation of plane through (1,1,1) and the line of intersection of planes $x+2y+3z=4$ and $4x+3y+z+1=0$

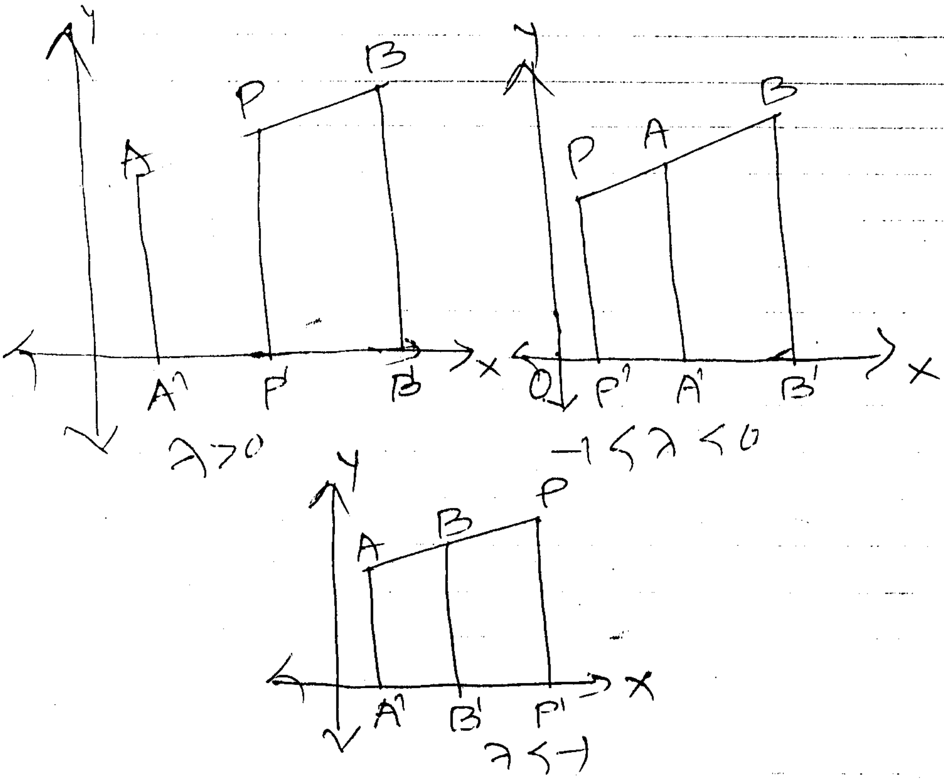
—x—x—

Solution of paper set: 2
Mathematics I (050) (E)

(1)

M-I Soln. ~~B. A. Patel~~ M.K.~~Patel~~

Q:1 A (1) Co ordinate of the point dividing $\overline{AB}$ from A in the ratio λ, if A is (x_1, y_1) and B is (x_2, y_2)



Suppose $P(x, y)$ divides $\overline{AB}$ from A in the ratio λ. Let $\overleftrightarrow{AB}$ not be horizontal or vertical. The feet of the $\perp$s from A, P, B on the X-axis are resply $A'(x_1, 0)$, $P'(x, 0)$ and $B'(x_2, 0)$. If none of A, B, P is on the X axis, then $\overleftrightarrow{AA'} \parallel \overleftrightarrow{PP'} \parallel \overleftrightarrow{BB'}$ and $\overleftrightarrow{AB}$ and the X-axis are their transversals

$$\therefore \frac{AP}{PB} = \frac{A'P'}{P'B'} \quad (1)$$

(2)

If any of A, B, P is on the X-axis, then also (1) is certainly true using similarity of triangles.

Case (1) $\lambda > 0$

As $\lambda > 0$ we have A-P-B

and so A'-P'-B'. The ratio of the division $\lambda = \frac{AP}{PB} = \frac{A'P'}{P'B'} = \frac{x - x_1}{x_2 - x}$ (∵ (1))

Case 2 : $-1 < \lambda < 0$

Now P-A-B and so P'-A'-B'

Hence ratio $\lambda = -\frac{AP}{PB} = -\frac{A'P'}{P'B'}$ (∵ (1))

$$= -\frac{x_1 - x}{x_2 - x} = \frac{x - x_1}{x_2 - x}$$

Case 3 : $\lambda < -1$

Now A-B-P, so A'-B'-P'. Here $\lambda < -1$

and the ratio $\lambda = -\frac{AP}{PB} = -\frac{A'P'}{P'B'}$ (∵ (1))

$$= -\frac{x - x_1}{x - x_2}$$

$$= \frac{x - x_1}{x_2 - x}$$

Hence in all cases $\lambda = \frac{x - x_1}{x_2 - x}$

$\therefore \lambda x_2 - \lambda x = x - x_1$

$\therefore (\lambda + 1)x = \lambda x_2 + x_1$

$\therefore x = \frac{\lambda x_2 + x_1}{\lambda + 1}$ $(\lambda \neq -1)$

(3)

If $\overline{AB}$ is vertical, then $x = x_1 = x_2$, so

$\frac{\lambda x_2 + x_1}{\lambda+1} = x$ remains valid

y - co-ordinate of P can be obtained similarly by using feet of ⊥s from A, B, P to Y-axis provided $\overline{AB}$ is not ~~vertical~~ horizontal.

$\therefore y = \frac{\lambda y_2 + y_1}{\lambda+1}$

If $\overline{AB}$ is horizontal $y = y_1 = y_2 = \frac{\lambda y_2 + y_1}{\lambda+1}$

Thus the co-ordinates of the point dividing $\overline{AB}$ from A in the ratio λ are $\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}\right)$, $\lambda \neq -1, 0$.

Q 1 A (2) Soln: For A, B, P there are two possibilities A-B-P and B-A-P.

If A-B-P then $AB + BP = AP$

$\Rightarrow AB + BP = 3AB$

$\Rightarrow BP = 2AB$

$\therefore \lambda = -\frac{AP}{PB} = -3:2$ Ans

If B-A-P then $\lambda = -\frac{AP}{BP}$

$= -\frac{3AB}{4AB} = -\frac{3}{4}$ Ans.

Q 1 (B) (1) Here, for $\triangle PBC$ $D_1 = \begin{vmatrix} x & y & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= x(5+2) - y(-3-4) + 1(6-20)$

$= 7(x+y-2)$

(7)

∴ the are of Δ PBC $= A_1 = \frac{1}{2}|D_1|$

$\therefore A_1 = \frac{7|x+y-2|}{2}$

Lly, for the Δ ABC, $D_2 = \begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= 6(5+2) - 3(-7) + 1(-14)$

$= 42 + 21 - 14$

$= 49$

∴ the area of Δ ABC, $\Delta_2 = \frac{1}{2}|D_2|$

$= \frac{49}{2}$

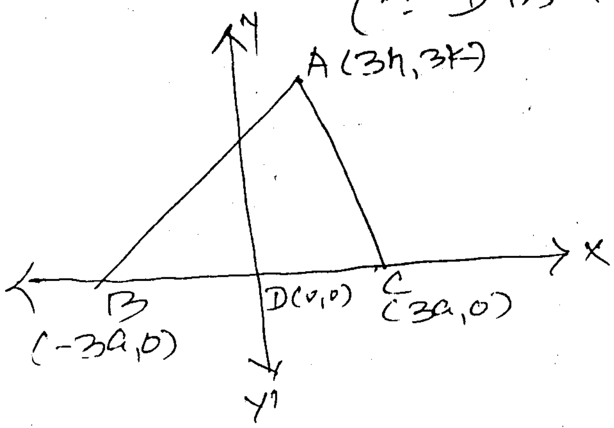
Now, $\frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{7|x+y-2|}{2} \times \frac{2}{49}$.

$\therefore \frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{|x+y-2|}{7}$

(2) Let the co-ordinates of A, D and C be $(3h, 3k)$, $(0,0)$ and $(3a, 0)$ respty.

∴ co-ordinate of B are $(-3a, 0)$

(∵ D is the midpoint of BC)



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Now, centroid $G = \left(\frac{3h-3a+3a}{3}, \frac{3k+0+0}{3}\right)$

$= (h, k)$

$AB^2 = (3h+3a)^2 + (3k)^2 = 9h^2 + 18ha + 9a^2 + 9k^2$ — (1)

$BC^2 = (3a+3a)^2 = 36a^2$ — (2)

$AC^2 = (3h-3a)^2 + (3k)^2 = 9h^2 - 18ha + 9a^2 + 9k^2$ — (3)

$GA^2 = (3h-h)^2 + (3k-k)^2 = 4h^2 + 4k^2$ (4)

$GB^2 = (h+3a)^2 + k^2 = h^2 + 6ha + 9a^2 + k^2$ (5)

$GC^2 = (h-3a)^2 + k^2 = h^2 - 6ha + 9a^2 + k^2$ (6)

Now,

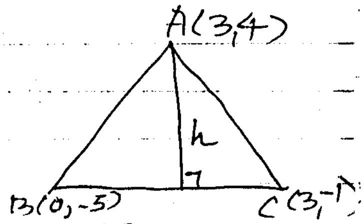
$L.H.S = AB^2 + BC^2 + CA^2$

$= 18h^2 + 18k^2 + 54a^2$

$R.H.S = 3(GA^2 + GB^2 + GC^2)$

$= 18h^2 + 18k^2 + 54a^2.$

$\therefore L.H.S = R.H.S$ $\therefore AB^2 + BC^2 + AC^2 = 3(GA^2 + GB^2 + GC^2)$



(3) Soln:

$$D = \begin{vmatrix} 3 & 4 & 1 \\ 0 & -5 & 1 \\ 3 & -1 & 1 \end{vmatrix}$$

$= 3(-5+1) - 4(0-3) + 15$

$= -12 + 12 + 15$

$= 15$

$\text{Area} = \frac{1}{2}|D| = \frac{1}{2} \times 15 = \frac{15}{2}$

$BC = \sqrt{(0-3)^2 + (-5+1)^2} = \sqrt{9+16} = 5$

Let the length of the segment from A to $\overleftrightarrow{BC}$ be h.

(6)

$\therefore$ Area $= \frac{1}{2} BC \cdot h$

$\therefore \frac{15}{2} = \frac{1}{2} \cdot 5h$

$\therefore h = \frac{15}{2} \times 2/5$

$\therefore h = 3$

Q.1(C) Soⁿ

(1) Slope of $\sqrt{3}x - 3y + 16 = 0$ is $\frac{1}{\sqrt{3}} = m_2$ (Say)

The lines which make the angle of 60^0 with $\sqrt{3}x - 3y + 16 = 0$ can form an equilateral triangle with that line. Let m_1 be the slope of the line which make an angle of 60^0 with $\sqrt{3}x - 3y + 16 = 0$

$\therefore \tan 60 = \left| \frac{m_1 - 1/\sqrt{3}}{1 + m_1 \cdot \frac{1}{\sqrt{3}}} \right| \qquad \therefore \sqrt{3} = \left| \frac{\sqrt{3}m_1 - 1}{\sqrt{3} + m_1} \right|$

$\therefore 3 + \sqrt{3}m_1 = \sqrt{3}m_1 - 1$ or $3 + \sqrt{3}m_1 = -\sqrt{3}m_1 + 1$

(1) but $3 + \sqrt{3}m_1 = \sqrt{3}m_1 - 1$ is not possible.

$\therefore m_1$ is not defined $\therefore$ line is parallel to Y-axis. It is passing through $(-2, 3)$

$\therefore$ its eqⁿ is $x = -2$ i.e. $x + 2 = 0$ --(1)

(2) From $3 + \sqrt{3}m_1 = -\sqrt{3}m_1 + 1$,

$\Rightarrow 2\sqrt{3}m_1 = -2$

$\therefore m_1 = -1/\sqrt{3}$

Line passes through $(-2, 3)$

$\therefore$ The eqⁿ of line : $y - 3 = -\frac{1}{\sqrt{3}}(x + 2)$

i.e. $x + \sqrt{3}y - 3\sqrt{3} + 2 = 0$

7

(2) Line passes through a fixed point $(x_1, y_1) = (2, 6)$

it's eqn can be written in the form

$a(x-x_1) + b(y-y_1) = 0, \quad a^2+b^2 \neq 0$

i.e. $a(x-2) + b(y-6) = 0$ i.e. $ax+by-(2a+6b)=0$

The length of $\perp$r from origin on this line is $\dfrac{|2a+6b|}{\sqrt{a^2+b^2}} = 2$

$\therefore 4(a+3b)^2 = 4(a^2+b^2) \quad \therefore 6ab + 8b^2 = 0$

$\therefore 2b(3a+4b) = 0 \quad \therefore b=0$ or $\dfrac{a}{b} = -\dfrac{4}{3}$

For $b=0$, the eqn of line is (when $b=0$, $a \neq 0$, since $a^2+b^2 \neq 0$)

$\therefore a(x-2) + b(y-6) = 0$

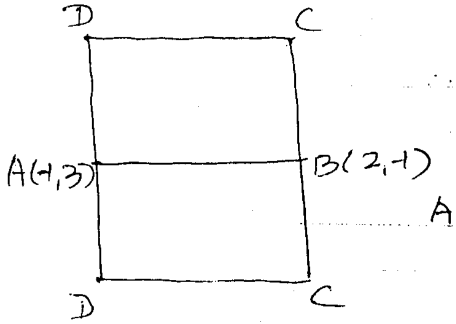
$\therefore a(x-2) = 0$ i.e. $x = 2 \quad (\because a \neq 0) \quad (1)$

For $\dfrac{a}{b} = -\dfrac{4}{3}$, the eqn of line is

$-\dfrac{4}{3}(x-2) + (y-6) = 0$ i.e. $4x - 3y + 10 = 0 \cdots (2)$

(3) Let $A(-1,3)$, $B(2,-1)$ be the adjacent vertices of a square ABCD

slope of $\overleftrightarrow{AB} = \dfrac{3-(-1)}{-1-2} = \dfrac{4}{-3}$



$\therefore$ Slope of $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC} = \dfrac{3}{4}$ (slope of $\perp$r lines)

$AB = BC = CD = DA$

$= \sqrt{(-1-2)^2 + (3+1)^2} = 5$

(8)

(D) is a point at distance 5 from A on line $\overleftrightarrow{AD}$ (There are two such points)

$\therefore$ Co-ordinates of D are
$(x_1 + r\cos\theta, y_1 + r\sin\theta)$ or
$(x_1 - r\cos\theta, y_1 - r\sin\theta)$
$\therefore (x_1, y_1) = (-1, 3)$ and $\tan\theta$ = slope of $\overleftrightarrow{AD}$
$= \frac{3}{4}$

$\therefore \cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}$

$\therefore$ Co-ordinate of D are $\left(-1+5\left(\frac{4}{5}\right), 3+5\left(\frac{3}{5}\right)\right)$
$= (3, 6)$

or $\left(-1-5\left(\frac{4}{5}\right), 3-5\left(\frac{3}{5}\right)\right) = (-5, 0)$

C is a point at distance 5 from B on $\overleftrightarrow{BC}$
slope of $\overleftrightarrow{BC}$ = $\tan\theta = \frac{3}{4}$ $\sin\theta = 3/5$ $\cos\theta = \frac{4}{5}$

$\therefore$ Co-ordinate of C are $(x_1 + r\cos\theta, y_1 + r\sin\theta)$
$= \left(2+5\left(\frac{4}{5}\right), -1+5\left(\frac{3}{5}\right)\right) = (6, 2)$

or $(x_1 - r\cos\theta, y_1 - r\sin\theta)$
$= \left(2-5\left(\frac{4}{5}\right), -1-5\left(\frac{3}{5}\right)\right) = (-2, -4)$
$\therefore$ With A and B, C (6, 2), D (3, 6)
or C (-2, -4), D (-5, 0)

(D) then. Accordingto text.

(9)

Q : 2 Soⁿ: (A) (1) theo. According to text
Pg. No. 52

(2) Here $p = 10$ and $\alpha = -\frac{5\pi}{6}$

$\therefore$ Eqⁿ of the line is

$\therefore x\cos\left(-\frac{5\pi}{6}\right) + y\sin\left(-\frac{5\pi}{6}\right) = 10$

$\therefore -\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 10$ or $\sqrt{3}x + y + 20 = 0$

(B) (1) theo. Ch: 4. According to text.
Pg. No. 76.

(2) the given lines are con-current.

$$\therefore \begin{vmatrix} a_1 & b_1 & -1 \\ a_2 & b_2 & -1 \\ a_3 & b_3 & -1 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$$

$\therefore (a_1, b_1), (a_2, b_2), (a_3, b_3)$ are collinear.

(3) $l_1 : 2x + 2y + 5 = 0$ —— (1)

$l_2 : x + y - 15 = 0$

for $l_2 \Leftrightarrow$ $2x + 2y - 30 = 0$ — (2)

Now $p = \frac{|c - c'|}{\sqrt{a^2 + b^2}}$

$= \frac{|-30 - 5|}{\sqrt{4 + 4}} = \frac{35}{2\sqrt{2}}$

(10)

Q. 2 (c) (1) Soln.

Let $A(a,0)$, $A'(-a',0)$, $B(0,b)$, $B'(0,-b')$

be the points on the circle $x^2+y^2+2gx+2fy+c=0$

Co-ordinates of A, A', B, B' Satisfy the eqn of Circle

$\therefore a^2+2ga+c=0 \quad \cdots (1) \qquad a'^2-2ga'+c=0 \quad \cdots (2)$

$b^2+2bf+c=0 \quad \cdots (3) \qquad b'^2-2b'f+c=0 \quad \cdots (4)$

From (1) and (2) $a^2-a'^2+2g(a+a')=0 \quad \therefore 2g=a'-a$

From (3) and (4), $2f=b'-b$

From (1), $c=-a^2-a(2g)=-a^2-a(a'-a)=-aa'$

From (3), $c=-b^2-b(2f)=-b^2-b(b'-b)=-bb'$

$\therefore c=-aa'=-bb' \quad \therefore aa'=bb' \quad \cdots\cdots \quad (a)$

Equation of the circle: $x^2+y^2+2gx+2fy+c=0$

i.e. $x^2+y^2+(a'-a)x+(b'-b)y-aa'=0 \quad \cdots (5)$

i.e. $x^2+y^2+(a'-a)x+(b'-b)y-bb'=0 \quad \cdots\cdots (6)$

adding (5) and (6)

$2x^2+2y^2+2(a'-a)x+2(b'-b)y-(aa'+bb')=0$

$2x^2+2y^2+2(a'-a)x+2(b'-b)y-2aa'=0 \quad (7)$

All the equations (5), (6), (7) represent the required circle.

OR

Soln: Consider the equation

$$\lambda_1(x+y-6)(2x+y-4)+\lambda_2(2x+y-4)(x+2y-5)$$
$$+\lambda_3(x+2y-5)(x+y-6)=0 \quad (1)$$

The point of intersection of any two lines from the given three lines satisfy this eqn. i.e all the three vertices of the triangle satisfy this eqn.

Of eqn (1) represents a circle, then

(1) The co-efficient of x^2 = The co eff of y^2

and (2) The co efficient of $xy = 0$

i.e. (i) $2\lambda_1 + 2\lambda_2 + \lambda_3 = \lambda_1 + 2\lambda_2 + 2\lambda_3$

$\therefore \lambda_1 - \lambda_3 = 0$ (2)

and (ii) $\Rightarrow 3\lambda_1 + 5\lambda_2 + 3\lambda_3 = 0$ (3)

From (2) and (3) $\lambda_1 : \lambda_2 : \lambda_3 = 5 : -6 : 5$

$\therefore \lambda_1 = 5K, \quad \lambda_2 = -6K, \quad \lambda_3 = 5K \quad K \in R$

Putting these values in (1)

$$5K(x+y-6)(2x+y-4) - 6K(2x+y-4)(x+2y-5) + 5K(x+2y-5)(x+y-6) = 0$$

$\Rightarrow K(x^2+y^2-17x-19y+50) = 0$, where $K \neq 0$

The eqn of the circle is $x^2+y^2-17x-19y+50=0$

(2) Soln: As the circle touches Y axis

so $r = |x$ co-ordinate of the centre of the circle$|$

$= |-2|$

$= 2$

(3) Soln: Comparing $x^2-y^2-2xy\tan\theta+2ay\sec\theta-a^2=0$ to

$Ax^2+2Hxy+By^2+2Gx+2Fy+C=0$

$A=1, \quad B=-1, \quad H=-\tan\theta, \quad G=0, \quad F=a\sec\theta,$

$C=-a^2$

$\therefore H^2-AB = \tan^2\theta+1 = \sec^2\theta > 0$

$G^2-AC = a^2 > 0$ and

(P.T.O)

(12)

$$\Delta = \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix} = \begin{vmatrix} 1 & -\tan\theta & 0 \\ -\tan\theta & -1 & a\sec\theta \\ 0 & a\sec\theta & -a^2 \end{vmatrix}$$

$$= 1(a^2 - a^2\sec^2\theta) + \tan\theta\,(a^2\tan\theta)$$

$$= a^2(-\tan^2\theta) + a^2\tan^2\theta = 0$$

$\therefore H^2 - AB > 0,\ G^2 - AC \geq 0$ and $\Delta = 0$

$\therefore$ Given eqn represents a pair of lines.

Co-ordinate of the point,

$$(\alpha, \beta) = \left(\frac{HF - BG}{AB - H^2}, \frac{GH - AF}{AB - H^2}\right)$$

$$= \left(\frac{-a\sec\theta\tan\theta}{-\sec^2\theta}, \frac{-a\sec\theta}{-\sec^2\theta}\right)$$

$$= (a\sin\theta,\ a\cos\theta)$$

OR.

Here X cut of the line $x - y - 2 = 0$ $= +2$

Y " " $x - y - 2 = 0$ $= -2$

$\therefore$ Area of the triangle formed by the line $x - y - 2 = \frac{1}{2}(2)(2) \neq 8$ Unit

$\therefore$ $x - y - 2 = 0$ is not the required line

Let the required line be

$$(3x + 2y + 4) + \lambda(x - y - 2) = 0 \quad \cdots (1)$$

13

$\therefore (3+\lambda)x + (2-\lambda)y + (4-2\lambda) = 0$

$\therefore X_{int} = -\dfrac{4-2\lambda}{3+\lambda} = \dfrac{2\lambda-4}{3+\lambda}, \quad \lambda \neq -3$

$Y_{int} = -\dfrac{4-2\lambda}{2-\lambda} = \dfrac{2\lambda-4}{2-\lambda}, \quad \lambda \neq 2$

Now $\Delta = \frac{1}{2}(X_{int})(Y_{int})$

$\therefore 8 = \dfrac{4(\lambda-2)(\lambda-2)}{2(3+\lambda)(2-\lambda)}$

$\therefore 4(3+\lambda) = -(\lambda-2)$

$\therefore 12 + 4\lambda = -\lambda + 2$

$\therefore 5\lambda = -10 \Rightarrow \lambda = -2$, put in eqn ①

$\therefore$ Required eqn of the line is

$(3x+2y+4) - 2(x-y-2) = 0$

$\therefore 3x+2y+4-2x+2y+4 = 0$

$\therefore x+4y+8 = 0$

Ans:3 (A) (1) So: ~~Stap~~ theory Page-94 Text.

(2) Here, the Y-axis is the axis of the parabola, $-4a = -8 \Rightarrow a = 2$

So the co-ordinate of the focus are $(0,-2)$, eqn of the directrix is the line $y-2=0$ and the length of the latus rectum is $4|a| = 8$.

OR

The end points of the latus rectum are $(a, 2a)$, and $(a, -2a)$

The eqn of the tangents passing through (x_1, y_1) is $yy_1 = 2a(x+x_1)$

$\therefore$ The eqn of the tangent at L is

$2ay = 2a(x+a) \Rightarrow y = x+a \Rightarrow x-y+a=0$

And eqn of the tangent at L' is $-2ay = 2a(x+a)$

$\Rightarrow -y = x+a \Rightarrow x+y+a=0$.

(14)

Q: 3(B) (1) theo Ch: 6. Page – 104

Ans:

(2) (i) Comparing $\frac{x^2}{100} + \frac{y^2}{25} = 1$ with

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a = 10$, $b = 5$

For point $(a\cos\theta, b\sin\theta)$ of the ellipse θ is known as the eccentric angle of that point

$\therefore$ Comparing $P(-6, 4)$ with $(a\cos\theta, b\sin\theta)$;

$a = 10$, $b = 5$ $\quad a\cos\theta = -6$ and $b\sin\theta = 4$

$\therefore 10\cos\theta = -6$, $\quad 5\sin\theta = 4$

$\therefore \cos\theta = -3/5$ and $\sin\theta = \frac{4}{5}$ $\therefore \cot\theta = -\frac{3}{4}$

$\therefore \theta = \cot^{-1}\left(-\frac{3}{4}\right) = \pi - \cot^{-1}\frac{3}{4} = \pi - \tan^{-1}\frac{4}{3}$

$(-\pi < \theta \leq \pi)$

(ii) Soln: Foci $(2, 0)$, $(-2, 0)$ and $e = 1/3$

$(ae, 0) = (2, 0)$, $(-ae, 0) = (-2, 0)$ and $e = \frac{1}{3}$

$\therefore ae = 2$ $\quad \therefore a(1/3) = 2$ $\quad \therefore a = 6$ $\quad \therefore a^2 = 36$

Now, $b^2 = a^2(1 - e^2) = 36\left(1 - \frac{1}{9}\right) = 32$

$\therefore$ Standard eqn of the ellipse: $\frac{x^2}{36} + \frac{y^2}{32} = 1$

OR

Ans:

The eqn of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

at the point $P(\theta)$ is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

Here the focii are $S(ae, 0)$ and $S'(-ae, 0)$

$\therefore SL = \frac{|e\cos\theta - 1|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$ & $S'L' = \frac{|-e\cos\theta - 1|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$

15

$\therefore SL \cdot S'L' = \dfrac{1-e^2\cos^2\theta}{\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta}{b^2}}$ $(\because e<1, \cos^2\theta<1)$

$= \dfrac{a^2b^2\left(1-\dfrac{a^2-b^2}{a^2}\cdot\cos^2\theta\right)}{b^2\cos^2\theta+a^2\sin^2\theta}$ $(\because b^2=a^2(1-e^2))$

$= \dfrac{b^2(a^2-a^2\cos^2\theta+b^2\cos^2\theta)}{b^2\cos^2\theta+a^2\sin^2\theta}$

$= b^2$

(C) (1) theo: Ch:7 Page: 129

Ans(2) Sol^n: $P(\alpha,\beta)$ lies on $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

$\therefore \dfrac{\alpha^2}{a^2}-\dfrac{\beta^2}{b^2}=1$

$\therefore \dfrac{\alpha^2}{a^2}-1=\dfrac{\beta^2}{b^2}$

$\therefore \dfrac{\alpha^2-a^2}{a^2}=\dfrac{\beta^2}{b^2}$

$\therefore b^2=\dfrac{a^2\beta^2}{\alpha^2-a^2}$

Here, $b^2=a^2(e^2-1)$

$\therefore \dfrac{a^2\beta^2}{\alpha^2-a^2}=a^2(e^2-1)$

$\therefore \dfrac{\beta^2}{\alpha^2-a^2}=e^2-1$

$\therefore e^2=1+\dfrac{\beta^2}{\alpha^2-a^2}$

$= \dfrac{\alpha^2-a^2+\beta^2}{\alpha^2-a^2} \Rightarrow e=\left(\dfrac{\alpha^2+\beta^2-a^2}{\alpha^2-a^2}\right)^{\frac{1}{2}}$

(16)

Ans 3(D) (1) Here the eq^n of the asymptotes is $x^2-2y^2=0$

If the angle b/w^n them is θ, then

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|}$$

Here $a=1$, $h=0$, $b=-2$

$$\therefore \tan\theta = \frac{2\sqrt{0-(-2)}}{|1-2|} = 2\sqrt{2}$$

$$\therefore \theta = \tan^{-1} 2\sqrt{2}$$

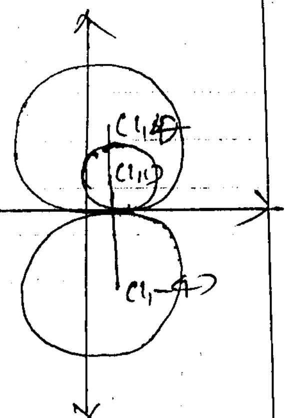
(2) So^n: The circle $x^2+y^2-2x-2y+1=0$ intersects x- axis

$\therefore y=0 \Rightarrow x^2-2x+1=0$

$\Rightarrow (x-1)^2=0$

$\Rightarrow x=1$

$\Rightarrow$ The circle intersects Xaxis at $(1,0)$



$\therefore$ The circle of radius 4 touches Xaxis at $(1,0)$

$\therefore$ its centre is $(1,4)$ or $(1,-4)$

$\therefore$ The eq^{n}s of the circles are

$(x-1)^2 + (y \pm 4)^2 = 16$

$\therefore x^2-2x+1+y^2 \pm 8y+16 = 16$

$\therefore x^2+y^2-2x \pm 8y+1=0$

Ans (4) (A) Here $a=1=b \Rightarrow \theta = \frac{\pi}{4}$

$\therefore$ Rotating the axes by $\frac{\pi}{4}$ we get

$$x = \frac{x'-y'}{\sqrt{2}} \quad \text{and} \quad y = \frac{x'+y'}{\sqrt{2}} \quad \cdots\cdots (1)$$

(17)

$$\therefore \left(\frac{x'-y'}{\sqrt{2}}\right)^2 + \left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) + \left(\frac{x'+y'}{\sqrt{2}}\right)^2 + \frac{x'-y'}{\sqrt{2}} - 4\left(\frac{x'+y'}{\sqrt{2}}\right) + 1 = 0$$

$$\therefore \frac{x'^2 - 2x'y' + y'^2 + x'^2 - y'^2 + x'^2 + 2x'y' + y'^2}{2} + \frac{x'-y'}{\sqrt{2}} - \frac{4x'+4y'}{\sqrt{2}} + 1 = 0$$

$$\therefore 3x'^2 + y'^2 + \sqrt{2}x' - \sqrt{2}y' - 4\sqrt{2}x' - 4\sqrt{2}y' + 2 = 0$$

$$\therefore 3x'^2 - 3\sqrt{2}x' + y'^2 - 5\sqrt{2}y' = -2$$

$$\therefore 3x'^2 - 3\sqrt{2}x' + \frac{3}{2} + y'^2 - 5\sqrt{2}y' + \frac{25}{2} = -2 + \frac{3}{2} + \frac{25}{2}$$

$$\therefore 3\left(x'^2 - \sqrt{2}x' + \frac{1}{2}\right) + \left(y'^2 - 5\sqrt{2}y' + \frac{25}{2}\right) = 12$$

$$\therefore 3\left(x' - \frac{1}{\sqrt{2}}\right)^2 + \left(y' - \frac{5}{\sqrt{2}}\right)^2 = 12$$

$\therefore$ Displacing origin to $\left(\frac{1}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ we get

$$x' = x + \frac{1}{\sqrt{2}} \quad \text{and} \quad y' = y + \frac{5}{\sqrt{2}} \quad \ldots (2)$$

$$\therefore 3x^2 + y^2 = 12$$

$\therefore \frac{x^2}{4} + \frac{y^2}{12} = 1$ is an eqn of ellipse.

Here $a^2 = 4$ and $b^2 = 12 \Rightarrow a = 2,\ b = 2\sqrt{3}$

$\therefore$ Length of the major axis $= 2b = 4\sqrt{3}$

and length of minor axis $= 2a = 4$

$a^2 = b^2(1-e^2) \Rightarrow 4 = 12(1-e^2) \Rightarrow \frac{1}{3} = 1 - e^2$

$\Rightarrow e^2 = 1 - \frac{1}{3} = 2/3 \Rightarrow e = \sqrt{2/3}$

Foci $= (0, \pm be)$

$= \left(0, \pm 2\sqrt{3}\,\frac{\sqrt{2}}{\sqrt{3}}\right)$

$\therefore (x, y) = (0, \pm 2\sqrt{2})$

Directrix: $y = \pm \frac{b}{e}$

$\therefore y = \pm \frac{2\sqrt{3}\cdot\sqrt{3}}{\sqrt{2}}$

$\therefore y = \pm 3\sqrt{2}$

$\therefore y' - \frac{5}{\sqrt{2}} = \pm 3\sqrt{2}$ (from (2))

(10)

foci ---- Continue

$\therefore (x', y') = \left(X + \frac{1}{\sqrt{2}}, Y + \frac{5}{\sqrt{2}}\right)$ from (2)

$= \left(0 + \frac{1}{\sqrt{2}}, \pm 2\sqrt{2} + \frac{5}{\sqrt{2}}\right)$

$= \left(\frac{1}{\sqrt{2}}, \frac{\pm 4 + 5}{\sqrt{2}}\right)$

$\therefore (x, y) = \left(\frac{x' - y'}{\sqrt{2}}, \frac{x' + y'}{\sqrt{2}}\right)$ (from (1))

$= \left(\frac{\frac{1}{\sqrt{2}} - \frac{\pm 4 + 5}{\sqrt{2}}}{\sqrt{2}}, \frac{\frac{1}{\sqrt{2}} + \frac{\pm 4 + 5}{\sqrt{2}}}{\sqrt{2}}\right)$

$= \left(\frac{1 \mp 4 - 5}{2}, \frac{1 \pm 4 + 5}{2}\right)$

$= (-4, 5)$ and $(0, 1)$

Directrix --- Conti---

$\therefore \frac{-x + y}{\sqrt{2}} - \frac{5}{\sqrt{2}} = \pm 3\sqrt{2}$

$\therefore -x + y - 5 = \pm 6$

$\therefore x - y + 11 = 0$ and $x - y - 1 = 0$

Centre :

$(X, Y) = (0, 0)$

$\therefore (x', y') = \left(X + \frac{1}{\sqrt{2}}, Y + \frac{5}{\sqrt{2}}\right)$

$= \left(\frac{1}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$

$\therefore (x, y) = \left(\frac{x' - y'}{\sqrt{2}}, \frac{x' + y'}{\sqrt{2}}\right)$

$= \left(\frac{\frac{1}{\sqrt{2}} - \frac{5}{\sqrt{2}}}{\sqrt{2}}, \frac{\frac{1}{\sqrt{2}} + \frac{5}{\sqrt{2}}}{\sqrt{2}}\right)$

$= (-2, 3)$

OR

Soln (1) Shifting the origin to $(1, -2)$,
we get $x = x' + 1$ and $y = y' - 2$

$\therefore x'y' = 2$

Here $a = b = 0$ $\theta = \pi/4$

Rotating the axes by $\frac{\pi}{4}$, we get

$x' = \frac{X - Y}{\sqrt{2}}$ and $y' = \frac{X + Y}{\sqrt{2}}$

(19)

$\therefore \left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) = 2$

$\therefore x^2 - y^2 = 4$ is an eq of a rectangular hyperbola.

Soⁿ (2) Here $a = b = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\therefore$ Rotating the axes by $\frac{\pi}{4}$, we get

$x = \frac{x'-y'}{\sqrt{2}}, \quad y = \frac{x'+y'}{\sqrt{2}}$

$\therefore \left(\frac{x'-y'}{\sqrt{2}}\right)^2 + 2\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) + \left(\frac{x'+y'}{\sqrt{2}}\right)^2 + \sqrt{2}\left(\frac{x'-y'}{\sqrt{2}}\right) - \sqrt{2}\left(\frac{x'+y'}{\sqrt{2}}\right) = 0$

$\therefore \frac{x'^2 - 2x'y' + y'^2 + 2x'^2 - 2y'^2 + x'^2 + 2x'y' + y'^2}{2} + x' - y' - x' - y' = 0$

$\therefore 2x'^2 - 2y' = 0$

$\therefore x'^2 = y'$ is an eqⁿ of a parabola.

Q: 4

Ans (B) (1) Ch - 9 Page : 194 theo.

(2) Soⁿ: The measure of angle betⁿ $\bar{a}$ and $\bar{b}$ is θ

$\therefore (\hat{\bar{a}, \bar{b}}) = \theta$, then $\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$,

but $|\bar{a}| = |\bar{b}| = 1$ ($\because \bar{a}, \bar{b}$ are unit vectors)

$\therefore \cos\theta = \bar{a}\cdot\bar{b}$ .... (1)

$|\bar{a}-\bar{b}|^2 = (\bar{a}-\bar{b})\cdot(\bar{a}-\bar{b}) = \bar{a}\cdot\bar{a} + \bar{b}\cdot\bar{b} - 2\bar{a}\cdot\bar{b}$

$= |\bar{a}|^2 + |\bar{b}|^2 - 2\bar{a}\cdot\bar{b}$

$= 1 + 1 - 2\cos\theta$ (using (1))

$\therefore 2 - 2\cos\theta = |\bar{a}-\bar{b}|^2 \quad \therefore 2(1-\cos\theta) = |\bar{a}-\bar{b}|^2$

$\therefore 4\sin^2\theta/2 = |\bar{a}-\bar{b}|^2$

$\therefore \sin\theta/2 = \frac{1}{2}|\bar{a}-\bar{b}|$ $\left(0 < \theta < \pi \quad \therefore 0 < \theta/2 < \pi/2 \quad \therefore \sin\theta/2 > 0\right)$

(20)

Ans 4 (C)

(1) theorm Ch: 10. Page: 176

(2) In $\triangle ABC$, D, E, F are the mid points of $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ respectly.
Let the $\perp_r$ bisectors of $\overline{BC}$ and $\overline{AC}$ intersects at point P(O).
Let $A = A(\bar{x})$, $B = B(\bar{y})$, $C = C(\bar{z})$

then $D\left(\frac{\bar{y}+\bar{z}}{2}\right)$, $E\left(\frac{\bar{z}+\bar{x}}{2}\right)$, $F\left(\frac{\bar{x}+\bar{y}}{2}\right)$

$\overrightarrow{BC} = \bar{z}-\bar{y}$, $\overrightarrow{CA} = \bar{x}-\bar{z}$, $\overrightarrow{AB} = \bar{y}-\bar{x}$

$\overrightarrow{PD} = \frac{\bar{y}+\bar{z}}{2}$, $\overrightarrow{PE} = \frac{\bar{z}+\bar{x}}{2}$, $\overrightarrow{PF} = \frac{\bar{x}+\bar{y}}{2}$

$\overrightarrow{PD} \perp \overrightarrow{BC} \Rightarrow \overrightarrow{PD}\cdot\overrightarrow{BC} = 0$

$\Rightarrow \left(\frac{\bar{y}+\bar{z}}{2}\right)\cdot(\bar{z}-\bar{y}) = 0$

$\Rightarrow \bar{y}\cdot\bar{z} + \bar{z}\cdot\bar{z} - \bar{y}\cdot\bar{y} - \bar{z}\cdot\bar{y} = 0$

$\Rightarrow |\bar{y}|^2 = |\bar{z}|^2$ ---(1)

$\overrightarrow{PE} \perp \overrightarrow{AC} \Rightarrow \overrightarrow{PE}\cdot\overrightarrow{AC} = 0 \Rightarrow \left(\frac{\bar{z}+\bar{x}}{2}\right)(\bar{z}-\bar{x}) = 0$

$\Rightarrow \bar{z}\cdot\bar{z} + \bar{x}\cdot\bar{z} - \bar{z}\cdot\bar{x} - \bar{x}\cdot\bar{x} = 0 \Rightarrow |\bar{z}|^2 = |\bar{x}|^2$ --(2)

from (1) and (2), $|\bar{x}|^2 = |\bar{y}|^2$

$\Rightarrow |\bar{x}|^2 - |\bar{y}|^2 = 0$

$\Rightarrow \left(\frac{\bar{x}+\bar{y}}{2}\right)\cdot(\bar{x}-\bar{y}) = 0$

$\Rightarrow \overrightarrow{PF}\cdot\overrightarrow{BA} = 0$ $\therefore \overrightarrow{PF} \perp \overrightarrow{BA}$

and F is the mid point of $\overline{AB}$
Thus, $\overrightarrow{PF}$ is the $\perp_r$ bisector of $\overline{AB}$
$\therefore$ All the three bisectors of the sides of a Triangle are concurrent.

Ans. 4 (D)

(1) Here velocity of the boat is $\bar{u}$
$\therefore \bar{u} = 10\sqrt{2}\,i + 0\cdot j$
Suppose the velocity of the wind is $\bar{v} = a\bar{i} + b\bar{j}$

(21)

Now the velocity of wind relative to boat is 5 kms from South-East

hence $\bar{v}-\bar{u} = 5\cos\frac{3\pi}{4}\hat{i} + 5\sin\frac{3\pi}{4}\hat{j}$

$= -\frac{5}{\sqrt{2}}\hat{i} + \frac{5}{\sqrt{2}}\hat{j}$

but $\bar{v}-\bar{u} = (a-10\sqrt{2})\hat{i} + b\hat{j}$

$\therefore a-10\sqrt{2} = -\frac{5}{\sqrt{2}}, \quad b = \frac{5}{\sqrt{2}}$ So $a = \frac{15}{\sqrt{2}}$

and $b = \frac{5}{\sqrt{2}}$

Thus, $|\bar{v}| = \sqrt{\frac{225}{2} + \frac{25}{2}} = \sqrt{125} = 5\sqrt{5}$

and $\hat{v} = \frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j}$

Hence the speed of wind is $5\sqrt{5}$ kms and the direction of wind is at an angle $\cos^{-1}\frac{3}{\sqrt{10}}$ with east, towards north.

(2) let $\bar{x} \neq 0, \bar{y} \neq 0$, Since $\bar{x}, \bar{y}$ are non-collinear $\bar{x}\times\bar{y} \neq 0$

Now $\bar{x}\cdot[\bar{y}\times(\bar{x}\times\bar{y})] = (\bar{x}\times\bar{y})\cdot(\bar{x}\times\bar{y})$

$= |\bar{x}\times\bar{y}|^2 \neq 0$

$\therefore \bar{x}, \bar{y}$ and $\bar{x}\times\bar{y}$ are not coplaner.

Ans:5 (A) (1) theo ch: 11 Page: 192

(2) theo ch - 12

or

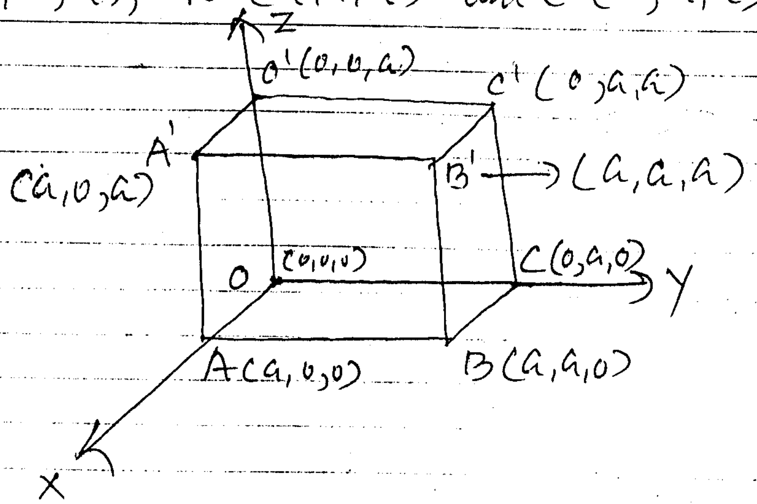
theo ch: 12

(B) (1) theo ch: 13

(22)

(2) Soln: Let the length of each side of the cube be a.

$\therefore$ Its vertices are O (0,0,0), A(a,0,0), B (a,a,0), C (0,a,0) & O' (0,0,a), A' (a,0,a), B' (a,a,a) and C' (0,a,a)



Now the diagonals are $\overline{OB'}$, $\overline{O'B}$, $\overline{AC'}$ and $\overline{A'C}$

$\therefore \overrightarrow{OB'} = (a,a,a) - (0,0,0) = (a,a,a)$ &

$\overrightarrow{O'B} = (a,a,0) - (0,0,a) = (a,a,-a)$

Uy $\overrightarrow{AC'} = (-a,a,a)$ and $\overrightarrow{A'C} = (-a,a,-a)$

If the angle btn the diagonals $\overrightarrow{OB'}$ and $\overrightarrow{O'B}$ is α, then

$$\cos\alpha = \left|\frac{\overrightarrow{OB'} \cdot \overrightarrow{O'B}}{|\overrightarrow{OB'}||\overrightarrow{O'B}|}\right|$$

$$= \left|\frac{a^2+a^2-a^2}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+a^2}}\right|$$

$$= \left|\frac{a^2}{\sqrt{3a^2}\sqrt{3a^2}}\right| = \frac{a^2}{3a^2} = \frac{1}{3}$$

$$\therefore \alpha = \cos^{-1} 1/3$$

(23)

~~OC~~

OR

Soln: Here $\bar{r} = (1,2,1) + k(-1,-2,1)$

$\therefore \bar{r} = (1,2,1) + (-k,-2k,k)$

$\therefore \bar{r} = (1-k,\ 2-2k,\ 1+k)$

This point is at distance of $\sqrt{6}$ from $(2,4,0)$ for some k.

$\therefore (1-k-2)^2 + (2-2k-4)^2 + (1+k-0)^2 = 6$

$\therefore (-1-k)^2 + (-2-2k)^2 + (1+k)^2 = 6$

$\therefore 1+2k+k^2+4+8k+4k^2+1+2k+k^2-6=0$

$\therefore 6k^2+12k=0$

$\therefore 6k(k+2)=0$

$\therefore k=0$ or $k=-2$

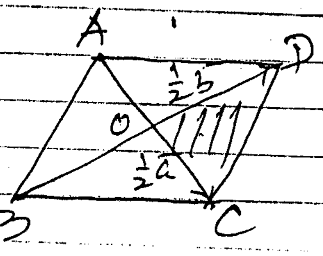
Now, $\bar{r} = (1-k,\ 2-2k,\ 1+k)$

$\therefore \bar{r} = (1-0,\ 2-2\cdot 0,\ 1+0)$ or for $k=-2$

$= (1,2,1)$ $\quad \bar{r} = (3,6,-1)$

A: 5

(C) (1) Soln: Suppose that diagonal intersects in O



$\therefore \overrightarrow{OC} = \frac{1}{2}\overrightarrow{AC} = \frac{1}{2}\bar{a}$

and $\overrightarrow{OD} = \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}\bar{b}$

Area of $\Delta ODC = \frac{1}{2}\left|\overrightarrow{OC} \times \overrightarrow{OD}\right|$

$= \frac{1}{2}\left|\frac{1}{2}\bar{a} \times \frac{1}{2}\bar{b}\right| = \frac{1}{8}|\bar{a}\times\bar{b}|$

$\therefore$ Area of $\square^m ABCD = 4\ ODC = 4 \times \frac{1}{8}|\bar{a}\times\bar{b}|$

$= \frac{1}{2}|\bar{a}\times\bar{b}|$

P.T.O.

(24)

(2) Solution: Let $\alpha x+\beta y+\gamma z+1=0$ be the eq of the plane

It passes through (2, 3, 4)

$\Rightarrow 2\alpha+3\beta+4\gamma+1=0$ --- (1)

The plane intersects x-axis, y-axis and z axes in A, B and C

$\therefore A\left(-\frac{1}{\alpha}, 0, 0\right)$, $B=\left(0, -\frac{1}{\beta}, 0\right)$

$C=\left(0, 0, -\frac{1}{\gamma}\right)$

Let the eq of the Sphere be

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$O \in$ Sphere $\Rightarrow d=0$

$A \in$ Sphere $\Rightarrow \frac{1}{\alpha^2}-\frac{2u}{\alpha}=0 \Rightarrow u=\frac{1}{2\alpha}$

lly $v=\frac{1}{2\beta}$ and $w=\frac{1}{2\gamma}$

$\therefore$ Centre $=(-u, -v, -w)=\left(-\frac{1}{2\alpha}, -\frac{1}{2\beta}, -\frac{1}{2\gamma}\right)$

$=(x, y, z)$

$\therefore x=-\frac{1}{2\alpha}$ $y=-\frac{1}{2\beta}$ $z=-\frac{1}{2\gamma}$

$\therefore$ From (1), $-\frac{2}{2x}-\frac{3}{2y}-\frac{4}{3z}+1=0$

$\Rightarrow 2/x+\frac{3}{y}+\frac{4}{z}=2$

the required point set

Ans. Q. 5 (D)

Here the lines are parallel since their eqns can be rewritten in the forms

$$\frac{x-1}{3} = \frac{y+2}{5} = \frac{z-1}{1} \text{ and}$$

$$\frac{x-\frac{1}{2}}{3} = \frac{y+3}{5} = \frac{z+\frac{1}{2}}{1},$$

Both are parallel & their direction are same

$\therefore \bar{a} = (1, -2, 1), \quad \bar{b} = (\frac{1}{2}, -3, -\frac{1}{2})$

$\bar{l} = (3, 5, 1).$

The required eq of the plane is

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ -\frac{1}{2} & -1 & -\frac{3}{2} \\ 3 & 5 & 1 \end{vmatrix} = 0$$

i.e.

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ -1 & -2 & -3 \\ 3 & 5 & 1 \end{vmatrix} = 0$$

i.e. $(x-1)(13) - (y+2)(8) + (z-1)(1) = 0$

i.e. $13x - 8y + z - 30 = 0.$

P.T.O.

OR

Soln: The eqn of the plane passing through the intersection of the planes $x+2y+3z-4=0$ and $4x+3y+z+1=0$ is

$\lambda(x+2y+3z-4)+4x+3y+z+1=0$

it passes through $(1,1,1)$

$\therefore \lambda(1+2+3-4)+4+3+1+1=0$

$\therefore 2\lambda+9=0$

$\Rightarrow \lambda=-9/2$

$\therefore$ from (1) $-\frac{9}{2}(x+2y+3z-4)+4x+3y+z+1=0$

$\therefore -9x-18y-27z+36+8x+6y+2z+2=0$

$\therefore -x-12y-25z+38=0$

$\therefore x+12y+25z=38$

Q. Paper set No. 3

1

MANINAGAR

TIME: 3 Hrs MATHS - I (050) MAX MARKS - 75

XII - SCI

1A 1) Obtain incentre of a triangle. (3)

2) If A is (2, 3) and B is (0, 7). In what ratio does the X-axis divides $\overline{AB}$ from A. (1)

B [B] Answer any two: (4)

1) If A, B, C, P are distinct and non collinear point of the plane then prove that Area of ΔPAB + Area of ΔPBC + Area of $\Delta PCA \geq$ Area of ΔABC

2) Find point C on $\overleftrightarrow{AB}$ such that $3AC = AB$ where A(0, 1) B(2, 9)

3) If (3, 2) (4, 5) and (2, 3) are three of the four vertices of a parallelogram. What is co-ordinate of fourth vertices

C [C] Attempt any two:- (4)

1) If A (3, 2) B (5, 6) $\in R^2$ and $P(x, y) \in \overline{AB}$ then P. that $17 \leq 3x + 4y \leq 42$

2) Find the equation of line which passes through (3, 4) and which makes an angle of $\pi/4$ with the line $3x + 4y = 2$.

3 Prove that the points (3,4) and (-2,1) $\boxed{2}$
are on opposite side of the line $3x-y+6=0$

D] Graph of linear equation represents a straight line (3)

2 A] Prove that if $ax^2+2hxy+by^2+2gx+2fy+c=0$ (3)
represents a pair of lines then this pair
is parallel to the pair of lines represented
by the pair of lines represted by ax^2+2hxy
$+by^2=0$ where $a^2+b^2+h^2\neq 0$

2. Find the angle between the lines (1)
represented by $x^2-2xy\sec\alpha+y^2=0$ $0<\alpha<\pi/2$

B] (1) Find the condition for the line (2)
$y=mx+c$ be tangent to the circle
$x^2+y^2=r^2$ and the point of contact.

2) The sides of a triangle are along the (2)
lines $2x-3y+5=0$ and $3x+2y+7=0$. Find
ortho centre and $x=2$

3) If $px^2+3y^2+(q-3)xy+2px+3qy-3=0$ (1)
represent a circle then find centre and
radius

C(1)] Find the equation of the circle passing through the points (5, -8) (-2, 9) and (2, 1) (3)

[OR]

Find the equation of tangents to the circle $x^2+y^2=17$ from the point (5, 3)

2) For $\lambda \in R-\{a\}$ show that line $\frac{x}{a-\lambda}+\frac{y}{b}=1$ (1) passes through a fixed point

D). Find the area of the parallelogram whose sides are along the line $y=mx+a$, $y=mx+b$, $y=nx+c$ and $y=nx+d$ (3)

[OR]

Prove that if $a+b+c=0$ and $b^2\neq ac$, $c^2\neq ab$ and $a^2\neq bc$, then the lines $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ are concurrent and find the point of concurrence.

3/, A] (1) Obtain standard equation of parabola (2)

(2) If the focus of the parabola $y^2=4ax$ divides a focal chord in the ratio 1 : 2 then find the equation of the line containing the focal chord. (2)

[OR]

Show that the line $3y=6x+2$ touches the parabola $3y^2=16x$. Find the point of contact.

B] (1) Obtain the equation of the tangent at the point (x_1, y_1) of the ellipse and hence obtain the equation of the tangent at 'θ' point of the ellipse (2) [4]

2) If the difference of the eccentric angles of P and Q is $\pi/2$ and O is the origin then prove that the area of ΔPOQ is $\frac{1}{2}ab$ for ellipse (2)

[OR]

The tangent at the point P intersect a directrix at F. Prove that PF forms a right angle at the corresponding focus

C] (1). Define rectangular hyperbola. Obtain its standard equation and eccentricity (2)

(2) Show that the angle between two asymptotes of the hyperbola $x^2 - 2y = 1$ is $\tan^{-1} 2\sqrt{2}$ (2).

D] (1). If S (4, 0) and $e = 3/2$ find the equation of hyperbola. (1)

2) Find the set of all points P outside a circle, such that the tangents drawn to a circle from P are ⊥lar to each other (2)

$\boxed{5}$

4) A] which curve is represented by the equation $3x^2+8xy-3y^2-20x+10y-15=0$ find the coordinates of focci, equations of directrix, and eccentricities (4)

$\boxed{OR}$

Identify the following curves by obtaining their standard form:

1) $x^2+y^2-4x-6y-2=0$
2) $x^2-y^2+4x+2y+3=0$

(2)

B] (1) Obtain necessary and sufficent condition for two vectors $\bar{x}=(x_1,x_2)$ $\bar{y}=(y_1,y_2)$ to be collinear $(\bar{x},\bar{y}\neq\theta)$

(2)

2) If $\bar{x},\bar{y},\bar{z}$ are non collinear, prove that $\bar{x}+\bar{y},\ \bar{y}+\bar{z},\ \bar{z}+\bar{x}$ are also non collinear.

(2)

C] (1) Obtain formula for that volume of prism

(2)

2) If A-P-B and if $\frac{AP}{PB}=\frac{m}{n}$ then, for any point O in space prove that
$n\,\overline{OA}+m\,\overline{OB}=(m+n)\,\overline{OP}$

[6]

D (1) A boat speeds in the north at $6\sqrt{2}$ kms, A man on the boat feels that the wind is blowing from the south-east at 5 kms. Find the true velocity of the wind. (2)

2) Find a, if $(2a\bar{i}+\bar{j}-4\bar{k}) \perp (a\bar{i}-2\bar{j}+\bar{k})$ (1)

5 A (1) In usual notations obtain the distance between given point and given line in R^3 (2)

2) Obtain equation of a plane passing through two intersecting lines. (2)

[OR]

Obtain equation of plane passing through two parallel lines.

B (1) Find vector and cartesian equation of a sphere having centre $C(\bar{c})$ and radius r. (1)

2) If the direction cosines l, m, n of the two lines satisfy $l+m+n=0$ and $l^2-m^2+n^2=0$ show that the angle between the two lines is $\pi/2$ (3)

[OR]

[7]

Obtain shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-2}{3}$

C] (1) Show that $(4,5,1)$ $(0,-1,-1)$, $(3,9,4)$, $(-4,4,4)$ cannot be vertices of any tetrahedron. (2)

2) obtain the equation, the centre and radius of the sphere through $(0,0,0)$ $(a,0,0)$, $(0,b,0)$ & $(0,0,c)$ (2)

D) Obtain the equation of plane passing through $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and (3)

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

[OR]

Obtain the intersection of the plane $x+y+2z=4$ and $2x-y+z+1=0$.

Solution of paper set No. 3 1]

Mathematics · I (050) (E)

1 A

(1) Text page – 19

(2) Here, A is (2, 3) and B is (0,7)

Supp. the pt. $P(x,0)$ of the X-axis divides $\overline{AB}$ from A in ratio $m:n$; where $m+n \neq 0$

∴ according to the y-coordinate,

$$y = \frac{my_2 + ny_1}{m+n} \text{ of } P,$$

$$0 = \frac{m(7) + n(3)}{m+n}$$

$\therefore 7m + 3n = 0$

$\therefore 7m = -3n$

$\therefore \frac{m}{n} = \frac{-3}{7}$

$\therefore m : n = -3 : 7$

∴ the x-axis divides $\overline{AB}$ from A at point $P(x, 0)$ in the ratio $m:n = -3:7$

1 B

(1) Supp. $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $P(0,0)$ are the distinct noncollinear pts. of a plane

Here, for $\triangle ABC$, $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

and if the determine corresponding to $\triangle PAB$, $\triangle PBC$ and $\triangle PCA$ are D_1, D_2 and D_3 resp., then

$D_1 + D_2 + D_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$$

$$= (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)$$

$$= x_1y_2 - x_1y_3 - x_2y_1 + x_3y_1 + x_2y_3 - x_3y_2$$

$$= x_1(y_2 - y_3) - y_1(x_2 - x_3) + 1(x_2y_3 - x_3y_2)$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = D$$

2]

Thus $D = D_1 + D_2 + D_3$ is obtained

$\therefore |D| = |D_1 + D_2 + D_3|$

$\therefore |D| \leq |D_1| + |D_2| + |D_3|$

$\therefore \frac{1}{2}[|D_1| + |D_2| + |D_3|] \geqslant \frac{1}{2}|D|$

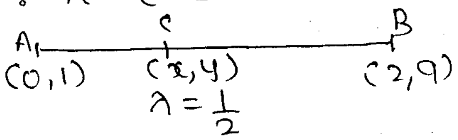
$\therefore$ the area of ΔPAB + the area of ΔPBC + area of ΔPCA $\geqslant$ the area of ΔABC

(2) Here $A(0,1)$ and $B(2,9)$ and supp. C is (x,y)

Now, A, B and C are collinear and $AB = 3AC$

$\therefore$ there are two possibilities

<u>Case-1</u> : A – C – B



Here, if $AC = x$ then $BC = 2x$

$\therefore$ the division ratio $\lambda = \frac{AC}{BC} = \frac{x}{2x} = \frac{1}{2}$

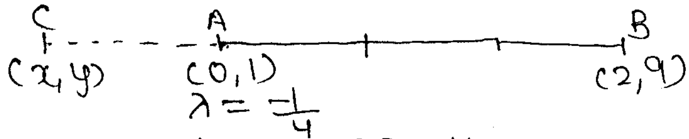
Thus, if A – C – B then C divides $\overline{AB}$ from A in the ratio $1:2$

$\therefore$ using $x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$ the Coordinates of C are $x = \frac{1(2) + 2(0)}{1+2} = \frac{2}{3}$

and $y = \frac{1(9) + 2(1)}{1+2} = \frac{11}{3}$

$\therefore$ the coordinates of C are $\left(\frac{2}{3}, \frac{11}{3}\right)$

<u>Case-2</u> : C – A – B



Here, $AC = x$ then $CB = 4x$

$\therefore$ the division ratio $\lambda = -\frac{AC}{BC} = -\frac{x}{4x} = \frac{-1}{4}$

Thus, if C – A – B then C divides $\overline{AB}$ from A in ratio $-1:4$

$\therefore$ the coordinates of C are

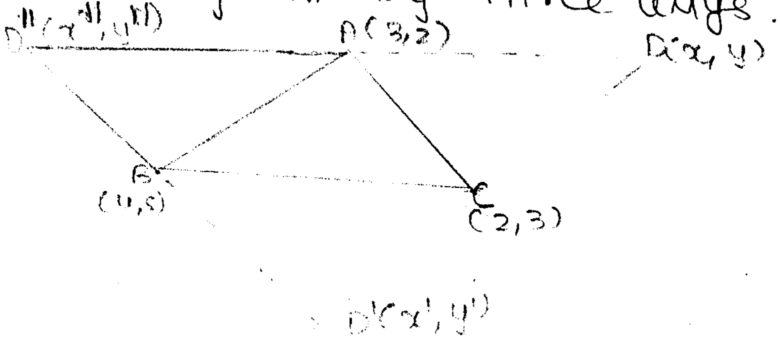
3]

$x = \frac{-1(2)+4(0)}{-1+4} = \frac{-2}{3}$ and $y = \frac{-1(9)+4(1)}{-1+4} = \frac{-5}{3}$

$\therefore$ the coordinates of C are $\left(\frac{-2}{3}, \frac{-5}{3}\right)$

Thus the coordinates of C are $\left(\frac{2}{3}, \frac{11}{3}\right)$ or $\left(\frac{-2}{3}, \frac{-5}{3}\right)$

(3) Supp. A is (3, 2), B is (4, 5) and C is (2, 3)
Here, we can get the fourth vertex of the parallelogram by three ways.



(1) □ABCD is a parallelogram and if the coord. of D are (x, y) then the midpoints of the diagonals $\overline{AC}$ and $\overline{BC}$ are same

$\therefore \frac{x+4}{2} = \frac{3+2}{2}$ and $\frac{y+5}{2} = \frac{2+3}{2}$

$\therefore x = 1$ and $y = 0$

$\therefore$ we get D(1, 0)

(2) □ABD'C is a parallelogram and the coordinates of D' are (x', y') then the midpts of the diagonals $\overline{AD'}$ and $\overline{BC}$ are same

$\therefore \frac{x'+3}{2} = \frac{4+2}{2}$ and $\frac{y'+2}{2} = \frac{5+3}{2}$

$\therefore x' = 3$ and $y' = 6$

$\therefore$ we get D'(3, 6)

(3) □ABCD'' is a parallelogram and if the coordinates of D'' are (x'', y'') then the

midpoints of the diagonals $\overline{AB}$ and $\overline{CD''}$ (4)
are same.

$\therefore \frac{x''+2}{2} = \frac{3+4}{2}$ and $\frac{y''+3}{2} = \frac{2+5}{2}$

$\therefore x'' = 5$ and $y'' = 4$

$\therefore$ we get $D''(5,4)$

Thus, the fourth vertex of the given parallelogram is $(1,0)$ or $(3,6)$ or $(5,4)$

C
(1) For $A(3,2)$, $B(5,6)$
Parametric eq^n of $\overleftrightarrow{AB}$

$x = tx_2 + (1-t)x_1$ $\quad$ $y = ty_2 + (1-t)y_1$

$= 5t + (1-t)3$ $\quad$ $= 6t + (1-t)2$

$= 2t + 3$ $\quad$ $= 4t + 2$

$\therefore 3x + 4y = 6t + 9 + 16t + 8$

$= 25t + 17$

But $P(x,y) \in \overline{AB}$

$\therefore 0 \leq t \leq 1$

$\therefore 0 \leq 25t \leq 25$

$\therefore 17 \leq 25t + 17 \leq 42$

$\therefore 17 \leq 3x + 4y \leq 42$

(2) Here the slope of line $3x + 4y - 2 = 0$ is
$m = -\frac{3}{4}$

Supp., the slope of required line is m_2
Also, the measure of the angle bet^n these two lines is $\alpha = 45°$

Now, acc. to $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore \tan 45° = \left|\frac{-3/4 - m_2}{1 + (-3/4)m_2}\right|$

$\therefore 1 = \left|\frac{-3 - 4m_2}{4 - 3m_2}\right|$

(5)

$\therefore \dfrac{-3-4m_2}{4-3m_2} = \pm 1$

$\therefore \dfrac{-3-4m_2}{4-3m_2} = 1$ or $\dfrac{-3-4m_2}{4-3m_2} = -1$

$\therefore -3-4m_2 = 4-3m_2$ $\quad\therefore -3-4m_2 = -4+3m_2$

$\therefore -7 = m_2$ $\quad\therefore 1 = 7m_2$

$\therefore m_2 = -7$ $\quad\therefore m_2 = \frac{1}{7}$

$\therefore$ two lines are possible

Now, these two lines pass through pt. $(3,4)$

$\therefore$ their equations, acc. to $y-y_1 = m(x-x_1)$ are,

$y-4 = -7(x-3)$ or $y-4 = \frac{1}{7}(x-3)$

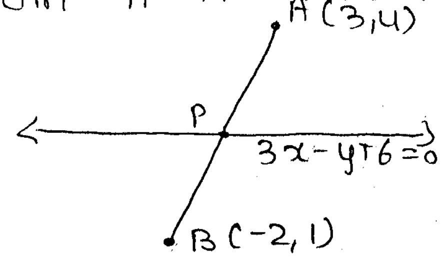
$\therefore y-4 = -7x+21$ $\quad\therefore 7y-28 = x-3$

$\therefore 7x+y-25=0$ $\quad\therefore x-7y+25=0$

Thus, the required lines are $x-7y+25=0$ and $7x+y-25=0$

(3) Supp. A is $(3,4)$ and B is $(-2,1)$

Here, supp. the line $3x-y+6=0$ divides $\overline{AB}$ from A in ratio λ $(\lambda \neq 1)$



Here the coord. of the pt. P, using $\left(\frac{\lambda x_2+x_1}{\lambda+1}, \frac{\lambda y_2+y_1}{\lambda+1}\right)$ are $\left(\frac{\lambda(-2)+3}{\lambda+1}, \frac{\lambda(1)+4}{\lambda+1}\right) = \left(\frac{-2\lambda+3}{\lambda+1}, \frac{\lambda+4}{\lambda+1}\right)$

Now, P is the element of line $3x-y+6=0$

$\therefore 3\left(\frac{-2\lambda+3}{\lambda+1}\right) - \left(\frac{\lambda+4}{\lambda+1}\right) + 6 = 0$

$\therefore -6\lambda+9-\lambda-4+6\lambda+6=0$

$\therefore -\lambda+11=0$

$\therefore \lambda = 11$

6]

Here, $\lambda > 0$

$\therefore$ we get A - P - B

$\therefore$ the pts. A and B are in the opposite half planes of line $3x - y + 6 = 0$

ie the pts $(3, 4)$ and $(-2, 1)$ are on the opp. side of line $3x - y + 6 = 0$.

1 D Text page 36.

2 A

(1) Text Pg 63

(2) Comparing the eq^n $x^2 - 2xy\sec\alpha + y^2 = 0$ with the general quadratic equation of a pair of lines $ax^2 + 2hxy + by^2 = 0$, $a = 1$, $h = -\sec\alpha$ and $b = 1$.

Now, if the measure of the angle bet^n the lines is θ then acc. to

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}, \quad \tan\theta = \frac{2\sqrt{\sec^2\alpha - 1}}{1 + 1}$$

$$= \sqrt{\tan^2\alpha}$$

$$= \tan\alpha$$

$(\because \tan\alpha > 0 \text{ for } 0 < \alpha < \frac{\pi}{2})$

$\therefore \theta = K\pi + \alpha, K \in Z$

$\therefore \theta = \alpha$ ($\because \theta$ is acute angle $\therefore K = 0$)

Thus the measure of the required angle is α unit

(B)

(1) Text Pg 76

(2) Supp. lines $l_1 : 2x - 3y + 5 = 0$

$l_2 : 3x + 2y + 7 = 0$

Here slope of l_1, $m_1 = 2/3$

slope of l_2, $m_2 = -3/2$

$m_1 m_2 = -1$

(7)

$\therefore l_1 \perp l_2$

$\therefore$ solving l_1 and l_2 we get

orthocentre $(x, y) = \left(\frac{-21-10}{4+9}, \frac{-14+15}{4+9}\right)$

$= \left(\frac{-31}{13}, \frac{1}{13}\right)$

(3) eqn represents a circle

$\therefore$ coefficient of $xy = 0$

$\therefore q - 3 = 0$

$\therefore q = 3$

and coeff. of x^2 = coeff. of $y^2 \neq 0$

$\therefore p = 3$

$\therefore$ circle : $3x^2 + 3y^2 + 6x + 9y - 3 = 0$

$\therefore x^2 + y^2 + 2x + 3y - 1 = 0$

$\therefore g = 1, \; f = \frac{3}{2}, \; c = -1$

$\therefore$ Centre $(-g, -f) = (-1, -3/2)$

and radius $r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1 + \frac{9}{4} + 1}$

$= \frac{\sqrt{17}}{2}$ units

C

(1) Eqn of a circle whose diametrically opp. pts. are $(-2, 9)$ and $(2, 1)$ is

$(x+2)(x-2) + (y-9)(y-1) = 0$

ie $x^2 + y^2 - 10y + 5 = 0 \quad \dots (1)$

The eqn of the line passing through $(-2, 9)$ and $(2, 1)$ is

$y - 9 = \frac{1-9}{2+2}(x+2)$

ie $2x + y - 5 = 0 \quad \dots (2)$.

The gen. eqn of the circle passing through the pt. of intersection of (1) & (2) is

8]

$x^2+y^2-10y+5+\lambda(2x+y-5)=0$

If this circle passes through $(5,-8)$ then

$25+64+80+5+\lambda(10-8-5)=0$

$\therefore \lambda = 58$

$\therefore$ substituting value in (3)

Required circle is

$x^2+y^2+116x+48y-285=0$

OR

Here the pt. $(5,3)$ is not on the circle $x^2+y^2=17$

So we will take the tangents to circle

$x^2+y^2=r^2$ with slope m as $y=mx\pm r\sqrt{1+m^2}$

Which is passing through $(5,3)$ & $r=\sqrt{17}$

$\therefore 3=5m\pm\sqrt{17}\sqrt{1+m^2}$

$\therefore (3-5m)^2=17(1+m^2)$

$\therefore 9-30m+25m^2=17+17m^2$

$\therefore 8m^2-30m-8=0$

$\therefore 4m^2-15m-4=0$

$\therefore 4m^2-16m+m-4=0$

$\therefore (4m+1)(m-4)=0$

$\therefore m=-\frac{1}{4}$ or $m=4$

Now,

(1) taking $m=4$ and $r=\sqrt{17}$

the tangents to circle are $y=4x\pm\sqrt{17}\sqrt{1+16}$

$\therefore y=4x\pm 17$

$\therefore 4x-y\pm 17=0$

Here the pt. $(5,3)$ is not on line $4x-y+17=0$

but it is on line $4x-y-17=0$

$\therefore$ we will take tangent as $4x-y-17=0$

(2) taking $m=-\frac{1}{4}$ & $r=\sqrt{17}$

the tangents to circle are $y=-\frac{1}{4}\pm\sqrt{17}\sqrt{1+\frac{1}{16}}$

$\therefore 4y = -x \pm 17$ 9]

$\therefore x + 4y \pm 17 = 0$

Here, the pt. $(5, 3)$ is not on line $x+4y+17=0$ but it is on line $x+4y-17=0$

$\therefore$ we will take the tangent as $x+4y-17=0$

Thus, the eq^n of the tangent to the given circle from the given pt. are $4x-y-17=0$ and $x+4y-17=0$

(2) line $\dfrac{x}{a-\lambda} + \dfrac{y}{b} = 1$

$\therefore \dfrac{y}{b} - 1 = \dfrac{-x}{a-\lambda}$

$\therefore y - b = \dfrac{-b}{a-\lambda}(x-0)$

$\therefore y - b = m(x-0)$ where $m = \dfrac{-b}{a-\lambda}$

Comparing with $y - y_1 = m(x - x_1)$

Given line passes through fixed pt.

$(x_1, y_1) = (0, b)$

: D

Here solving eq^n $y = mx + a$ and $y = nx + c$, also $y = mx + a$ and $y = nx + d$

we get $A\left(\dfrac{c-a}{m-n}, \dfrac{mc-na}{m-n}\right)$ and $B\left(\dfrac{d-a}{m-n}, \dfrac{md-na}{m-n}\right)$

$$AB^2 = \left(\frac{c-a}{m-n} - \frac{d-a}{m-n}\right)^2 + \left(\frac{mc-na}{m-n} - \frac{md-na}{m-n}\right)^2$$

$$= \left(\frac{c-d}{m-n}\right)^2 + m^2\left(\frac{c-d}{m-n}\right)^2$$

$$= \left(\frac{c-d}{m-n}\right)^2 (1+m^2)$$

$$\therefore AB = \left|\frac{c-d}{m-n}\right| \cdot \sqrt{1+m^2}$$

Also, the $\perp$r dist. bet^n $\overline{AB}$ & $\overline{CD}$ is $p_1 = \dfrac{|a-b|}{\sqrt{1+m^2}}$

10]

Now, the area of parallelogram $= AB \cdot p_1$

$$= \left|\frac{c-d}{m-n}\right| \cdot \sqrt{1+m^2} \cdot \frac{|a-b|}{\sqrt{1+m^2}}$$

$$= \left|\frac{(a-b)(c-d)}{m-n}\right|.$$

OR.

Here the lines are $ax+by+c=0 \cdots (1)$

$bx+cy+a=0 \cdots (2)$

& $cx+ay+b=0 \cdots (3)$

and $b^2 \neq ac$, $c^2 \neq ab$ and $a^2 \neq bc$

Now, $a_1b_2 - a_2b_1 = ca - b^2 \neq 0$

$a_2b_3 - a_3b_2 = ab - c^2 \neq 0$

and $a_3b_1 - a_1b_3 = bc - a^2 \neq 0$

and $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$(\because$ by applying $R_{21}(1), R_{31}(1))$

$$= \begin{vmatrix} 0 & 0 & 0 \\ b & c & a \\ c & a & b \end{vmatrix} \quad (\because a+b+c=0)$$

$= 0$

$\therefore$ given lines are concurrent

Now, to get pt. of concurrence, solving eqn (1) and (2) using cramer's rule

Here, $\dfrac{x}{\begin{vmatrix} b & c \\ c & a \end{vmatrix}} = \dfrac{-y}{\begin{vmatrix} a & c \\ b & a \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}}$

$\therefore \dfrac{x}{ab-c^2} = \dfrac{-y}{-(bc-a^2)} = \dfrac{1}{ac-b^2}$

$\therefore x = \dfrac{ab-c^2}{ac-b^2}$ and $y = \dfrac{bc-a^2}{ac-b^2}$

11]

$$= \frac{b(-b-c)-c^2}{c(-b-c)-b^2} \qquad = \frac{b(-b-a)-a^2}{a(-a-b)-b^2}$$

$$= \frac{-b^2-bc-c^2}{bc-c^2-b^2} \qquad = \frac{-ab-b^2-a^2}{-a^2-ab-b^2} \qquad (\because a+b+c=0 \Rightarrow a=-b-c \text{ and } c=-a-b)$$

$$= 1 \qquad = 1$$

$\therefore$ the coordinates of pt. of concurrence are $(1, 1)$

3 A

(1) Text Pg 87

(2) Supp. $\overline{PQ}$ is focal chord of parabola $y^2=4ax$ and the coord. of P and Q are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ resp. and focus $S(a,0)$.
Here, S, P and Q are collinear pts. we get $t_1t_2=-1$

$\therefore$ taking $t_2 = \frac{-1}{t_1}$, we get the coord. $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$ of Q

Now, $S(a,0)$ divides $\overline{PQ}$ from P in ratio 2:1

$\therefore$ acc. to $y = \frac{\lambda y_2 + y_1}{\lambda + 1}$

The y-coord of P is

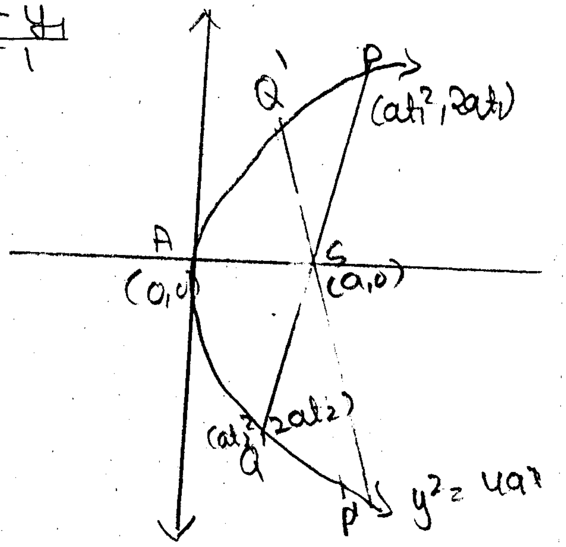
$$0 = \frac{2\left(-\frac{2a}{t_1}\right) + 2at_1}{2+1}$$

$$\therefore \frac{4a}{t_1} = 2at_1$$

$$\therefore \frac{2}{t_1} = t_1$$

$$\therefore t_1^2 = 2$$

$$\therefore t_1 = \pm\sqrt{2}$$



Now, 12]

(1) Taking $t_1 = \sqrt{2}$, we get coord. $(2a, 2\sqrt{2}a)$ of P

Here, the focal chord is passing through S and P

$\therefore$ eqn of line containing the focal chord is,

$$\begin{vmatrix} x & y & 1 \\ a & 0 & 1 \\ 2a & 2\sqrt{2}a & 1 \end{vmatrix} = 0$$

$\therefore -2\sqrt{2}ax + ay + 2\sqrt{2}a^2 = 0$

$\therefore 2\sqrt{2}x - y - 2\sqrt{2}a = 0$

$\therefore y = 2\sqrt{2}(x-a)$

(2) Taking $t_1 = -\sqrt{2}$, we get coord. $(2a, -2\sqrt{2}a)$ of P

Hence, the second eqn of line containing focal chord is $y = -2\sqrt{2}(x-a)$

Thus, the two eqn of focal chord are $y = \pm 2\sqrt{2}(x-a)$

OR

Here, line $3y = 6x + 2$

$y = 2x + \frac{2}{3}$

Parabola $3y^2 = 16x$

$\therefore y^2 = \frac{16}{3}x$

$\therefore a = \frac{4}{3}$

$c = \frac{2}{3}$ & $\frac{a}{m} = \frac{4/3}{2} = \frac{2}{3}$

$\therefore c = \frac{a}{m}$

$\therefore$ given line touches given parabola.

(4) pt. of contact. [3]

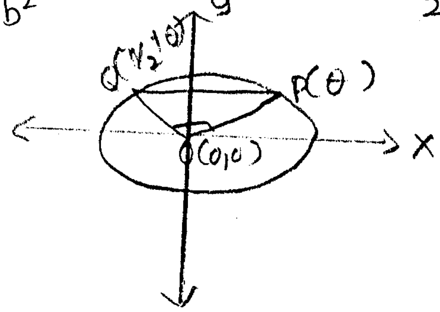
$\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{4/3}{4}, \frac{2 \cdot 4/3}{2}\right)$

$= \left(\frac{1}{3}, \frac{4}{3}\right)$

(B)

(1) Text Pg 103

(2) Here, the diff. of eccentric angles of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is $\frac{\pi}{2}$



$\therefore$ we will take $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ on ellipse

Here the coord. of pt. $P(\theta)$ are $(a\cos\theta, b\sin\theta)$ and the coord. of pt. $Q\left(\frac{\pi}{2}+\theta\right)$ are $\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$

ie $(-a\sin\theta, b\cos\theta)$

Also, $O(0,0)$ is the centre of ellipse

The vertices of $\triangle OPQ$ are $(0,0)$ $(a\cos\theta, b\sin\theta)$ and $(-a\sin\theta, b\cos\theta)$

$$\therefore D = \begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \end{vmatrix}$$

$= 1[ab(\cos^2\theta + \sin^2\theta)]$

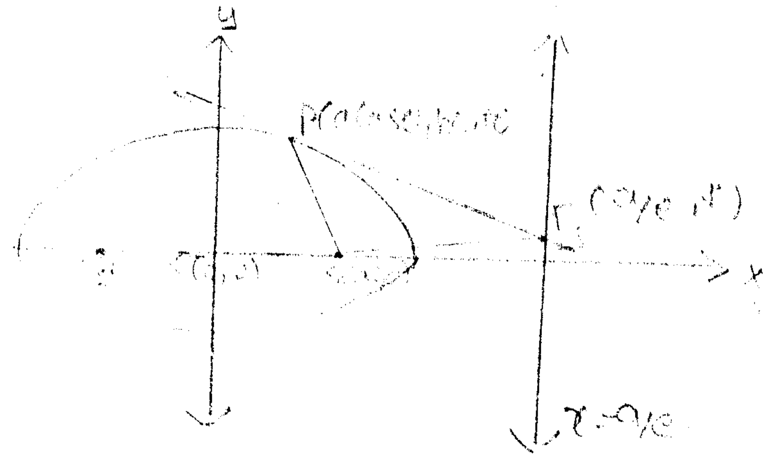
$= ab$

$\therefore$ The area of $\Delta OPQ\ A = \frac{1}{2}|D|$ [4]

$$\Delta = \frac{1}{2}|ab| = \frac{1}{2}ab\ (\because a, b > 0)$$

Thus, the area of $\Delta OPQ = \frac{1}{2}ab$ is proved

OR.



The tangent $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ at pt. $P(a\cos\theta, b\sin\theta)$ to ellipse intersects the directrix $x = \frac{a}{e}$ of ellipse at pt. $F\left(\frac{a}{e}, k\right)$

$\therefore \frac{a}{e}\cdot\frac{\cos\theta}{a} + \frac{K}{b}\sin\theta = 1$

$\therefore \frac{K}{b}\sin\theta = 1 - \frac{\cos\theta}{e}$

$\therefore K = \frac{b(e-\cos\theta)}{e\sin\theta}$

$\therefore \left(\frac{a}{e}, \frac{b(e-\cos\theta)}{e\sin\theta}\right)$ are coord. of F & S is $(ae, 0)$

$\therefore$ the slope m_1 to $\overleftrightarrow{SF} = \dfrac{\frac{b(e-\cos\theta)}{e\sin\theta} - 0}{a/e - ae}$

$$= \frac{b(e-\cos\theta)}{\sin\theta\cdot a(1-e^2)} \quad \ldots(1)$$

and slope of m_2 of $\overleftrightarrow{SP} = \frac{b\sin\theta - 0}{a\cos\theta - ae} = \frac{-b\sin\theta}{a(e-\cos\theta)}$ ...(2)

15

Now, from (1) and (2),

the slope $\overleftrightarrow{SF}$ · slope of $\overleftrightarrow{SP}$

$$= \frac{b(e-\cos\theta)}{\sin\theta \cdot a(1-e^2)} \cdot \frac{(-b\sin\theta)}{a(e-\cos\theta)}$$

$$= \frac{-b^2}{a^2(1-e^2)} = -\frac{b^2}{b^2} = -1$$

$\therefore \overleftrightarrow{SF} \perp \overleftrightarrow{SP}$

$\therefore \overline{PF}$ subtends a right angle at focus S.

(1) Text page 121

(2) Here the eqn of asymptotes $x^2-2y^2=0$

if the angle betn them is θ, then

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|}$$

Here, $a=1, h=0, b=-2$

$$\therefore \tan\theta = \frac{2\sqrt{0-(-2)}}{|1-2|} = 2\sqrt{2}$$

$$\therefore \theta = \tan^{-1}(2\sqrt{2})$$

(2) $S(4,0)$, $e=3/2$

$ae = 4 \quad e = 3/2$

$a \cdot \frac{3}{2} = 4 \Rightarrow a = \frac{8}{3}$

Now, $b^2 = a^2(e^2-1)$

$$\therefore b^2 = \frac{64}{9}\left(\frac{9}{4}-1\right)$$

$$= 16 - \frac{64}{9}$$

$$= \frac{144-64}{9}$$

$$\therefore b^2 = 80/9$$

16]

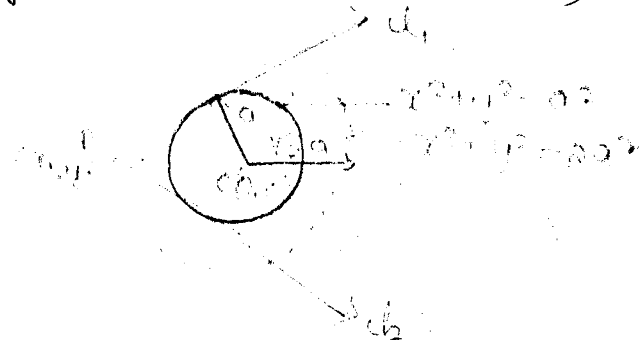
$\therefore$ Eq^n of hyperbola will be

$$\frac{9x^2}{64} - \frac{9y^2}{80} = 1.$$

(c) Supp. $y = mx \pm a\sqrt{1+m^2}$. are the tangents to the circle $x^2+y^2=a^2$ which are passing through pt. $P(x_1, y_1)$ outside the circle

$\therefore y_1 = mx_1 \pm a\sqrt{1+m^2}$

$\therefore (y_1 - mx_1)^2 = a^2(1+m^2)$



$\therefore y_1^2 - 2x_1y_1m + m^2x_1^2 = a^2 + a^2m^2$

$\therefore (a^2 - x_1^2)m^2 + 2x_1y_1m + (a^2 - y_1^2) = 0$

If m_1 & m_2 are roots of quadratic eq^n in m then $m_1m_2 = \dfrac{a^2 - y_1^2}{a^2 - x_1^2}$.

Now, tangent lines drawn from P are ⊥r to each other

$\therefore$ taking $m_1m_2 = -1$, $\dfrac{a^2 - y_1^2}{a^2 - x_1^2} = -1$

$\therefore a^2 - y_1^2 = x_1^2 - a^2$

$\therefore x_1^2 + y_1^2 = 2a^2$.

In general, this eq^n can be written as $x^2 + y^2 = 2a^2$.

Thus locus of P is concentric circle $x^2+y^2=2a^2$ with rad. $\sqrt{2}\,a$.

17

4(A) Here, $a \neq b$

$\therefore$ On rotating the axes by an angle θ

$\tan 2\theta = \frac{2h}{a-b} = \frac{4}{3}$

$\therefore \cos 2\theta = \frac{3}{5}$

$\therefore \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+3/5}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$

$\therefore \sin\theta = \frac{1}{\sqrt{5}}$

Here $x = x'\cos\theta - y'\sin\theta = \frac{2x'-y'}{\sqrt{5}}$

and $y = y'\cos\theta + x'\sin\theta = \frac{x'+2y'}{\sqrt{5}}$

$\therefore$ Equation of the curve is

$3\left(\frac{2x'-y'}{\sqrt{5}}\right)^2 + 8\left(\frac{2x'-y'}{\sqrt{5}}\right)\left(\frac{x'+2y'}{\sqrt{5}}\right) - 3\left(\frac{x'+2y'}{\sqrt{5}}\right)^2$

$- 20\left(\frac{2x'-y'}{\sqrt{5}}\right) + 10\left(\frac{x'+2y'}{\sqrt{5}}\right) - 15 = 0$

$\therefore \left(\frac{12x'^2}{5} + \frac{16x'^2}{5} - \frac{3x'^2}{5}\right) + \left(\frac{3y'^2}{5} - \frac{16y'^2}{5} - \frac{12y'^2}{5}\right)$

$+ \left(\frac{-12x'y'}{\sqrt{5}} + \frac{24x'y'}{\sqrt{5}} - \frac{12x'y'}{\sqrt{5}}\right) + \left(\frac{-40x'}{\sqrt{5}} + \frac{10x'}{\sqrt{5}}\right)$

$+ \left(\frac{20y'}{\sqrt{5}} + \frac{20y'}{\sqrt{5}}\right) - 15 = 0$

$\therefore 5x'^2 - 5y'^2 - 6\sqrt{5}x' + 8\sqrt{5}y' - 15 = 0$

$\therefore 5\left(x'^2 - \frac{6x'}{\sqrt{5}} + \frac{9}{5}\right) - 5\left(y'^2 - \frac{8}{\sqrt{5}}y' + \frac{16}{5}\right) = 8$

18

$\therefore \left(x' - \frac{3}{\sqrt{5}}\right)^2 - \left(y' - \frac{4}{\sqrt{5}}\right)^2 = \frac{8}{5}$

Now on shifting origin to $\left(\frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

$x' - \frac{3}{\sqrt{5}} = X$ and $y' - \frac{4}{\sqrt{5}} = Y$

$\therefore$ The equation of the curve $X^2 + Y^2 = \left(2\frac{\sqrt{2}}{\sqrt{5}}\right)^2$, which represent the rectangular hyperbola

Here $a = 2\sqrt{\frac{2}{5}}$ $e = \sqrt{2}$

$\therefore$ the focci (x, y) system, according to $(\pm ae, 0)$ are $\left(\pm \frac{4}{\sqrt{5}}, 0\right)$ and the directrix, according to $X = \pm a/e$ $X = \pm 2/\sqrt{5}$ and

the length of both the axes $2a = 4\sqrt{\frac{2}{5}}$

Now in $(x'y')$ system the focci according to $\left(X + \frac{3}{\sqrt{5}}, Y + \frac{5}{\sqrt{4}}\right)$ are

$\left(\frac{4}{\sqrt{5}} + \frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right) = \left(\frac{7}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$ and $\left(\frac{-4}{\sqrt{5}} + \frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

$= \left(-\frac{1}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

Now in $(x'y')$ system the focci according to $\left(\frac{\cancel{2x' + 3y'}}{\sqrt{5}}, \cancel{y'} + \frac{2\cancel{5}'}{\sqrt{4}}\right)$ are $\left(\frac{2x' - y'}{\sqrt{5}}, \frac{x' + 2y'}{\sqrt{5}}\right)$ are

$\left(\frac{\frac{14}{\sqrt{5}} - \frac{4}{\sqrt{5}}}{\sqrt{5}}, \frac{\frac{7}{\sqrt{5}} + \frac{8}{\sqrt{5}}}{\sqrt{5}}\right) = (2, 3)$ and

19

$\left(\frac{\frac{-2}{\sqrt{5}}-\frac{3}{\sqrt{5}}}{\sqrt{5}}, \frac{\frac{-1}{\sqrt{5}}+\frac{2}{\sqrt{5}}}{\sqrt{5}}\right) = \left(\frac{-6}{5}, \frac{7}{5}\right)$ and the

directrixs in $(x'y')$ system, according to $X = x' - \frac{3}{\sqrt{5}}$ are $x' - \frac{3}{\sqrt{5}} = \pm\frac{2}{\sqrt{5}}$ ie $x' - \sqrt{5} = 0$ and $x' - \frac{1}{\sqrt{5}} = 0$

Now for the directrixes in (x,y) system, substituting $x = x'\cos\theta + y\sin\theta = \frac{2x+y}{\sqrt{5}}$,

the original directrixs are $\frac{2x+y}{\sqrt{5}} - \sqrt{5} = 0$ and $\frac{2x+y}{\sqrt{5}} - \frac{1}{\sqrt{5}} = 0$ ie $2x+y-5=0$ and

$2x+y-1=0$

Thus the given second degree equation represents rectangular hyperbola

The eccentricity is $e = \sqrt{2}$

The given co-ordinate of the focci $(2,3)$ and $\left(-\frac{6}{5}, \frac{7}{5}\right)$.

The equation of the directrixs are $2x+y-5=0$ and $2x+y-1=0$

and the length of both axes are $4\sqrt{\frac{2}{5}}$ units.

[OR] 20]

11) Here $a = b$
$\theta = \pi/4$

$\therefore$ On rotating axes by $\theta = \pi/4$

$$x = \frac{x' - y'}{\sqrt{2}} \quad y = \frac{x' + y'}{\sqrt{2}}$$

$\therefore$ the equation of the curve are

$$\left(\frac{x' - y'}{\sqrt{2}}\right)^2 + \left(\frac{x' + y'}{\sqrt{2}}\right)^2 - 4\left(\frac{x' - y'}{\sqrt{2}}\right) - 6\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)$$
$$-2 = 0$$

$\therefore x'^2 + y'^2 - 5\sqrt{2}\,x' - \sqrt{2}\,y' - 2 = 0$

$\therefore x'^2 - 5\sqrt{2}\,x' + \frac{25}{2} + y'^2 - \sqrt{2}\,y' + \frac{1}{2} = 15$

$\therefore \left(x' - \frac{5}{\sqrt{2}}\right)^2 + \left(y' - \frac{1}{\sqrt{2}}\right)^2 = 15$

Now, on shifting the origin to $\left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$x' - \frac{5}{\sqrt{2}} = X$ and $y' - \frac{1}{\sqrt{2}} = Y$

$\therefore$ the equation of the curve is
$X^2 + Y^2 = (\sqrt{15})^2$, which represent a circle

21

2) Here $a \neq b$

$\therefore$ On rotating the axes by an angle θ

$\tan 2\theta = \dfrac{2h}{a-b} = \dfrac{0}{1+1} = 0$

$\therefore \dfrac{2\tan\theta}{1-\tan^2\theta} = 0$

$\therefore \tan\theta = 0$

$\therefore \theta = 0$

$\therefore$ Hence there is no need to rotates the axes

Here the equation is,

$x^2 + 4x + 4 - y^2 + 2y - 1 = 0$

$\therefore (x+2)^2 - (y-1)^2 = 0$

Now, on shifting the origin to $(-2, 1)$

$x+2 = x'$ and $y-1 = y'$

$\therefore$ the equation of the curve is

$x'^2 - y'^2 = 0$ ie $(x'+y')(x'-y') = 0$, which represent a pair of lines.

Here we get the lines $x'+y' = 0$ & $x'-y' = 0$

Now, the original lines in (x, y) system are

$x+2+y-1 = 0$ and $x+2-y+1 = 0$

ie $x+y+1 = 0$ and $x-y+3 = 0$

Thus, the given second degree equ. represent a pair of lines, whose equation are $x+y+1=0$ & $x-y+3=0$

B (1) Text page 154 Theorem - 5 22]

2) Suppose $\bar{x} = (x_1\ x_2\ x_3)$ $\bar{y} = (y_1\ y_2\ y_3)$ and $\bar{z} = (z_1\ z_2\ z_3) \in R^3$

Here, we want to show that $\bar{x}+\bar{y}, y+\bar{z}, \bar{z}+\bar{x}$ are not coplanar vectors..

Also $\bar{x} \neq \theta, \bar{y} \neq \theta, \bar{z} \neq \theta$, and x, y and $\bar{z}$ are non-coplanar is given

$$\therefore \bar{x}\cdot(\bar{y}\times\bar{z}) = [x\ y\ \bar{z}] = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} \neq 0$$

Now, $(\bar{x}+\bar{y})\cdot[(y+\bar{z})\times(\bar{z}+\bar{x})]$

$$= (\bar{x}+\bar{y})\ [y\times\bar{z} + y\times\bar{x} + \bar{z}\times\bar{z} + \bar{z}\times\bar{x}]$$

$$= (\bar{x}+\bar{y})\ [y\times\bar{z} + y\times\bar{x} + \theta + \bar{z}\times\bar{x}]$$

$$= (\bar{x}+\bar{y})\ [y\times\bar{z} + y\times\bar{x} + \bar{z}\times\bar{x}]$$

$$= x\cdot(\bar{y}\times\bar{z}) + \bar{x}\cdot(y\times\bar{x}) + \bar{x}\cdot(\bar{z}\times\bar{x}) + y\ (\bar{y}\times\bar{z}) + y\ (\bar{y}\times\bar{x}) + y\ (\bar{z}\times\bar{x})]$$

$$= [\bar{x}\ y\ \bar{z}] + [\bar{x}\ y\ \bar{x}] + [\bar{x}\ \bar{z}\ \bar{x}] + [y\ \bar{y}\ \bar{z}] + [y\ y\ \bar{x}] + [y\ \bar{z}\ \bar{x}]$$

$$= [\bar{x}\ y\ \bar{z}] + [\bar{x}\ y\ \bar{z}]$$

$$= 2[\bar{x}\ y\ \bar{z}]$$

$\neq 0$ ($\because$ data

$\therefore$ the vector $x+y$, $y+\bar{z}$ and $\bar{z}+\bar{x}$ are non coplan

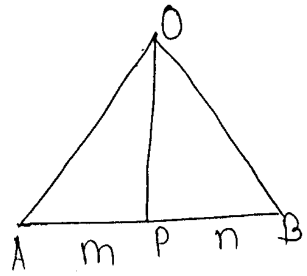
23

C (1) Text page 175

2) Here the direction of ~~rays~~ vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are same and $\frac{AP}{PB} = \frac{m}{n}$. Hence $n\overrightarrow{AP} = m\overrightarrow{PB}$

$\therefore n(\overrightarrow{OP} - \overrightarrow{OA}) = m(\overrightarrow{OB} - \overrightarrow{OP})$

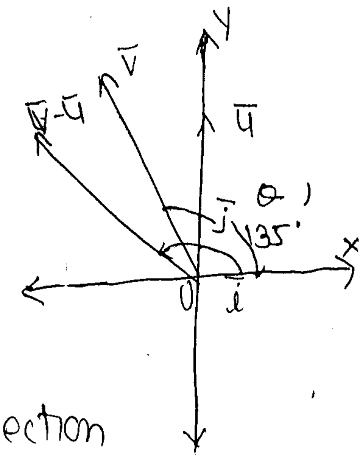
$\therefore (m+n)\overrightarrow{OP} = n(\overrightarrow{OA}) + m(\overrightarrow{OB})$



B (1) Here, the velocity of the boat is $\bar{u} = 0\cdot\bar{i} + 6\sqrt{2}\,\bar{j}$

$= \frac{12}{\sqrt{2}}\bar{j}$

Suppose the true velocity of the wind is $\bar{v}$. The wind blows from the south-east i.e. It seems to go in the direction North-West and the velocity of the wind relative of the boat is



$\bar{v} - \bar{u} = 5\cos 135^\circ\,\bar{i} + 5\sin 135^\circ\,\bar{j}$

$\therefore \bar{v} - \bar{u} = -\frac{5}{\sqrt{2}}\bar{i} + \frac{5}{\sqrt{2}}\bar{j}$

Now the true velocity of wind $\bar{v} = (\bar{v} - \bar{u}) + \bar{u}$

$\therefore \bar{v} = \frac{-5}{\sqrt{2}}\bar{i} + \frac{17}{\sqrt{2}}\bar{j}$

Now $|\bar{v}| = \sqrt{\frac{25}{2} + \frac{289}{2}} = \sqrt{157}$

24.

~~Or~~ and if $\bar{v}$ makes an angle θ with $\overrightarrow{OX}$, then

$$\cos\theta = \frac{\bar{v}\cdot\hat{i}}{|\bar{v}|\,|\hat{i}|} = \frac{-5}{\sqrt{2}\times\sqrt{157}\times 1} = \frac{-5}{\sqrt{314}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-5}{\sqrt{314}}\right) = \pi - \cos^{-1}\left(\frac{5}{\sqrt{314}}\right)$$

$\therefore$ the magnitude of the true velocity of the wind is $\sqrt{157}$ ~~units~~ units and its direction ~~is~~ at angle $\pi - \cos^{-1}\left(\frac{5}{\sqrt{314}}\right)$ with the East towards the North.

2) Here $(2a, a, -4)\cdot(a, -2, 1) = 0$

$\therefore 2a^2 - 2a - 4 = 0$

$\therefore a^2 - a - 2 = 0$

$\therefore (a-2)(a+1) = 0$

$\therefore a - 2 = 0$ or $a + 1 = 0$

$\therefore a = 2$ or $a = -1$

5(A)(1) Text Page 191

(2) Text Page 2.4

OR

Text Page 203

B (i) Test Page 214 (3)

B (2) 25

Here $l+m+n=0$ ... (1)

and $l^2-m^2+n^2=0$ ... (2)

Now from the equation (1), we get

$m=-(l+n)$.

Substituting it in the equation (2),

$l^2-[-(l+n)]^2+n^2=0$

$\therefore l^2-(l+n)^2+n^2=0$

$\therefore l^2-(l^2+2ln+n^2)+n^2=0$

$\therefore l^2-l^2-2ln-n^2+n^2=0$

$\therefore -2ln=0$

$\therefore ln=0$

$\therefore l=0$, or $n=0$

Now:-

1) If $l=0$, then from the equation.

(1) $m=-n$. and so we get the direction ratios of the first line $0, -n$, and n.

2) If $n=0$ then from the equation (2)

2) $m=-l$ and so we get the direction ratios of the second line $l, -l$ and 0.

Now, for angle θ bet^n two lines 26)

$$\cos\theta = \frac{(0,-n,n)\cdot(l,-l,0)}{\sqrt{2n^2}\ \sqrt{2l^2}}$$

$$= \frac{nl}{2nl}$$

$$= \frac{1}{2}$$

$$\therefore \theta = \pi/3$$

OR

Here vector eq^n from given cartesian eq^n of line is $\bar{r} = (3,-15,9) + K(2,-7,5)$ and $\bar{r} = (-1,1,9) + K(2,+1,-3)$

comparing them with vector eq^n $\bar{r} = \bar{a} + K\bar{l}$ and $\bar{r} = \bar{b} + K\bar{m}$, we get

$\bar{a} = (3,-15,9)$ $\bar{b} = (-1,1,9)$ $\bar{l} = (2,-7,5)$ and $\bar{m} = (2,1,-3)$

$$\therefore \bar{l}\times\bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \bar{i}(21-5) - \bar{j}(-6-10) + \bar{k}(2+14)$$

$$= 16\bar{i} + 16\bar{j} + 16\bar{k}$$

$$= (16,16,16)$$

$$\therefore |\bar{l}\times\bar{m}| = \sqrt{(16)^2 + (16)^2 + (16)^2}$$

$$= 16\sqrt{3}$$

Now, $$\bar{u} = \frac{\bar{l}\times\bar{m}}{|\bar{l}\times\bar{m}|}$$

$$= \frac{(16,16,16)}{16\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}(1,1,1)$$

and $\bar{a} - \bar{b} = (3, -15, 9) - (-1, 1, 9)$ 27

$= (4, -16, 0)$

$\therefore$ $\perp$r dist. bet^n two lines is

$|(\bar{a} - \bar{b}) \cdot \bar{u}| = |(4, -16, 0) \cdot \frac{1}{\sqrt{3}}(1, 1, 1)|$

$= \frac{1}{\sqrt{3}} |4 - 16 + 0|$

$= \frac{1}{\sqrt{3}} |-12| = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

$\therefore$ Thus, the shortest dist. bet^n two lines is $4\sqrt{3}$ units

C

(1) Here supp. $V(4, 5, 1)$ $A(0, -1, -1)$ $B(3, 9, 4)$ & $C(-4, 4, 4)$

$\therefore \overrightarrow{VA} = (-4, -6, -2)$, $\overrightarrow{VB} = (-1, 4, 3)$, $\overrightarrow{VC} = (-8, -1, 3)$

Now, $\overrightarrow{VA} \cdot (\overrightarrow{VB} \times \overrightarrow{VC}) = [\overrightarrow{VA}\ \overrightarrow{VB}\ \overrightarrow{VC}]$

$$= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$= -4(12+3) + 6(-3+24) - 2(1+32)$

$= -60 + 126 - 66$

$= 0$

$\therefore \overrightarrow{VA}, \overrightarrow{VB}$ & $\overrightarrow{VC}$ are collinear

$\therefore$ the pts. V, A, B & C are coplanar

$\therefore$ they cannot be vertices of any tetrahedron.

(2) Let required eq^n of a sphere

$S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

$(0, 0, 0) \in S \quad \therefore d = 0$

$(a, 0, 0) \in S \quad \therefore a^2 + 2ua = 0 \Rightarrow u = -a/2 \ (a \neq 0)$

28

$(0,b,0) \in S \quad \therefore b^2 + 2bv = 0 \Rightarrow v = -\frac{b}{2} \; (b \neq 0)$

$(0,0,c) \in S \quad \therefore c^2 + 2wc = 0 \Rightarrow w = -\frac{c}{2}$

$\therefore$ The eq^n of sphere will be

$$x^2 + y^2 + z^2 - ax - by - cz = 0$$

centre $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

& $r = \frac{1}{2}\sqrt{a^2+b^2+c^2}$.

(D)

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

Here eq^n of lines are $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

$\therefore \bar{a} = (4,-3,-1),\ \bar{b} = (1,-1,-10),\ \bar{l} = (1,-4,7)$

and $\bar{m} = (2,-3,8)$

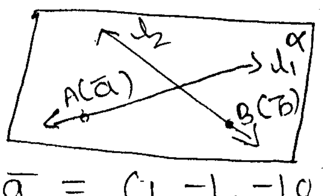
Now, $\bar{l} \times \bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix}$

$= \bar{i}(-32+21) - \bar{j}(8-14) + \bar{k}(-3+8)$

$= \bar{i}(-11) - \bar{j}(-6) + \bar{k}(5)$

$= -11\bar{i} + 6\bar{j} + 5\bar{k}$

$= (-11, 6, 5)$



and $\bar{b} - \bar{a} = (1,-1,-10) - (4,-3,-1)$

$= (-3, 2, -9)$

29

$\therefore (\bar{b}-\bar{a})\cdot(\bar{l}\times\bar{m}) = (-3,2,-9)\cdot(-11,6,5)$

$= 33+12-45=0$

$\therefore$ both lines are intersecting lines

Now, the eq^n of the plane passing through these lines, acc. to

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0 \quad \text{is,}$$

$$\begin{vmatrix} x-4 & y+3 & z+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = 0$$

$\therefore (x-4)(-32+21) - (y+3)(8-14) + (z+1)(-3+8) = 0$

$\therefore (x-4)(-11) - (y+3)(-6) + (z+1)5 = 0$

$\therefore -11x+44+6y+18+5z+5=0$

$\therefore -11x+6y+5z+67=0$

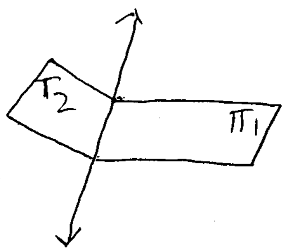
OR

Here the planes are $x+y+2z=4$ and $2x-y+z=-1$

ie $(x,y,z)\cdot(1,1,2)=4$ and $(x,y,z)\cdot(2,-1,1)=-1$

Comparing these eq^n with gen. eq^n $\bar{r}\cdot\bar{n}=d$ of the plane.

For first plane $\bar{n_1}=(1,1,2)$ and for the second plane, $\bar{n_2}=(2,-1,1)$



Now, $\bar{n}=\bar{n_1}\times\bar{n_2}$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}.$$

$= \bar{i}(1+2) - \bar{j}(1-4) + \bar{k}(-1,-2)$

$= 3\bar{i}+3\bar{j}-3\bar{k}$

$= (3,3,-3)$

Now, to obtain common pt. of intersection for both the planes taking $z=0$ in their eq^n we get $x+y=4$ & $2x-y=1$ [307]

On solving eq^n we get $x=1$ & $y=3$

Thus one common pt. of intersection is $\bar{a}=(1,3,0)$

Also, eq^n of intersecting line of planes passing through $\bar{a}=(1,3,0)$ & having dir^n $\bar{n}=(3,3,-3)$

acc. to $\bar{r}=\bar{a}+K_1\cdot\bar{n}$ $(K_1\in R)$ is

$$\bar{r}=(1,3,0)+K_1(3,3,-3)\ (K\in R)$$
$$=(1,3,0)+3K_1(1,1,-1);\ (K\in R)$$
$$=(1,3,0)+K(1,1,-1);\ (K\in R)$$
$$(\because \text{taking } 3K_1=K)$$

Thus eq^n of intersecting line of planes is $\bar{r}=(1,3,0)+K(1,1,-1);\ K\in R$.

Q. Paperset No. 4
Mathematics I (050) (E) 1

Maximum Marks 75.

Time : 3 Hrs.

Q1.(A) (1) Using co-ordinate geometry in R^2 obtain the co-ordinates of incentre of tringle. (3)

(2) If A is (2, 3) and B is (0, 7) in what ratio does the X - axis divides $\overline{AB}$ from B. (1)

(B) Answer any Two :- (4)

(1) If A, B, C, P are distinct and non-collineal points of the plane then prove that area of ΔPAB + area of ΔPBC + area of $\Delta PCA \geq$ area of ΔABC.

(2) Find point C on the $\overleftrightarrow{AB}$ such that $AB = 3AC$. where A (0, 1), B (2, 9).

(3) If (3, 2), (4, 5) and (2, 3) are three of the four vertices of a parallelogram, find the co-ordinates of fourth vertex. (4)

(C) Attempt any two :-

(1) If A (3, 2), B (5, 6) and $P(x, y) \in \overline{AB}$ then prove that $17 \leq 3x + 4y \leq 39$.

(2) Find the equation of line which passes through (3, 4) and which makes an angle of $\frac{\pi}{4}$ with the line $3x + 4y - 2 = 0$.

(3) Prove that the points (3, 4) and (-2, 1) are on opposite side of the line $3x - y + 6 = 0$ in R^2 (3)

(D) Using slopes of two intersecting lines obtain the formula for measure of angle between them. If one out of two intersecting line is vertical then what is the form of measure of angle between them? (3)

Q2. (A) (1) Prove that if general quadratic equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines then this pair is parallel to the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ (where $a^2 + h^2 + b^2 \neq 0$)

(2) Find the angle between the lines represented by $x^2 - 2xy\sec\alpha + y^2 = 0$, $(0 < \alpha < \frac{\pi}{2})$. (1)

2.

(B) ① Find the condition for the line $y = mx + c$ be tangent to the circle $x^2 + y^2 = r^2$ and the point of contact. (2)

② The sides of a triangle are along the lines $2x - 3y + 5 = 0$ and $3x + 2y + 7 = 0$ and $x = 2$ find the point of concurrence of all the three altitudes of such triangle. ①

③ If $px^2 + 3y^2 + (q - 3)xy + 2px + 3qy - 3 = 0$ represents a circle then find centre and radius. ①

(C) ① Find the equation of the circle passing through the points $(5, -8)$, $(-2, 9)$ and $(2, 1)$. (3)

OR

① Find the equation of the tangents to the circle $x^2 + y^2 = 17$ from the point $(5, 3)$.

② Show that for $\lambda \in R - \{a\}$ line $\frac{x}{a-\lambda} + \frac{y}{b} = 1$ passes through a fixed point. ①

(D) Find the area of the parallelogram whose sides are along the lines $y = mx + a$, $y = mx + b$, $y = nx + c$, $y = nx + d$. (3)

$\boxed{OR}$

(D) Show that lines $ax + by + c = 0$, $bx + cy + a = 0$, $cx + ay + b = 0$ are concurrent and also find point of concurrence. $(b^2 \neq ac$, $c^2 \neq ab$, $a^2 \neq bc)$

Q-3 (A) ① Obtain standard equation of parabola. (2)

② If the focus of the parabola $y^2 = 4ax$ divides a focal chord in the ratio $1:2$ then find the equation of the line containing the focal chord. (2)

OR

② Show that the line $3y = 6x + 2$ touches the parabola $3y^2 = 16x$. Find the point of contact.

(B) ① Obtain the equation of the tangent at the point (x_1, y_1) of the ellipse and hence obtain the equation of the tangent at θ-point of the ellipse. (2)

② If the difference of the eccentric angle of P and Q points on the ellipse is $\pi/2$ and O is the origin then prove that the area of ΔPOQ is $\frac{1}{2}ab$. (2)

OR

3.

OR

(2) The tangent at the point P intersects a directrix at F. Prove that $\overline{PF}$ forms a right angle at the corresponding focus.

(C) (1) Define rectangular hyperbola. Obtain its standard equation and eccentricity. (2)

(2) Show that the angle between two asymptotes of the hyperbola $x^2-2y^2=1$ is $\tan^{-1}2\sqrt{2}$. (2)

(D) (1) If $S(4,0)$ and $e=3/2$, find the equation of a hyperbola. (1)

(2) Find the set of all points P outside a circle, such that the tangent drawn to a circle from P are perpendicular to each other. (2)

Q4 (A) Which curve is represented by the equation $3x^2+8xy-3y^2-20x+10y-15=0$. Find the co-ordinates of focii, equation of directrices and eccentricity. (4)

OR

Identify the following curves by obtaining their standard form of equation.

(i) $x^2+y^2-4x-6y-2=0$

(ii) $x^2-y^2+4x+2y+3=0$.

(B) (1) Obtain necessary and sufficient condition for two vectors $\bar{x}, \bar{y} \in R^2$ to be collinear. $(\bar{x}\neq 0, \bar{y}\neq 0)$. (2)

(2) If $\bar{x}, \bar{y}, \bar{z}$ are linearly independent then prove that $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are also linearly independent (2)

(C) (1) Obtain formula for the volume of a prism. (using vectors) (2)

(2) If A - P - B and if AP : PB = m : n then, for any point O in space prove that
$n\overrightarrow{OA}+m\overrightarrow{OB}=(m+n)\overrightarrow{OP}$. (2)

(D) (1) A boat speeds in the north at $6\sqrt{2}$ kms. A man on the boat feels that the wind is blowing from the south-east at 5 kms. Find the true velocity of the wind. (2)

(2) Find the value of a, if $(2a\bar{i}+a\bar{j}+4\bar{k})\perp(a\bar{i}-2\bar{j}-\bar{k})$ (1)

4.

Q.5 (A) ① In usual notations obtain the distance between given point and given line not containing that point in R^3. (2)

② Obtain equation of a plane passing through two intersecting lines in R^3. (2)

OR

② Obtain equation of a plane passing through two parallel lines in R^3.

(B) ① Find vector and cartesian equation of a sphere having centre $C(\bar{c})$ and radius r. (1)

② If the direction cosines l, m, n of two lines satisfy $l+m+n=0$ and $l^2+n^2=m^2$, show that the angle between the two lines is $\pi/3$. (3)

OR

② obtain shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{9-z}{3}$.

(C) ① Show that $(4, 5, 1)$, $(0, -1, -1)$, $(3, 9, 4)$, $(-4, 4, 4)$ can not be vertices of any tetrahedron. (2)

② obtain the equation, the centre and radius of the sphere through $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$. (2)

(D) obtain the equation of the plane passing through $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. (3)

OR

Obtain the intersection of the plane $x+y+2z=4$ and $2x-y+z+1=0$.

— x — x —

Solution of paper set No. 4 ①

Mathematics I (050)(E)

A.1 (A) ① Theory (Text) page No. 19.

② Here A is (2, 3) and B is (0, 7)

Suppose the point $p(x, 0)$ of the X-axis divides $\overline{AB}$ from B in the ratio $m:n$ where $m+n \neq 0$

$\therefore$ according to the y- co-ordinate,

$$y = \frac{my_2 + ny_1}{m+n} \text{ of } P,$$

$$\therefore 0 = \frac{3m+7n}{m+n}$$

$$\therefore 3m = -7n$$

$$\therefore m:n = -7:3$$

$\therefore$ The X-axis divides $\overline{AB}$ from B at point $p(x, 0)$ in the ratio $-7:3$.

(B) ① Suppose co-ordinates of point p are (0, 0) and corresponding to that co-ordinates of A, B & C are (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.

For $\triangle ABC$, $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

If determinants corresponding to $\triangle PAB$, $\triangle PBC$ and $\triangle PCA$ are D_1, D_2 and D_3 respectively, then

$$D_1 + D_2 + D_3$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$$

$$= (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3).$$

$$= x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3$$

$$= x_1(y_2 - y_3) - y_1(x_2 - x_3) + 1(x_2y_3 - x_3y_2).$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= D.$$

Thus, $D = D_1 + D_2 + D_3$.

$$\therefore |D| = |D_1 + D_2 + D_3|$$

$$\therefore |D| \leq |D_1| + |D_2| + |D_3|$$

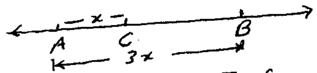
$$\therefore \frac{1}{2}|D| \leq \frac{1}{2}|D_1| + \frac{1}{2}|D_2| + \frac{1}{2}|D_3|.$$

$\therefore$ The area of $\triangle ABC \leq$ area of $\triangle PAB$ + area of $\triangle PBC$ + area of $\triangle PCA$.

2.

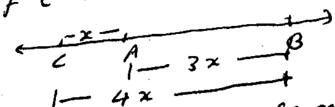
(2) Here $A(0, 1)$ and $B(2, 9)$ and suppose $C(x, y)$.
Now A, B and C are collinear and $AB = 3AC$
$\therefore$ There are two possibilities.

Case(i) If $A - C - B$ then,



$\therefore$ C divides $\overline{AB}$ from A in the ratio $\lambda = \frac{AC}{CB} = \frac{x}{2x} = \frac{1}{2}$.
Using division pt. co-ordinates
$x = \frac{mx_2 + nx_1}{m+n}$ & $y = \frac{my_2 + ny_1}{m+n}$
Co-ordinates of C are
$\left(\frac{1(2)+2(0)}{1+2}, \frac{1(9)+2(1)}{1+2}\right)$
$= \left(\frac{2}{3}, \frac{11}{3}\right)$,

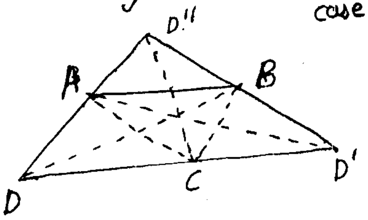
Case (ii)
If $C - A - B$ then,



$\therefore$ C divides $\overline{AB}$ from A in the ratio $\lambda = -\frac{AC}{CB} = -\frac{x}{4x}$
$\therefore \lambda = -\frac{1}{4}$
$\therefore$ Co-ordinates of C are
$\left(\frac{-1(2)+4(0)}{-1+4}, \frac{-1(9)+4(1)}{-1+4}\right)$
$= \left(-\frac{2}{3}, -\frac{5}{3}\right)$.

Thus the co-ordinates of C are $\left(\frac{2}{3}, \frac{11}{3}\right)$ or $\left(-\frac{2}{3}, -\frac{5}{3}\right)$.

(3) Suppose A is $(3, 2)$, B is $(4, 5)$ and C is $(2, 3)$.
We can get the fourth vertex of the parallelogram by three ways.



case ① $\square ABCD$ is a parallelogram and if the co-ordinates of D are (x, y) then the mid-points of the diagonals $\overline{AC}$ and $\overline{BC}$ are same. $\therefore$ $\frac{x+4}{2} = \frac{5}{2}$ and $\frac{y+5}{2} = \frac{5}{2}$
$\therefore$ $x = 1$ and $y = 0$
$\therefore$ $D(1, 0)$.

case ② $\square ABD'C$ is a parallelogram.
$\therefore$ midpt. diagonal $\overline{AD'}$ = mid pt. of diagonal $\overline{BC}$.
If co-ordinates of D' are (x', y')
$\frac{x'+3}{2} = \frac{6}{2}$ and $\frac{y'+2}{2} = \frac{8}{2}$
$\therefore$ $x' = 3$ and $y' = 6$
$\therefore$ $D'(3, 6)$.

case ③ $\square ACBD''$ is parallelogram and if co-ordinates of D'' are (x'', y'') then the mid pt. of diagonal $\overline{CD''}$ = mid pt. of diagonal $\overline{AB}$
$\therefore$ $\frac{x''+2}{2} = \frac{7}{2}$ and $\frac{y''+3}{2} = \frac{7}{2}$
$\therefore$ $x'' = 5$ and $y'' = 4$.
$\therefore$ we get $D''(5, 4)$.

$\therefore$ Thus, the fourth vertex of the given parallelogram is $(1, 0)$ or $(3, 6)$ or $(5, 4)$.

3.

(C) (1) For $A(3, 2)$, $B(5, 6)$

Parametric equation of $\overleftrightarrow{AB}$:-

$x = tx_2 + (1-t)x_1$, $y = ty_2 + (1-t)y_1$

$= 5t + (1-t)3$, $= 6t + (1-t)2$

$\therefore x = 2t + 3$ $\therefore y = 4t + 2$, $t \in R$.

$\therefore 3x + 4y = 6t + 9 + 16t + 8$

$\therefore 3x + 4y = 22t + 17$

But $P(x, y) \in \overline{AB}$

$\therefore 0 \le t \le 1$

$\therefore 0 \le 22t \le 22.$

$\therefore 17 \le 22t + 17 \le 39$

$\therefore 17 \le 3x + 4y \le 39.$

(2) Here, the slope of the line $3x + 4y - 2 = 0$ is $m_1 = -\frac{3}{4}$

Suppose, the slope of the required line is m_2.

Also, the measure of the angle between these two lines is $\alpha = 45°$

Now by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\tan 45° = \left|\frac{-3/4 - m_2}{1 + (-3/4)m_2}\right|$$

$$\therefore 1 = \left|\frac{-3 - 4m_2}{4 - 3m_2}\right|$$

$\therefore -3 - 4m_2 = 4 - 3m_2$ or $-3 - 4m_2 = -4 + 3m_2$

$\therefore m_2 = -7$ or $m_2 = \frac{1}{7}$

$\therefore$ two lines are possible.

Now by slope point equation of a line

$y - y_1 = m(x - x_1)$

$y - 4 = -7(x - 3)$ or $y - 4 = \frac{1}{7}(x - 3)$

$\therefore 7x + y - 25 = 0$ $\therefore x - 7y + 25 = 0.$

Thus, the required lines are $x - 7y + 25 = 0$ and $7x + y - 25 = 0$.

(3) Suppose A is $(3, 4)$ & B is $(-2, 1)$

Suppose $P(x, y) \in l : 3x - y + 6 = 0$ divides $\overline{AB}$ from A in the ratio λ. $(\lambda \ne -1, 0)$

By division point co-ordinates of a division point of line segment co-ordinates of point P are $\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right) = \left(\frac{-2\lambda + 3}{\lambda + 1}, \frac{\lambda + 4}{\lambda + 1}\right) \in l$.

$\therefore 3\left(\frac{-2\lambda + 3}{\lambda + 1}\right) - \left(\frac{\lambda + 4}{\lambda + 1}\right) + 6 = 0$

$\therefore -6\lambda + 9 - \lambda - 4 + 6\lambda + 6 = 0$

$\therefore \lambda = 11 > 0$

$\therefore$ we get $A - P - B$. $\therefore$ The points $A(3, 4)$ and $B(-2, 1)$ are on the opposite side of the line $3x - y + 6 = 0$.

5

(C) (1) Equation of a circle whose diametrically opposite (end) points are $(-2, 9)$ and $(2, 1)$ is

$(x+2)(x-2) + (y-9)(y-1) = 0$

$\therefore x^2+y^2-10y+5=0. \leftarrow (1)$

The equation of the line passing through points $(-2, 9)$ and $(2, 1)$ is

$$y-9 = \frac{1-9}{2+2}(x+2).$$

$\therefore 2x+y-5=0 \leftarrow (2)$

The general equation of the circle passing through the point of intersections of (1) and (2) is

$x^2+y^2-10y+5+\lambda(2x+y-5)=0 \leftarrow (3)$

If this circle is passes through $(5, -8)$

$(25+64+80+5)+\lambda(10-8-5)=0$

$\therefore 174 - 3\lambda = 0$

$\therefore \lambda = +58.$

Now substituting this value of λ in (3)

Equation of desired circle is

$(x^2+y^2-10y+5) \cancel{+} 58(2x+y-5)=0$

$\therefore x^2+y^2+116x+48y-285=0$

$\boxed{OR}$

Here $P(5, 3) \notin S: x^2+y^2=17$ $(\because 25+9=34 \neq 17)$.

From equation of circle $x^2+y^2=r^2$ given circle have centre $O(0,0)$ and radius $r=\sqrt{17}$

If line $y=mx \pm r\sqrt{1+m^2}$ is tangent to circle passes through $(5, 3)$ then,

$3 = 5m \pm \sqrt{17}\sqrt{1+m^2}.$

$\therefore (3-5m)^2 = 17(1+m^2)$

$\therefore 4m^2-15m-4=0$

$\therefore (4m+1)(m-4)=0$

$\therefore m = -\frac{1}{4}$ or $m=4$

Now by a slope point equation of a line equation of required tangents to circle are

$y-y_1 = m(x-x_1)$ | $y-3=4(x-5)$

$\therefore y-3 = -\frac{1}{4}(x-5)$ | $\therefore 4x-y-17=0$

$\therefore x+4y-17=0$

Thus the equations of the tangents to the given circle from the given point are $4x-y-17=0$ and $x+4y-17=0$.

(4)

A.1.D. Theory (Text) page No. 36.

A.2.A ① Theory (Text) page No. 63.

② Comparing the equation $x^2 - 2xy\sec\alpha + y^2 = 0$ with the homogeneous quadratic equation $ax^2 + 2hxy + by^2 = 0$, $a = 1$, $h = -\sec\alpha$, $b = 1$.

Here $h^2 - ab = \sec^2\alpha - 1 = \tan^2\alpha > 0$ $(\because 0 < \alpha < \pi/2)$.

$\therefore$ Given equation represents two distinct lines passes through $O(0,0)$.

If the measure of angle between the lines is θ then according to,

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|} = \frac{2\sqrt{\sec^2\alpha - 1}}{2} = \tan\alpha. \ (\because 0 < \alpha < \pi/2)$$

$\therefore \theta = \alpha$.

Thus, the measure of the required angle is α unit.

(B) ① Text page. 76.

② Suppose lines $l_1 : 2x - 3y + 5 = 0$ ...(1), $l_2 : 3x + 2y + 7 = 0$ ...(2)

Slope of line l_1, $m_1 = \frac{2}{3}$

" " " l_2, $m_2 = -3/2$

$\therefore m_1 \cdot m_2 = -1$.

$\therefore l_1 \perp l_2$.

Solving equations (1) and (2) we get orthocentre H of the triangle.

$$H(x, y) = \left(\frac{-21-10}{4+9}, \frac{15-14}{4+9}\right)$$

$$\therefore H(x, y) = \left(-\frac{31}{13}, \frac{1}{13}\right).$$

③ Since given equation $px^2 + 3y^2 + (q-3)xy + 2px + 3qy - 3 = 0$ represents a circle then

Co-efficient of $xy = 0$

i.e. $(q - 3) = 0$

$\therefore q = 3$

& Co-efficient of x^2 = Co-efficient of y^2

$\therefore p = 3$.

$\therefore$ Equation of the circle is,

$3x^2 + 3y^2 + 6x + 9y - 3 = 0$

$\therefore x^2 + y^2 + 2x + 3y - 1 = 0$

$\therefore g = 1, \ f = \frac{3}{2}, \ c = -1$

$\therefore$ Centre $C(-g, -f) = (-1, -\frac{3}{2})$

& radius $= r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1 + 9/4 + 1}$

$\therefore r = \frac{\sqrt{17}}{2}$ units.

6

(2) Here for given line : $\frac{x}{a-\lambda} + \frac{y}{b} = 1$

$$\therefore \frac{y}{b} - 1 = -\frac{x}{a-\lambda}$$

$$\therefore y - b = -\frac{b}{a-\lambda}(x - 0).$$

Comparing with $y - y_1 = m(x - x_1)$.

$m = -\frac{b}{a-\lambda}$ and fixed point on the given line is $(x_1, y_1) = (0, b)$.

(D) Here solving $y = mx + a$ and $y = nx + c$ also $y = mx + a$ and $y = nx + d$ we get

$$A\left(\frac{c-a}{m-n}, \frac{mc-na}{m-n}\right) \text{ and } B\left(\frac{d-a}{m-n}, \frac{md-na}{m-n}\right)$$

Now $AB^2 = \left(\frac{c-a}{m-n} - \frac{d-a}{m-n}\right)^2 + \left(\frac{mc-na}{m-n} - \frac{md-na}{m-n}\right)^2.$

$$= \left(\frac{c-d}{m-n}\right)^2 + m^2\left(\frac{c-d}{m-n}\right)^2$$

$$= \left(\frac{c-d}{m-n}\right)^2(1 + m^2).$$

$$\therefore AB = \left|\frac{c-d}{m-n}\right|\sqrt{1+m^2}.$$

Also, the perpendicular distance between $\overline{AB}$ and $\overline{CD}$ is $P_1 = \frac{|a-b|}{\sqrt{1+m^2}}$.

Now the area of the parallelogram

$$= AB \cdot P_1$$

$$= \left|\frac{c-d}{m-n}\right|\sqrt{1+m^2} \cdot \frac{|a-b|}{\sqrt{1+m^2}}$$

$$= \left|\frac{(a-b)(c-d)}{m-n}\right|.$$

<u>OR</u>

Here the lines are $ax + by + c = 0$ —(1)

$bx + cy + a = 0$ —(2)

and $cx + ay + b = 0$ —(3) and $b^2 \neq ac$, $c^2 \neq ab$

and $a^2 \neq bc$.

$$\left.\begin{aligned} \text{Now } a_1b_2 - a_2b_1 &= ac - b^2 \neq 0 \\ a_2b_3 - a_3b_2 &= ab - c^2 \neq 0 \\ a_1b_3 - a_3b_1 &= a^2 - bc \neq 0 \end{aligned}\right\} \longleftarrow (I)$$

and $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$(\because a+b+c=0)$

$\therefore$ Given lines are concurrent.

Now $a + b + c = 0$ then their point of concurrence is $(1, 1)$.

7

Q3. A (1) Theory Text. page No. 87

(2) Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are the end points of focal chord of parabola $y^2 = 4ax$. where $S(a, 0)$ divides $\overline{PQ}$ from P in ratio 2:1.

$\therefore$ according to $y = \frac{\lambda y_2 + y_1}{\lambda + 1}$, y - co-ordinates of S is $0 = \frac{2(-\frac{2a}{t_1}) + 2at_1}{2+1}$. $(\because t_1 t_2 = -1)$.

$\therefore \frac{4a}{t_1} = 2at_1$

$\therefore t_1 = \pm\sqrt{2} \quad \therefore t_2 = \mp\frac{1}{\sqrt{2}}$.

(i) For $t_1 = \sqrt{2}$, $P(at_1^2, 2at_1) = (2a, 2\sqrt{2}a)$

Equation of ~~foc~~ line $\overleftrightarrow{PQ}$ is

$$\begin{vmatrix} x & y & 1 \\ a & 0 & 1 \\ 2a & 2\sqrt{2}a & 1 \end{vmatrix} = 0 \quad (\because S(a,0) \in \overleftrightarrow{QP}).$$

$\therefore -2\sqrt{2}ax + ay + 2\sqrt{2}a^2 = 0$

$\therefore y = 2\sqrt{2}(x-a)$.

(ii) For $t_1 = -\sqrt{2}$, $P(2a, -2\sqrt{2}a)$.

$\therefore$ second equation of the line containing the focal - chord is $y = -2\sqrt{2}(x-a)$.

Thus, the two equation of the focal chord are $y = \pm 2\sqrt{2}(x-a)$.

OR

Here, line $3y = 6x + 2$

$\therefore y = 2x + \frac{2}{3}$.

Comparing with $y = mx + c$

$m = 2, \ c = \frac{2}{3}$

parabola $3y^2 = 16x \Rightarrow y^2 = \frac{16}{3}x \Rightarrow a = \frac{4}{3}$.

$c = \frac{2}{3}$ and $\frac{a}{m} = \frac{4/3}{2} = \frac{2}{3}$

$\therefore c = \frac{a}{m}$

$\therefore$ Given line touches the given parabola point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{1}{3}, \frac{4}{3}\right)$.

(B) (1) Text page No. 103

(2) Here the difference of the eccentric angles (8)
of ~~p~~ points on the ellipse is $\frac{\pi}{2}$.
we will take $P(\theta) = (a\cos\theta, b\sin\theta)$
and $Q(\frac{\pi}{2}+\theta) = (a\cos(\frac{\pi}{2}+\theta), b\sin(\pi/2+\theta))$
$= (-a\sin\theta, b\cos\theta)$

Also $O(0,0)$ is the centre.
The vertices of the ΔOPQ are $O(0,0)$
$P(a\cos\theta, b\sin\theta)$, $Q(-a\sin\theta, b\cos\theta)$.

$$\therefore D = \begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \end{vmatrix}$$
$$= ab(\cos^2\theta + \sin^2\theta)$$
$$= ab.$$

$\therefore$ The area of $\Delta OPQ = \Delta = \frac{1}{2}|D|$
$= \frac{1}{2}ab.$ $(a>0, b>0)$

OR

The tangent $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ at the point $P(a\cos\theta, b\sin\theta)$
to the ellipse intersects the directrix $x = \frac{a}{e}$
of the ellipse at the point $F(\frac{a}{e}, k)$.

$$\therefore \frac{a}{e}\cdot\frac{\cos\theta}{a} + \frac{k}{b}\sin\theta = 1.$$
$$k = \frac{b(e-\cos\theta)}{e\sin\theta}.$$
$$\therefore F\left(\frac{a}{e}, \frac{b(e-\cos\theta)}{e\sin\theta}\right) \;\&\; S(ae, 0).$$

Now the slope of $\overleftrightarrow{SF}$ $\times$ slope of $\overleftrightarrow{SP}$

$$= \left\{\frac{\frac{b(e-\cos\theta)}{e\sin\theta} - 0}{\frac{a}{e} - ae}\right\} \times \left\{\frac{b\sin\theta - 0}{a\cos\theta - ae}\right\}$$
$$= \frac{b(e-\cos\theta)}{a(1-e^2)\sin\theta} \times \frac{b\sin\theta}{-a(e-\cos\theta)}$$
$$= -\frac{b^2}{a^2(1-e^2)}$$
$$= -\frac{b^2}{b^2} \quad (\because \text{For } a>b,\ b^2 = a^2(1-e^2))$$
$$= -1. \quad \therefore \overleftrightarrow{SF} \perp \overleftrightarrow{SP}$$

$\therefore \overline{PF}$ subtends a right angle at the focus S.

9

C (1) Text page 121.

(2) Here equation of the asymptotes $x^2-2y^2=0$ of the hyperbola $x^2-2y^2=1$.

If the angle between them is θ, then

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|}.$$

As $a=1$, $h=0$, $b=-2$.

$$\therefore \tan\theta = \frac{2\sqrt{0-(-2)}}{|1-2|} = 2\sqrt{2}.$$

$$\therefore \theta = \tan^{-1}2\sqrt{2}.$$

(D) ① Here focus $S(4,0)=(ae,0)$, $e=\frac{3}{2}$

$$\therefore ae = 4$$

$$\therefore a\cdot\frac{3}{2} = 4$$

$$\therefore a = \frac{8}{3} \quad \therefore a^2 = \frac{64}{9}$$

$$b^2 = -a^2(1-e^2) = a^2(e^2-1)$$

$$= -\frac{64}{9}\left(1-\frac{9}{4}\right) = \frac{64}{9}\left(\frac{9}{4}-1\right)$$

$$b^2 = \frac{80}{9}.$$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\therefore \frac{9x^2}{64} - \frac{9y^2}{80} = 1.$$

② Suppose $y = mx \pm a\sqrt{1+m^2}$ are the tangents to the circle $x^2+y^2=a^2$. which are passing through a point $P(x_1, y_1)$ out side the circle.

$$\therefore y_1 = mx_1 \pm a\sqrt{1+m^2}$$

$$\therefore (y_1 - mx_1)^2 = a^2(1+m^2).$$

$$\therefore (a^2-x_1^2)m^2 + 2x_1y_1m + (a^2-y_1^2) = 0.$$

If m_1 and m_2 are the roots of this quadratic equation in m, then $m_1 \cdot m_2 = \dfrac{a^2-y_1^2}{a^2-x_1^2}$. —①

Now tangents through P are perpendicular to each other taking $m_1 \cdot m_2 = -1$ in ①

$$\therefore \frac{a^2-y_1^2}{a^2-x_1^2} = -1 \quad \therefore a^2-y_1^2 = -a^2+x_1^2$$

$$\therefore x_1^2 + y_1^2 = 2a^2.$$

Thus, the locus of P is the concentric circle $x^2+y^2=2a^2$ with the radius $\sqrt{2}a$.

(10)

A-4(A) Here, $a \neq b$, $2h = 8$

$\therefore \tan 2\theta = \frac{2h}{a-b} = \frac{8}{6} = \frac{4}{3}$.

$\therefore$ On rotating the axes by an angle θ

where $\tan 2\theta = \frac{4}{3}$.

$\therefore \cos 2\theta = \frac{3}{5}$.

$\therefore \cos\theta = \sqrt{\frac{1+3/5}{2}} = \frac{2}{\sqrt{5}}$, $\sin\theta = \frac{1}{\sqrt{5}}$.

Here $x = x'\cos\theta - y'\sin\theta = \frac{2x'-y'}{\sqrt{5}}$

$y = x'\sin\theta + y'\cos\theta = \frac{x'+2y'}{\sqrt{5}}$

$\therefore$ Equation of curve in (x', y') co-ordinates system is.

$$3\left(\frac{2x'-y'}{\sqrt{5}}\right)^2 + 8\left(\frac{2x'-y'}{\sqrt{5}}\right)\left(\frac{x'+2y'}{\sqrt{5}}\right) - 3\left(\frac{x'+2y'}{\sqrt{5}}\right)^2 - 20\left(\frac{2x'-y'}{\sqrt{5}}\right) + 10\left(\frac{x'+2y'}{\sqrt{5}}\right) - 15 = 0.$$

$$\therefore \frac{1}{5}(12+16-3)x'^2 + \frac{1}{5}(3-16-12)y'^2 + \frac{1}{\sqrt{5}}(-40+10)x' + \frac{1}{\sqrt{5}}(20+20)y' - 15 = 0$$

$$\therefore 5x'^2 - 5y'^2 - 6\sqrt{5}x' + 8\sqrt{5}y' - 15 = 0$$

$$\therefore 5\left(x'^2 - \frac{6x'}{\sqrt{5}} + \frac{9}{5}\right) - 5\left(y'^2 - \frac{8y'}{\sqrt{5}} + \frac{16}{5}\right) = 8$$

$$\therefore \left(x' - \frac{3}{\sqrt{5}}\right)^2 - \left(y' - \frac{4}{\sqrt{5}}\right)^2 = \frac{8}{5}.$$

Now on shifting the origin to $\left(\frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$,

New co-ordinates are (X, Y) then

$x' - \frac{3}{\sqrt{5}} = X$ and $y' - \frac{4}{\sqrt{5}} = Y$.

$\therefore$ The equation of the curve

$$X^2 - Y^2 = \frac{8}{5} = \left(\frac{2\sqrt{2}}{\sqrt{5}}\right)^2 = a^2$$

$\therefore$ The given represents the rectangular hyperbola.

$\therefore a = 2\sqrt{\frac{2}{5}}$ & $e = \sqrt{2}$

In (X, Y) system

Focii : $(\pm ae, 0)$
$= \left(\pm \frac{4}{\sqrt{5}}, 0\right)$.

Directrices:-
$X = \pm \frac{a}{e}$
$X = \pm \frac{2}{\sqrt{5}}$.

Length of Transverse and conjugate axes
$= 2a = 4\sqrt{\frac{2}{5}}$ units.

In (x', y') System

Focii :- (x', y')
$= \left(X + \frac{3}{\sqrt{5}}, Y + \frac{4}{\sqrt{5}}\right)$
$= \left(\pm\frac{4}{\sqrt{5}} + \frac{3}{\sqrt{5}}, 0 + \frac{4}{\sqrt{5}}\right)$.
$= \left(\frac{7}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$ and $\left(-\frac{1}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$.

Directrices: -
$X = \pm\frac{2}{\sqrt{5}}$
$\therefore x' - \frac{3}{\sqrt{5}} = \pm\frac{2}{\sqrt{5}}$
$\therefore x' = \sqrt{5}$ & $x' = \frac{1}{\sqrt{5}}$

In (x, y) (i.e. original) system. :

Focii : (x, y)
$= \left(\frac{2x'-y'}{\sqrt{5}}, \frac{x'+2y'}{\sqrt{5}}\right)$
$= \left(\frac{2\left(\frac{7}{\sqrt{5}}\right) - \frac{4}{\sqrt{5}}}{\sqrt{5}}, \frac{\frac{7}{\sqrt{5}} + \frac{4}{\sqrt{5}}\times 2}{\sqrt{5}}\right)$
and $\left(\frac{2\left(-\frac{1}{\sqrt{5}}\right) - \frac{4}{\sqrt{5}}}{\sqrt{5}}, \frac{-\frac{1}{\sqrt{5}} + 2\left(\frac{4}{\sqrt{5}}\right)}{\sqrt{5}}\right)$
$= (2, 3)$ and $\left(-\frac{6}{5}, \frac{7}{5}\right)$.

As $x' = x\cos\theta + y\sin\theta = \frac{2x+y}{\sqrt{5}}$
Equation of directrices,
$x' = \sqrt{5}$ and $x' = \frac{1}{\sqrt{5}}$
$2x + y = 5$ and $2x + y = 1$.
$\therefore 2x + y - 5 = 0$
and $2x + y - 1 = 0$.

OR

(11)

(i) Here $a = b \Rightarrow \tan 2\theta = \infty \Rightarrow \theta = \pi/4$.

$\therefore$ On rotating the axes by $\theta = \pi/4$,

$x = \frac{x'-y'}{\sqrt{2}}$ and $y = \frac{x'+y'}{\sqrt{2}}$

$\therefore$ The equation of the curve

$\left(\frac{x'-y'}{\sqrt{2}}\right)^2 + \left(\frac{x'+y'}{\sqrt{2}}\right)^2 - 4\left(\frac{x'-y'}{\sqrt{2}}\right) - 6\left(\frac{x'+y'}{\sqrt{2}}\right) - 2 = 0$.

$\therefore x'^2 + y'^2 - 5\sqrt{2}x' - \sqrt{2}y' - 2 = 0$

$\therefore \cancel{x'^2 - 5\sqrt{2}x' + \frac{25}{2} + y'^2 - \sqrt{2}y' + \frac{1}{\sqrt{2}} = 10}$

$\therefore x'^2 - 5\sqrt{2}x' + \frac{25}{2} + y'^2 - \sqrt{2}y' + \frac{1}{2} = 15$.

$\therefore \left(x' - \frac{5}{\sqrt{2}}\right)^2 + \left(y' - \frac{1}{\sqrt{2}}\right)^2 = 15$.

Now, on shifting the origin to $\left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$x' - \frac{5}{\sqrt{2}} = X$, $y' - \frac{1}{\sqrt{2}} = Y$.

$\therefore$ The equation of the curve is

$X^2 + Y^2 = (\sqrt{15})^2$.

Which represents a circle.

centre $C(0, 0)$ & radius $r = \sqrt{15}$

In (x', y') system centre (x', y')

$= \left(X + \frac{5}{\sqrt{2}}, Y + \frac{1}{\sqrt{2}}\right)$

$= \left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

In (x, y) system centre $= (x, y)$

$= \left(\frac{x'-y'}{\sqrt{2}}, \frac{x'+y'}{\sqrt{2}}\right)$

$= \left(\frac{\frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\sqrt{2}}, \frac{\frac{5}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\sqrt{2}}\right)$

$= (2, 3)$.

(12) Here $a \neq b$ and $2h = 0$

$\tan 2\theta = 0$

$\therefore \theta = 0$

Hence, there is no need to rotate the axes. Here, the equation $x^2 + 4x + 4 - y^2 + 2y - 1 = 0$

$\therefore (x+2)^2 - (y-1)^2 = 0$

Now on shifting the origin $(-2, 1)$.

$x + 2 = x'$ and $y - 1 = y'$.

$\therefore$ The equation of the curve is $x'^2 - y'^2 = 0$.

i.e. $(x'+y')(x'-y') = 0$ which represents a pair of lines.

Here we get the lines $x' + y' = 0$ and $x' - y' = 0$

Now original lines are $x + y + 1 = 0$ and $x - y + 3 = 0$.

12.

B (1) Text page 154. Thm.5.

(2) Suppose $\bar{x} = (x_1, x_2, x_3)$, $\bar{y} = (y_1, y_2, y_3)$ and $\bar{z} = (z_1, z_2, z_3) \in R^3$

Here, we want to show that $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are linearly independent i.e. non-coplanes, ~~is given~~

Also $\bar{x}$, $\bar{y}$ & $\bar{z}$ are linearly independent non null vectors then $\bar{x}\cdot(\bar{y}\times\bar{z}) = 0$. —(1)

$\therefore [\bar{x}\ \bar{y}\ \bar{z}] = 0$.

Now $(\bar{x}+\bar{y})\cdot[(\bar{y}+\bar{z})\times(\bar{z}\times\bar{x})]$.

$= (\bar{x}+\bar{y})\cdot[\bar{y}\times\bar{z} + \bar{y}\times\bar{x} + \theta + \bar{z}\times\bar{x}]$.

$= (\bar{x}+\bar{y})\cdot[\bar{y}\times\bar{z} + \bar{y}\times\bar{x} + \bar{z}\times\bar{x}]$

$= (\bar{x}+\bar{y})\cdot[\bar{y}\times\bar{z} + \bar{y}\times\bar{x} + \bar{z}\times\bar{x}]$

$= \bar{x}\cdot(\bar{y}\times\bar{z}) + \bar{x}\cdot(\bar{y}\times\bar{x}) + \bar{x}\cdot(\bar{z}\times\bar{x}) + \bar{y}\cdot(\bar{y}\times\bar{z}) + \bar{y}\cdot(\bar{y}\times\bar{x}) + \bar{y}\cdot(\bar{z}\times\bar{x})$.

$= [\bar{x}\ \bar{y}\ \bar{z}] + [\bar{x}\ \bar{y}\ \bar{x}] + [\bar{x}\ \bar{z}\ \bar{x}] + [\bar{y}\ \bar{y}\ \bar{z}] + [\bar{y}\ \bar{y}\ \bar{x}] + [\bar{y}\ \bar{z}\ \bar{x}]$.

$= [\bar{x}\ \bar{y}\ \bar{z}] + 0 + 0 + 0 + 0 + [\bar{y}\ \bar{z}\ \bar{x}]$.

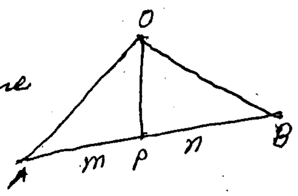
$= [\bar{x}\ \bar{y}\ \bar{z}] + [\bar{x}\ \bar{y}\ \bar{z}]$

$= 2[\bar{x}\ \bar{y}\ \bar{z}]$.

$\neq 0$. (By 1)

$\therefore$ The vectors $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are linearly independent vectors.

(C) (1) Text page No. 175.



(2) Here the direction of vectors $\vec{AP}$ and $\vec{PB}$ are same and $\frac{AP}{PB} = \frac{m}{n}$. Hence $n\vec{AP} = m\vec{PB}$

$\therefore n(\vec{OP} - \vec{OA}) = m(\vec{OB} - \vec{OP})$

$\therefore (m+n)\ \vec{OP} = n(\vec{OA}) + m(\vec{OB})$.

(D) (1) Here the velocity of the boat is

$\bar{u} = 0\cdot\bar{i} + 6\sqrt{2}\bar{j}$

$\therefore \bar{u} = \frac{12}{\sqrt{2}}\bar{j}$

Suppose the true velocity of the wind is $\bar{v}$.

The wind blows from the south-east.

i.e. it seems to go in the direction North-west and the velocity of the wind relative to the boat is $\bar{v} - \bar{u} = 5\cos 135°\bar{i} + 5\sin 135\bar{j}$

$\therefore \bar{v} - \bar{u} = -\frac{5}{\sqrt{2}}\bar{i} - \frac{5}{\sqrt{2}}\bar{j}$.

Now the true velocity of wind $\bar{v} = (\bar{v} - \bar{u}) + \bar{u}$

$\therefore \bar{v} = -\frac{5}{\sqrt{2}}\bar{i} + \frac{17}{\sqrt{2}}\bar{j}$. Now $|\bar{v}| = \sqrt{\frac{25}{2} + \frac{289}{2}} = \sqrt{157}$ units

and if $\bar{v}$ makes an angle θ with $\vec{OX}$, then

$\cos\theta = \frac{\bar{v}\cdot\bar{i}}{|\bar{v}||\bar{i}|} = -\frac{5}{\sqrt{2}\sqrt{157}\times 1} = -\frac{5}{\sqrt{314}}$

$\therefore \theta = \pi - \cos^{-1}\frac{5}{\sqrt{314}}$ with the East towards to the North.

13

(2) Here $(2a, a, 4) \perp (a, -2, -1)$

$\therefore (2a, a, 4) \cdot (a, -2, -1) = 0$

$\therefore 2a^2 - 2a - 4 = 0$

$\therefore a^2 - a - 2 = 0$

$\therefore (a-2)(a+1) = 0$

$\therefore a = 2$ or $a = -1$.

A.5.(A) (1) Text page No. 191.

(2) Text page No. 204.

OR

(2) Text page No. 203.

(B) (1) Text page 214.

(2) Here $l + m + n = 0$ — (1)

$l^2 - m^2 + n^2 = 0$ — (2)

By (1), $m = -(l+n)$ substituting in (2).

$l^2 - (l+n)^2 + n^2 = 0$

$\therefore -2ln = 0$

$\therefore l = 0$ or $n = 0$

(i) If $l = 0$ then from (1), $m = -n$

so we get direction ratio of the first diagonal vector $0, -n$ and n.

(ii) If $n = 0$ then from the equation (2) $m = -l$ and so direction ratio of the second diagonal vector is $l, -l, 0$.

$\therefore$ For the measure of angle θ between two diagonal vectors is,

$$\cos\theta = \frac{(0, -n, n) \cdot (l, -l, 0)}{\sqrt{2n^2} \cdot \sqrt{2l^2}}$$

$$= \frac{0 + nl + 0}{2 \cdot nl}$$

$\therefore \cos\theta = \frac{1}{2}$ $\quad (\because 0 < \theta < \pi/2)$

$\therefore \theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

OR

Here vector forms of given lines in R^3 is $\bar{r} = (3, -15, 9) + k(2, -7, 5)$ and $\bar{r} = (-1, 1, 9) + k(2, 1, -3)$ $k \in R$ comparing with the vector equations $\bar{r} = \bar{a} + k\bar{l}$, $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$, we get $\bar{a} = (3, -15, 9)$, $\bar{b} = (-1, 1, 9)$ $\bar{l} = (2, -7, 5)$ and $\bar{m} = (2, 1, -3)$.

$\therefore \bar{l} \times \bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = \bar{i}(21-5) - \bar{j}(-6-10) + \bar{k}(2+14)$. 14.

$= (16, 16, 16)$.

$\therefore |\bar{l} \times \bar{m}| = \sqrt{(16)^2+(16)^2+(16)^2} = 16\sqrt{3}$.

Now $\bar{u} = \dfrac{\bar{l} \times \bar{m}}{|\bar{l} \times \bar{m}|} = \dfrac{1}{\sqrt{3}}(1, 1, 1)$.

and $\bar{a} - \bar{b} = (3, -15, 9) - (-1, 1, 9)$

$= (4, -16, 0)$.

$\therefore$ The perpendicular distance between two lines is $|(\bar{a}-\bar{b}) \cdot \bar{u}| = |(4, -16, 0) \cdot \frac{1}{\sqrt{3}}(1, 1, 1)|$

$= \frac{1}{\sqrt{3}} |4 - 16 - 0|$

$= \frac{12}{\sqrt{3}}$

$= 4\sqrt{3}$

$\therefore$ Thus, the shortest distance between two lines is $4\sqrt{3}$ units.

(c) ① Here suppose $V(4, 5, 1)$, $A(0, -1, -1)$, $B(3, 9, 4)$ and $C(-4, 4, 4)$.

$\therefore \vec{VA} = (-4, -6, -2)$, $\vec{VB} = (-1, 4, 3)$ and $\vec{VC} = (-8, -1, 3)$.

Now $\vec{VA} \cdot (\vec{VB} \times \vec{VC}) = [\vec{VA}\ \vec{VB}\ \vec{VC}]$

$= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66$.

$= 0$

$\therefore \vec{VA}$, $\vec{VB}$ and $\vec{VC}$ are collinear. are coplaner.

$\therefore$ The points V, A, B, C are coplaner.

$\therefore$ They cannot be the vertices of any tetrahedron.

② Suppose equation of desired sphere through $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

$S: x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ — ①

As $(0, 0, 0) \in S$ $\therefore$ $d = 0$

$(a, 0, 0) \in S$ $\therefore$ $a^2 + 2ua = 0 \Rightarrow u = -\frac{a}{2}$

$(0, b, 0) \in S$ $\therefore$ $b^2 + 2vb = 0 \Rightarrow v = -\frac{b}{2}$

$(0, 0, c) \in S$ $\therefore$ $c^2 + 2wc = 0 \Rightarrow w = -\frac{c}{2}$.

$\therefore$ The equation of the sphere will be

$x^2 + y^2 + z^2 - ax - by - cz = 0$

$\therefore$ Centre $(-u, -v, -w) = (a/2, b/2, c/2)$

& radius $r = \sqrt{u^2+v^2+w^2-d} = \frac{1}{2}\sqrt{a^2+b^2+c^2}$.

15.

(D) (D) Here from equation of given lines

$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

Comparing with $\bar{r} = \bar{a} + k\bar{l}$, $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$

$\bar{a} = (4, -3, -1)$, $\bar{b} = (1, -1, -10)$, $\bar{l} = (1, -4, 7)$

and $\bar{m} = (2, -3, 8)$.

Now $\bar{l} \times \bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = \bar{i}(-11) - \bar{j}(-6) + \bar{k}(5)$

$\bar{l} \times \bar{m} = (-11, +6, 5)$

& $\bar{b} - \bar{a} = (1, -1, -10) - (4, -3, -1)$

$= (-3, 2, -9)$

$\therefore (\bar{b} - \bar{a}) \cdot (\bar{l} \times \bar{m}) = (-3, 2, -9) \cdot (-11, 6, 5)$

$= 33 + 12 - 45$

$= 0.$

$\therefore$ Both the lines are intersecting lines.

Now equation of plane passing through such lines according to $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ is

$\begin{vmatrix} x-4 & y+3 & z+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = 0$

$\therefore (x-4)(-32+21) - (y+3)(8-14) + (z+1)(-3+8) = 0$

$\therefore -11x + 44 + 6y + 18 + 5z + 5 = 0$

$\therefore 11x - 6y - 5z = 67.$

$\boxed{OR}$

Here the planes are $x + y + 2z = 4$ and $2x - y + z = -1$

i.e. $(x, y, z) \cdot (1, 1, 2) = 4$

& $(x, y, z) \cdot (2, -1, 1) = -1$

$\therefore$ Comparing with $\bar{r} \cdot \bar{n} = d$ their normal vectors are $\bar{n}_1 = (1, 1, 2)$, $\bar{n}_2 = (2, -1, 1)$.

Now $\bar{n} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 3\bar{i} + 3\bar{j} - 3\bar{k}$

$\therefore \bar{n} = (3, 3, -3)$.

Now for a common point of intersecting plane taking $z = 0$. $x + y = 4$ and $2x - y = -1$

Solving these equations $x = 1$ and $y = 3$

Thus, one common point of intersecting planes is $\bar{a}(1, 3, 0)$. Also the equation of the line of both the intersecting planes is $\bar{r} = \bar{a} + k\bar{n}$, $k \in R$

$\bar{r} = (1, 3, 0) + k'(3, 3, -3)$; $k' \in R$

$\therefore \bar{r} = (1, 3, 0) + k(1, 1, -1)$. where $k = 3k' \in R$.

Thus, the equation of the intersecting line of the planes is $\bar{r} = (1, 3, 0) + k(1, 1, -1)$, $k \in R$.

Question Paper set: 5
Maths - I

Sub: Maths - I (050E)
Std: XII

Marks: 75] [Time: 3hrs.

- Instructions:
  1) There are five questions. All are compulsory. Each question carry equal marks.
  2) The digits on R.H.S. in a bracket, indicates the marks of that particular question.
  3) Must show the calculations for the objectives.

Que: 1 (A) (1) By using division of line segment, obtain the necessary and sufficient condition for three distinct points of R^2 to be collinear. (03)

(2) For which value of 'a' would the points (0,0), (0,1) and (a,1) be the vertices of a right angled triangle. (01)

(B) Attempt (any 2) (04)

(1) Prove that the two lines joining the mid-pts. of the pairs of opposite sides and the line joining the mid-point of the diagonals of a quadrilateral are concurrent.

(2) If $A(x_1, x_1 \tan\theta_1)$, $B(x_2, x_2\tan\theta_2)$, $C(x_3, x_3\tan\theta_3)$ are the vertices of a $\triangle ABC$ & the circumcentre & centroid of $\triangle ABC$ are $(0,0)$ & (x,y) resp. then by accepting $\frac{x}{y} = \frac{\cos\theta_1+\cos\theta_2+\cos\theta_3}{\sin\theta_1+\sin\theta_2+\sin\theta_3}$ prove that $\frac{x}{y} = \frac{1+4\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2}}{4\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\cos\frac{\theta_3}{2}}$

where $\theta_1+\theta_2+\theta_3 = \pi$. $(0<\theta_1,\theta_2,\theta_3<\frac{\pi}{2})$

(3) Find the points which divide the line segment joining (1,2) & (2a, b) into n equal parts.

(C) Attempt (any 2) (04)

(1) If the sum of the intercepts on the axes of a line is constant, find the equation satisfied by mid-point of the segment of the line intercepted between the axes.

[P.T.O.

(2) $A(2,3)$, $B(5,-1)$ are the given points. If $(x,y) \in \overline{AB}$ then prove that $4 \le x+y \le 5$.

(3) An adjacent pair of vertices of a square is $(-1,3)$ & $(2,-1)$. Find the remaining vertices.

(D) Prove that graph of $S = \{(x,y) \mid ax+by+c=0;\ a^2+b^2 \neq 0\}$ represents a line (03)

Que 2 (A) (1) Obtain the angle between the pair of lines $ax^2+2hxy+by^2=0$. $(a^2+h^2+b^2 \neq 0)$ (03)

(2) If $\frac{n}{l+m}$ = constant then prove that the line $lx+my+n=0$ passes through a fixed pt. (01)

(B) (1) Prove that two tangents can be drawn to the circle from an outside pt. $P(x_1,y_1)$ (02)

(2) Find the coordinates of the foot of the perpendicular from the origin on the line $\frac{x}{a}+\frac{y}{b}=1$. (01)

(3) Find the equations of lines at distance 10 units from the pt. $(4,-3)$ on the line $\perp^{ar}$ to the line $3x+4y=0$. (01)

(C) (1) The lengths of the tangents drawn from a point P to two circles with centre at origin are inversely proportional to the corresponding radii. Show that all such points P lie on a circle with centre at origin. (03)

or

Find the equation of the circumscribed circle of the Δ formed by three lines $x+y-6=0$, $2x+y-4=0$ & $x+2y-5=0$.

(2) Show that the line $2x-3y+39=0$, contains a diameter of the circle $x^2+y^2+12x-18y-5=0$ (01)

(D) Prove that the lines $(a^2-3b^2)x^2 + 8abxy + (b^2-3a^2)y^2=0$ and $ax+by+c=0$, $c\neq 0$ contain the sides of an equilateral triangle whose area is $\frac{c^2}{\sqrt{3}(a^2+b^2)}$. (03)

OR

Obtain the equation of the line without finding the point of intersection of $2x-5y+1=0$ & $x-2y-2=0$ whose both intercepts are equal.

Que:3 (A)(i) Define: Latus-Rectum of a parabola. Find its length and also find the coordinates of the end-points of a latus-rectum. (02)

(ii) Show that the equation of the common tangent to the parabolas $y^2=4ax$ and $x^2=4by$ is $a^{1/3}x+b^{1/3}y+(ab)^{2/3}=0$ (02)

OR

Find the set of points P so that

(1) the sum of the slopes of the tangents drawn to the parabola from P is a constant k.

(2) the product of the slopes of the tangents drawn to the parabola from P is a constant k.

(B) (1) Explain: Auxilliary Circle of an ellipse (02)

(2) The line segment of any tangent, between the tangents at the end-points of the major axis, forms a right angle at any focus of the ellipse (02)

OR

The tangent at point P intersects a directrix at F then prove that PF forms right angle at the corresponding focus.

(C)(1) Define: Asymptotes. Find the equations of the Asymptotes of a hyperbola (02)

(2) Find the equation of the common tangent to the hyperbola $3x^2-4y^2=12$ and parabola $y^2=4x$. (02)

[P.T.O.

(D) (1) Find the equation of a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which cuts equal intercepts on the axes. (01)

(2) If circles $x^2+y^2+2gx+c^2=0$ and $x^2+y^2+2fy+c^2=0$ mutually touches each other, then show that $g^{-2}+f^{-2}=c^{-2}$. (02)

Que. 4 (A) Which curve is represented by $7x^2-2xy+7y^2+12\sqrt{2}x-36\sqrt{2}y+72=0$. Find the foci, directrices, eccentricity, lengths of the axes and the coordinates of the centre (04)

OR

Identify the following quadratic curves

(1) $x^2+y^2-10xy+18x+6y-14=0$

(2) $4x^2-y^2+4x+2y-3=0$.

(B) (1) Explain: Angle between two non-null vectors. (02)

(2) If $\bar{x}=(2,-6,3)$, $\bar{y}=(1,2,-2)$ and $\theta=(\widehat{\bar{x},\bar{y}})$ then find the value of $\sin\theta$ and also find a unit vector perpendicular to $\bar{x}$ & $\bar{y}$. (02)

(C) (1) Explain: Geometrical Interpretation of $|\bar{a}\times\bar{b}|$. (02)

(2) A river flows with a speed of 5 kms. One desires to cross the river in direction $\perp^{cr}$ to the flow, find in what direction should he swim if his speed is 8 kms. (02)

(D) (1) Show that $(1,2,4)$, $(-1,1,1)$, $(6,3,8)$, $(2,1,2)$ are the vertices of a trapezium. Find the area of this trapezium (02)

(2) If $\bar{x}$, $\bar{y}$ are non-collinear vectors of R^3 then prove that $\bar{x}$, $\bar{y}$ and $\bar{x}\times\bar{y}$ are non-coplanar (01)

Que. 5 (A) (1) If three distinct points $A(\bar{a})$, $B(\bar{b})$ & $C(\bar{c})$ of space are collinear then $l,m,n \in R-\{0\}$ can be obtained such that $l\bar{a}+m\bar{b}+n\bar{c}=\bar{0}$ & $l+m+n=0$. Prove. (02)

(2) Obtain the conditions for two planes to be parallel or $\perp^{cr}$. (02)

OR

2) Obtain a distance of a given plane from a given point in R^3. (02)

(B) (1) Define: sphere. (01)

(2) If a line makes with diagonals of a cube angles $\alpha, \beta, \gamma, \delta$ then show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 8/3$. (03)

OR

Show that $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{5}$ & $\frac{x}{3} = \frac{y-1}{2} = \frac{z-1}{1}$ are skew lines & also find the shortest distance between them

(C) (1) Show that centroid and incentre of an equilateral triangle are the same. Find the incentre of the triangle with vertices (6,4,6), (12,4,0), (4,2,-2) (02)

(2) Prove that the condition that the sphere $x^2+y^2+z^2=r^2$ touches the plane $ax+by+cz=p$ $(p\neq 0)$ is $r^2(a^2+b^2+c^2)=p^2$ (02)

(D) Find the length, the foot and the equation of $\perp^{ar}$ from $(2,-1,2)$ to the plane $2x-3y+4z=44$ (03)

OR

Find the common equation of the common section of $x+2y-3z=6$ and $2x-y+z=17$.

* * *

MATHS-I (050) (E) Solution of Paper Set: 5 (1)

Que: 1 (A) (1) NECESSARY PART:-
Let $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ be three points in R^2.
If A, B, C are collinear then A divides $\overline{BC}$ from B in some ratio λ. $(\lambda \neq 0, -1)$.

$\therefore x_1 = \dfrac{\lambda x_3 + x_2}{\lambda+1}$; $y_1 = \dfrac{\lambda y_3 + y_2}{\lambda+1}$

$\therefore \lambda x_1 + x_1 = \lambda x_3 + x_2$; $\lambda y_1 + y_1 = \lambda y_3 + y_2$

$\therefore x_1 - x_2 = \lambda(x_3 - x_1)$; $y_1 - y_2 = \lambda(y_3 - y_1)$

$\therefore \dfrac{x_1 - x_2}{x_3 - x_1} = \lambda$; $\dfrac{y_1 - y_2}{y_3 - y_1} = \lambda$.

$\therefore \dfrac{(x_1 - x_2)}{(x_3 - x_1)} = \dfrac{(y_1 - y_2)}{(y_3 - y_1)}$

$\therefore (x_1 - x_2)(y_3 - y_1) - (y_1 - y_2)(x_3 - x_1) = 0$

$\therefore x_1 y_3 - x_1 y_1 - x_2 y_3 + x_2 y_1 - x_3 y_1 + x_1 y_1 + x_3 y_2 - x_1 y_2 = 0$

$\therefore x_1 y_3 - x_2 y_3 + x_2 y_1 - x_3 y_1 + x_3 y_2 - x_1 y_2 = 0$

$\therefore (x_1 y_2 - x_1 y_3) + (x_3 y_1 - x_2 y_1) + (x_2 y_3 - x_3 y_2) = 0$

$\therefore x_1 (y_2 - y_3) - y_1 (x_2 - x_3) + (x_2 y_3 - x_3 y_2) = 0$

$$\therefore \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

SUFFICIENT PART:- let $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Thus as before $(x_1 - x_2)(y_3 - y_1) = (x_3 - x_1)(y_1 - y_2)$

If $x_1 = x_2$ then $(y_1 - y_2)(x_3 - x_1) = 0 \Rightarrow x_3 = x_1$. as $A \neq B$.
$\therefore x_1 = x_2 = x_3$ and $\overleftrightarrow{AB}$ is vertical and A, B, C are Colinear

Similarly $y_1 = y_2 = y_3$ then $\overleftrightarrow{AB}$ is horizontal
Hence we assume $x_1 \neq x_2$, $x_3 \neq x_1$, $y_1 \neq y_2$, $y_1 \neq y_3$

$\therefore \dfrac{y_3 - y_1}{y_1 - y_2} = \dfrac{x_3 - x_1}{x_1 - x_2} = \lambda$.

$\therefore (y_3 - y_1) = \lambda(y_1 - y_2)$; $(x_3 - x_1) = \lambda(x_1 - x_2)$

$\therefore \lambda y_1 + y_1 = y_3 + \lambda y_2$;

$\therefore y_1(\lambda + 1) = \lambda y_2 + y_3$

$\therefore y_1 = \dfrac{\lambda y_2 + y_3}{\lambda + 1}$; Similarly, $x_1 = \dfrac{\lambda x_2 + x_3}{\lambda + 1}$; $\lambda \neq 0, -1$.

Thus A divides $\overline{BC}$ from B in ratio λ. and A, B, C are colinear.

(2) Suppose A (0, 0), B (0, 1) and C (a, 1).

$\therefore AB^2 = 1$; $BC^2 = a^2$; $AC^2 = a^2 + 1$

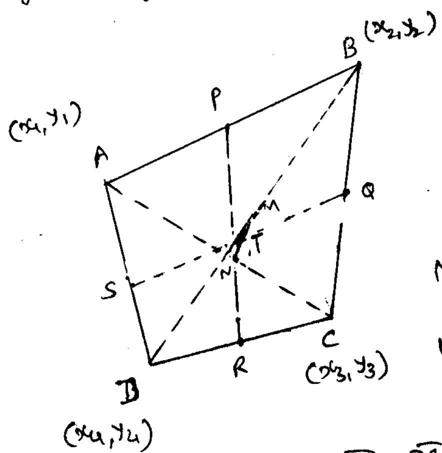
For $a = 0$; we get $B = C$ which is impossible.

$\therefore a \neq 0$.

Now $\forall a \in R - \{0\}$; $AB^2 + BC^2 = AC^2$ so $m\angle B = \frac{\pi}{2}$.

Thus $\forall a \in R - \{0\}$, the given points can be the vertices of a right angled triangle

(B) (1)



Let the vertices of a quadrilateral be A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) and D (x_4, y_4).

Now mid-point of $\overline{AC} = N = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

Mid-point of $\overline{BD} = M = \left(\frac{x_2+x_4}{2}, \frac{y_2+y_4}{2}\right)$

The midpoint of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ are $P\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $Q\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$, $R\left(\frac{x_3+x_4}{2}, \frac{y_3+y_4}{2}\right)$, $S\left(\frac{x_4+x_1}{2}, \frac{y_4+y_1}{2}\right)$

Thus, mid-point of $\overline{PR} = \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}\right)$

Mid-point of $\overline{QS} = \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}\right)$

Mid-point of $MN = \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}\right)$

$\therefore$ Thus $\overline{PR}$, $\overline{QS}$ and $\overline{MN}$ have the same point as mid-point.

$\therefore \overleftrightarrow{PR}$, $\overleftrightarrow{QS}$ and $\overleftrightarrow{MN}$ are concurrent

(2) Here $P(0,0)$ and $G(x, y)$ are the circumcentre and centroid resp. and $\theta_1 + \theta_2 + \theta_3 = \pi$. and we have also

$$\frac{x}{y} = \frac{\cos\theta_1 + \cos\theta_2 + \cos\theta_3}{\sin\theta_1 + \sin\theta_2 + \sin\theta_3}$$

$$= \frac{2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right) + 1 - 2\sin^2\frac{\theta_3}{2}}{2\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right) + 2\sin\frac{\theta_3}{2}\cos\frac{\theta_3}{2}}$$

(2)

$$\therefore \frac{x}{y} = \frac{2\cos\left(\frac{\pi-\theta_3}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)+1-2\sin^2\frac{\theta_3}{2}}{2\sin\left(\frac{\pi-\theta_3}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)+2\sin\frac{\theta_3}{2}\cos\frac{\theta_3}{2}}$$

$$= \frac{2\sin\frac{\theta_3}{2}\left[\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\sin\frac{\theta_3}{2}\right]+1}{2\cos\frac{\theta_3}{2}\left[\cos\left(\frac{\theta_1-\theta_2}{2}\right)+\sin\frac{\theta_3}{2}\right]}$$

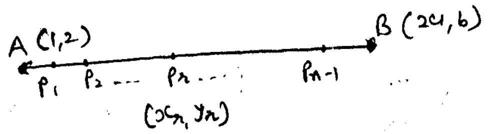
$$= \frac{2\sin\frac{\theta_3}{2}\left[\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\sin\left(\frac{\pi-(\theta_1+\theta_2)}{2}\right)\right]+1}{2\cos\frac{\theta_3}{2}\left[\cos\left(\frac{\theta_1-\theta_2}{2}\right)+\sin\left(\frac{\pi-(\theta_1+\theta_2)}{2}\right)\right]}$$

$$= \frac{2\sin\frac{\theta_3}{2}\left[\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\cos\left(\frac{\theta_1+\theta_2}{2}\right)\right]+1}{2\cos\frac{\theta_3}{2}\left[\cos\left(\frac{\theta_1-\theta_2}{2}\right)+\cos\left(\frac{\theta_1+\theta_2}{2}\right)\right]}$$

$$= \frac{2\sin\frac{\theta_3}{2}\left[-2\sin\frac{\theta_1}{2}\sin\left(-\frac{\theta_2}{2}\right)\right]+1}{2\cos\frac{\theta_3}{2}\left[2\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\right]}$$

$$\frac{x}{y} = \frac{1+4\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2}}{4\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\cos\frac{\theta_3}{2}}$$

(3)



→ Suppose P_1 divides $\overline{AB}$, from A in ratio $1:n-1$, P_2 divides $\overline{AB}$ from A in ratio $2:n-2$ and so on.

$\therefore$ $P_r(x_r, y_r)$ divides $\overline{AB}$ from A in ratio $r:n-r$ where $r = 1, 2, 3, \ldots (n-1)$ Hence $\lambda = \frac{r}{n-r}$, $(x_1, y_1) = (1,2)$

$$\therefore (x_r, y_r) = \left(\frac{\frac{r}{n-r}(2a)+1}{\frac{r}{n-r}+1}, \frac{\frac{r}{n-r}(b)+2}{\frac{r}{n-r}+1}\right)$$

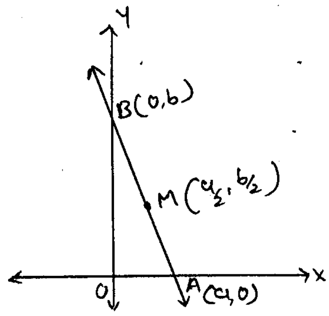
$$= \left(\frac{2ar+n-r}{n}, \frac{br+2n-2r}{n}\right)$$

$$= \left(\frac{(2a-1)r+n}{n}, \frac{(b-2)r+2n}{n}\right)$$

$$= \left(1+\frac{(2a-1)r}{n}, 2+\frac{(b-2)r}{n}\right)$$

where $r = 1, 2, 3, \ldots (n-1)$

(C)
(1) Suppose the line intersects the axes at A $(a,0)$ and B $(0,b)$.



$\therefore$ Mid-point of $\overline{AB}$ is M $\left(\frac{a}{2}, \frac{b}{2}\right)$

Now $a+b=2k$ where k is a constant

$\therefore \frac{a}{2}+\frac{b}{2}=k.$

This shows that $M\left(\frac{a}{2}, \frac{b}{2}\right)$ lies on the line $x+y=k$

$\therefore$ The reqd. equation of the line is $x+y=k$ where the sum of the intercepts is $2k$

(2) The parametric equations of $\overline{AB}$ are

$\left\{(x,y) \middle/ \begin{array}{l} x=tx_2+(1-t)x_1 \\ y=ty_2+(1-t)y_1 \end{array}, 0\le t\le 1\right\}.$

Here $(x_1,y_1)=(2,3)$, $(x_2,y_2)=(5,-1)$.

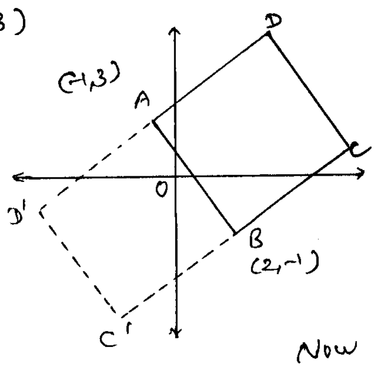
$\therefore x = t(5)+(1-t)(2)$
$= 5t-2t+2 = 3t+2$ and

$y = t(-1)+(1-t)(3)$
$= -4t+3$

Now $x+y = -t+5$. —(1)

We have $0\le t\le 1$
$\Rightarrow 0\ge -t\ge -1$
$\Rightarrow -1\le -t\le 0$
$\Rightarrow 5-1\le 5-t\le 0+5$
$\Rightarrow 4\le x+y\le 5$ ($\because$ by (1))

(3)



Here two squares are possible

Suppose A is $(-1,3)$ and B is $(2,-1)$.

$\therefore$ the slope of $\overleftrightarrow{AB} = \frac{3+1}{-1-2} = -\frac{4}{3}$

and $AB^2=(-1-2)^2+(3+1)^2=25$
$\Rightarrow AB=5$. (sup. $AB=r \Rightarrow r=5$)

Now $\overleftrightarrow{AD}\perp\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}\perp\overleftrightarrow{AB}$

$\therefore$ the slope of $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ is $\tan\theta=\frac{3}{4}>0 \Rightarrow 0<\theta<\frac{\pi}{2}$.

$\therefore \cos\theta=\frac{4}{5}$ and $\sin\theta=\frac{3}{5}$ and $r=5$.

$\therefore$ The coordinates of the point on $\overleftrightarrow{AD}$ at the distance of 5 units from the point $A(-1,3)$ are (x,y)

Then according to $x = x_1 + |r|\cos\theta$ and $y = y_1 + |r|\sin\theta$ ③

$x = -1 + 5\left(\frac{4}{5}\right) = 3$ and $y = 3 + 5\left(\frac{3}{5}\right) = 6$

and $x = x_1 - |r|\cos\theta$, $y = y_1 - |r|\sin\theta$

$x = -1 - 5\left(\frac{4}{5}\right) = -5$; $y = 3 - 5\left(\frac{3}{5}\right) = 0$

Thus we get $D(3,6)$ and $D'(-5,0)$

|||ly the coordinates of the point at the distance of 5 units from the point $B(2,-1)$ on $\overleftrightarrow{BC}$ are (x,y) then

$x = 2 + 5\left(\frac{4}{5}\right)$ and $y = -1 + 5\left(\frac{3}{5}\right) = 2$

and for other point, $x = 2 - 5\left(\frac{4}{5}\right) = -2$ and $y = -1 - 5\left(\frac{3}{5}\right) = -4$

$\therefore$ We get $C(6,2)$, $C'(-2,-4)$.

Thus the remaining vertices of the square are $(6,2)$ and $(3,6)$ or $(-2,-4)$ and $(-5,0)$.

(D) Prove that the graph of a linear equation represents a line

Here $S = \{(x,y) \mid ax + by + c = 0;\ a^2+b^2 \neq 0\}$ is a linear equation

Case: I $a = 0;\ b \neq 0 \Rightarrow a^2+b^2 \neq 0$

$\therefore ax + by + c = 0 \Leftrightarrow y = -\frac{c}{b}$

Thus S represents a horizontal line which is perpendicular to Y-axis.

Case: II $a \neq 0;\ b = 0 \Rightarrow a^2+b^2 \neq 0 \Leftrightarrow x = -\frac{c}{a}$

Thus S represents a vertical line which is $\perp^{ar}$ to X-axis.

Case: III $a \neq 0;\ b \neq 0$

Now, It is obvious that $P\left(-\frac{c}{a}, 0\right)$, $Q\left(0, -\frac{c}{b}\right)$ are in S

Hence S has at least two elements.

Sup. $A(x_1,y_1)$, $B(x_2,y_2) \in S$. $A \neq B$. then

$ax_1 + by_1 + c = 0$ — (1) and

$ax_2 + by_2 + c = 0$ — (2)

Sup. $P(x,y) \in \overleftrightarrow{AB}$ where $P \neq A;B$.

Thus $x = tx_2 + (1-t)x_1$, $y = ty_2 + (1-t)y_1$, $t \in R$

$\therefore$ LHS $= ax + by + c$

$= a(tx_2 + (1-t)x_1) + b(ty_2 + (1-t)y_1) + c$

$= t(ax_2 + by_2 + c) + (ax_1 + by_1 + c) - t(ax_1 + by_1 + c)$

$= 0$ = RHS

$\therefore \overleftrightarrow{AB} \subset S$ — (A)

Conversely, let $C(x_3, y_3) \in S$. $C \neq A; B$.

$\therefore ax_3 + by_3 + c = 0$ — (3)

we know that $a \neq 0$

Then divide (1), (2), (3) by a so we get

$\frac{x_1}{1} + \frac{b}{a} y_1 + \frac{c}{a} = 0$ — (4)

$x_2 + \frac{b}{a} y_2 + \frac{c}{a} = 0$ — (5) and

$x_3 + \frac{b}{a} y_3 + \frac{c}{a} = 0$ — (6)

Here if $y_1 = y_2$ then $x_1 = x_2 \Rightarrow (x_1, y_1) = (x_2, y_2)$

But $A \neq B \Rightarrow y_1 \neq y_2$

By solving (4) and (5) we get the values of b/a and c/a and substituting them in (6), we get

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

ie $A; B; C$ are collinear. so $C \in S \Rightarrow C \in \overleftrightarrow{AB}$

$\therefore S \subset \overleftrightarrow{AB}$ — (B)

By (A) and (B) $S = \overleftrightarrow{AB}$

$\therefore$ The graph of the linear equation $ax+by+c=0, a^2+b^2 \neq 0$ is a line

Que: 2 (A) Angle between pair of Lines:- $(ax^2+2hxy+by^2=0;\ a^2+h^2+b^2 \neq 0)$

Case I: $a=b=0;\ h \neq 0$

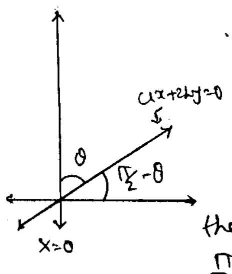
$\therefore$ The given equation will be $xy=0$.

$\therefore x=0$ or $y=0$.

ie x-axis and the y-axis. which are $\perp^{ar}$ to each other

$\therefore$ Angle between them is $\frac{\pi}{2}$.

Case: II $b=0;\ a \neq 0;\ h \neq 0$.



$\therefore$ The pair of lines will be $ax^2+2hxy=0$

$\therefore x(ax+2hy)=0$.

$\therefore x=0$ or $ax+2hy=0$.

If the angle between the lines has measure θ, then the line $ax+2hy=0$ makes an angle of measure $\frac{\pi}{2}-\theta$ with the x-axis. If $ax+2hy=0$ has slope m then

(9)

$\tan\left(\frac{\pi}{2}-\theta\right) = |m| = \left|-\frac{a}{2h}\right|$

$\therefore \cot\theta = \left|\frac{a}{2h}\right| \quad \therefore \quad \tan\theta = \frac{2|h|}{|a|}$

$= \frac{2\sqrt{h^2-0}}{|a+0|}$

$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|} \quad \left(\because b=0 \Rightarrow ab=0 \text{ and } a+b=a\right)$

case: (3) $b=0$; $h=0$, $a\neq 0$

Here lines are coincident. is $x=0$.

$\therefore$ The angle between them is zero.

Case: IV $b\neq 0$

Since the equation represents a pair of lines, $h^2-ab>0$

$\therefore ax^2+2hxy+by^2 = \frac{1}{b}\left(b^2y^2+2bhxy+abx^2\right)$

$= \frac{1}{b}\left[b^2y^2+2bhxy+h^2x^2-h^2x^2+abx^2\right]$

$= \frac{1}{b}\left[(by+hx)^2 - x^2(h^2-ab)\right]$

$= \frac{1}{b}\left[(by+hx)^2 - \left(x\sqrt{h^2-ab}\right)^2\right]$

$= \frac{1}{b}\left[\left(by+hx-x\sqrt{h^2-ab}\right)\left(by+hx+x\sqrt{h^2-ab}\right)\right]$

$= \frac{1}{b}\left[\left\{x\left(h-\sqrt{h^2-ab}\right)+by\right\}\left\{x\left(h+\sqrt{h^2-ab}\right)+by\right\}\right]$

$\therefore ax^2+2hxy+by^2=0$ represents two lines and which are

$\left(h-\sqrt{h^2-ab}\right)x+by=0 \quad \text{--- (I)}$

$\left(h+\sqrt{h^2-ab}\right)x+by=0 \quad \text{--- (II)}$

$\therefore$ The slope of line (1) is $m_1 = -\frac{\left(h-\sqrt{h^2-ab}\right)}{b}$ and

the slope of line (2) is $m_2 = -\frac{\left(h+\sqrt{h^2-ab}\right)}{b}$

Now, if the lines are perpendicular to each other then

$m_1 m_2 = -1$.

$\therefore \frac{\left(h-\sqrt{h^2-ab}\right)\left(h+\sqrt{h^2-ab}\right)}{b^2} = -1$

$\therefore h^2-(h^2-ab) = -b^2 \quad \therefore ab+b^2=0$

$\therefore b(a+b)=0$

but $b\neq 0 \quad \therefore \boxed{a+b=0}$

Otherwise if θ is the angle between them, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$$

$$= \left|\frac{\frac{-(h-\sqrt{h^2-ab})}{b} + \frac{(h+\sqrt{h^2-ab})}{b}}{1+\frac{a}{b}}\right| \quad \left(\because m_1m_2 = \frac{ab}{b^2} = \frac{a}{b}\right)$$

$$= \left|\frac{-h+\sqrt{h^2-ab}+h+\sqrt{h^2-ab}}{a+b}\right|$$

$$= \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

$$\therefore \tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|}$$

if the lines are $\perp^{ar}$ then $a+b=0 \Rightarrow \tan\theta$ will be undefined

$\therefore$ for each case the angle between the pair of lines is $\boxed{\theta = \tan^{-1}\frac{2\sqrt{h^2-ab}}{|a+b|}}$

(2) $\frac{n}{l+m}$ = Constant $= k$ (sup.)

$\therefore n = lk + mk$ — (1)

Now $lx+my+n=0$

$\therefore lx+my+lk+mk=0$

$\therefore l(x+k)+m(y+k)=0$.

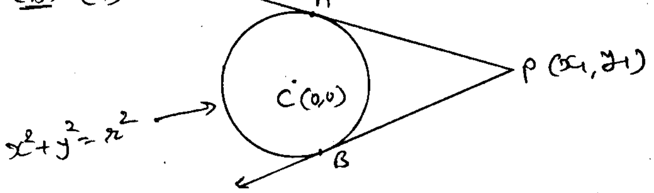
$\therefore x+k=0$ and $y+k=0$ are two lines ($\because$ theorem 1)

$\therefore x=-k$ and $y=-k$

$\therefore (x,y) = (-k,-k)$ ~~are~~ is a fixed point

$\therefore$ A line passing through a fixed point

(B) (1)



Suppose the point $P(x_1,y_1)$ is outside of a circle $x^2+y^2=r^2$

$\therefore CP > r \Leftrightarrow CP^2 > r^2$

$\Leftrightarrow x_1^2+y_1^2 > r^2$

$\Leftrightarrow x_1^2+y_1^2-r^2 > 0$ — (1)

Now the equation of tangents to the circle having slope m are ⑤

$$y = mx \pm r\sqrt{1+m^2}$$

If the point $P(x_1, y_1)$ is on this tangent then

$$y_1 = mx_1 \pm r\sqrt{1+m^2}$$

$$\therefore (y_1 - mx_1)^2 = r^2(1+m^2)$$

$$\therefore y_1^2 - 2mx_1y_1 + m^2x_1^2 = r^2 + r^2m^2$$

$$\therefore m^2(x_1^2 - r^2) - 2mx_1y_1 + (y_1^2 - r^2) = 0 \quad —(2)$$

If $x_1 \neq \pm r$ then this equation represents a quadratic equation in m and its discriminant will be

$$\Delta = (-2x_1y_1)^2 - 4(x_1^2 - r^2)(y_1^2 - r^2)$$

$$= 4x_1^2y_1^2 - 4(x_1^2y_1^2 - x_1^2r^2 - y_1^2r^2 + r^4)$$

$$= 4x_1^2r^2 + 4y_1^2r^2 - 4r^4$$

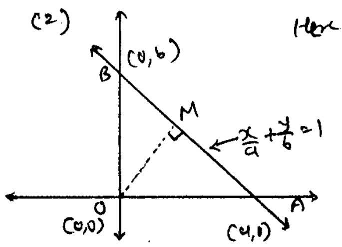
$$= 4r^2(x_1^2 + y_1^2 - r^2)$$

but by (1) we say that $\Delta > 0$

So equation (2) has two distinct real roots.

$\therefore$ Two tangents can be drawn to a circle from a point outside of a circle

(2)



Here, $\overline{OM}$ is the $\perp^{ar}$ drawn from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$.

is $bx + ay - ab = 0 \quad —(1)$

So the equation of the line containing $\overleftrightarrow{OM}$ is $ax - by = 0$. $\quad (\because c = 0) \quad —(2)$

by solving (1) and (2) we get

$$x = \frac{ab^2}{a^2+b^2} \quad \text{and} \quad y = \frac{a^2b}{a^2+b^2}$$

$\therefore$ The coordinates of the foot of the $\perp^{ar}$ drawn from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ is $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$

(3) Slope of given line $= -3/4$

$\therefore$ the slope of the line $\perp^{ar}$ to given line $= 4/3 = \tan\theta$

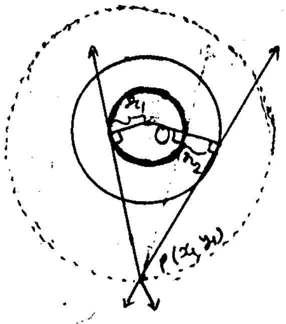
$\therefore$ the equation of this $\perp^{ar}$ line is

$$\frac{x-4}{\cos\theta} = \frac{y+3}{\sin\theta} \quad \text{ie} \quad (y+3) = \frac{4}{3}(x-4) \Rightarrow \boxed{4x - 3y - 25 = 0}$$

(C) (1) Sup., two concentric circles with the centre having radii r_1, r_2 resp. where $r_2 > r_1$, and $\overleftrightarrow{PT_1}$ and $\overleftrightarrow{PT_2}$ are the tangents to the circle from the point $P(x_1, y_1)$. outside the circle

So that $PT_1^2 = x_1^2 + y_1^2 - r_1^2$ and

$PT_2^2 = x_1^2 + y_1^2 - r_2^2$



but the length of these tangents are inversely proportional to their radii.

$\therefore \frac{PT_1}{PT_2} = \frac{r_2}{r_1}$

$\therefore r_1^2 PT_1^2 = r_2^2 PT_2^2$

$\therefore r_1^2 (x_1^2 + y_1^2 - r_1^2) = r_2^2 (x_1^2 + y_1^2 - r_2^2)$

$\therefore r_1^2 x_1^2 + r_1^2 y_1^2 - r_1^4 = r_2^2 x_1^2 + r_2^2 y_1^2 - r_2^4$

$\therefore (r_1^2 - r_2^2) x_1^2 + (r_1^2 - r_2^2) y_1^2 = (r_1^2 - r_2^2)(r_1^2 + r_2^2)$

$(\because r_1 \neq r_2 \Rightarrow (r_1 - r_2)^2 \neq 0)$

$\therefore x_1^2 + y_1^2 = r_1^2 + r_2^2$

$= r^2$ (where $r^2 = r_1^2 + r_2^2$)

$\therefore$ centre of this circle is $(0, 0)$

Thus the general equation of the set of pts. is $x^2 + y^2 = r^2$

$\therefore$ The locus of the point P is a circle with the centre $(0,0)$ and radius $r = \sqrt{r_1^2 + r_2^2}$

OR

OR (5)

If $l_1 : x+y-6=0$, $l_2 : 2x+y-4=0$ and $l_3 : x+2y-5=0$ are the sides of a Δ. equation of circle through the vertices is $\lambda_1 l_2 l_3 + \lambda_2 l_3 l_1 + \lambda_3 l_1 l_2 = 0$

$\therefore \lambda_1(2x+y-4)(x+2y-5) + \lambda_2(x+2y-5)(x+y-6) + \lambda_3(x+y-6)(2x+y-4) = 0$

$\therefore \lambda_1(2x^2+5xy+2y^2-14x-13y+20) + \lambda_2(x^2+3xy+2y^2-11x-17y+30)$ — (1)

$+\lambda_3(2x^2+3xy+y^2-16x-10y+24) = 0$

If (1) represents a ⊙ then,

$2\lambda_1 + \lambda_2 + 2\lambda_3 = 2\lambda_1 + 2\lambda_2 + \lambda_3$

$\therefore \lambda_3 = \lambda_2$ — (2)

and $5\lambda_1 + 3\lambda_2 + 3\lambda_3 = 0$ — (3) $\therefore \dfrac{\lambda_1}{\lambda_3} = -\dfrac{6}{5}$

but by (2) $5\lambda_1 = -6\lambda_3$

and so $\dfrac{\lambda_1}{\lambda_2} = -\dfrac{6}{5}$

$\lambda_1 : \lambda_2 : \lambda_3 = -6 : 5 : 5$.

$\therefore \lambda_1 = -6k,\ \lambda_2 = 5k,\ \lambda_3 = 5k$; $k \in R - \{0\}$

Substituting these values in (1) then

$-6k(2x^2+5xy+2y^2-14x-13y+20) + 5k(x^2+3xy+2y^2-11x-17y+30)$

$+5k(2x^2+3xy+y^2-16x-10y+24) = 0$

$\therefore 3kx^2 + 3y^2k - 51kx - 57ky + 150k = 0$

$\therefore 3k(x^2+y^2-17x-19y+50) = 0$ but $k \neq 0$

$\therefore x^2+y^2-17x-19y+50 = 0$ is the reqd. equation of the ⊙

(2) from the equation $x^2+y^2+12x-18y-5=0$ we get $g=6, f=-9; c=-5$

$\therefore$ centre of a ⊙ is $(-g,-f) = (-6, 9)$

Now the line is $2x-3y+39=0$

substituting the values of the coordinates of the centre of a ⊙ in the left side of a line

$LHS = 2x-3y+39$

$= 2(-6)-3(9)+39$

$= -12-27+39 = 0 = RHS.$

$\therefore$ the centre of the ⊙ lies on the line $2x-3y+39=0$.

$\therefore$ the line $2x-3y+39=0$ contains a diameter of the ⊙.

(D) Here slope of the line $ax+by+c=0$ is $-\dfrac{a}{b}$.

If the slope of the line making an angle of 60° with this line is m, then $\tan\dfrac{\pi}{3} = \left|\dfrac{m+a/b}{1-m\,a/b}\right|$ where $m = y/x$

here (x,y) is any pt. on line thro' origin

$\therefore \sqrt{3} = \left|\dfrac{y/x + a/b}{1 - \frac{y}{x}\frac{a}{b}}\right|$ $\therefore \sqrt{3} = \left|\dfrac{ax+by}{bx-ay}\right|$

$\therefore 3(bx-ay)^2 = (ax+by)^2$

$\therefore 3(b^2x^2 - 2abxy + a^2y^2) = a^2x^2 + 2abxy + b^2y^2$

$\therefore (a^2-3b^2)x^2 + 8abxy + (3a^2+b^2)y^2 = 0.$

Now the area of an equilateral $\Delta = \frac{p^2}{\sqrt{3}}$

where p = altitude of Δ.

Here one vertex of a Δ is origin and opposite side is

$ax+by+c=0$

$\therefore p = \frac{|c|}{\sqrt{a^2+b^2}}$

$\therefore$ Area $= \frac{p^2}{\sqrt{3}} = \frac{c^2}{\sqrt{3}(a^2+b^2)}$

OR

→ x-intercept of $2x-5y+1=0$ is $-\frac{1}{2}$

y- " is $\frac{1}{5}$

$\therefore -\frac{1}{2} \neq \frac{1}{5}$

$\therefore 2x-5y+1=0$ is not a reqd. line

Now, for $x-2y-2=0$, x intercept $= 2$

y- " $= -1$

$\therefore 2 \neq -1$

$\therefore x-2y-2=0$ is not a reqd line

sup. the reqd. equation of a line is

$(2x-5y+1)+\lambda(x-2y-2)=0$ — (A)

$\therefore (2+\lambda)x + (-5-2\lambda)y + (1-2\lambda)=0$ whose both intercepts are equal

$\therefore -\frac{(1-2\lambda)}{(2+\lambda)} = -\frac{(1-2\lambda)}{(-5-2\lambda)}$

$\therefore \frac{2\lambda-1}{\lambda+2} = \frac{1-2\lambda}{5+2\lambda}$

$\therefore 10\lambda + 4\lambda^2 - 5 - 2\lambda = \lambda - 2\lambda^2 + 2 - 4\lambda$

$\therefore 6\lambda^2 + 11\lambda - 7 = 0$

$\therefore 6\lambda^2 - 3\lambda + 14\lambda - 7 = 0$

$\therefore (3\lambda+7)(2\lambda-1) = 0$

$\therefore \lambda = -\frac{7}{3}$; $\lambda = \frac{1}{2}$

if $\lambda = \frac{1}{2}$ then by (A) $(2x-5y+1) + \frac{1}{2}(x-2y-2) = 0$

$\therefore \boxed{5x-12y=0}$

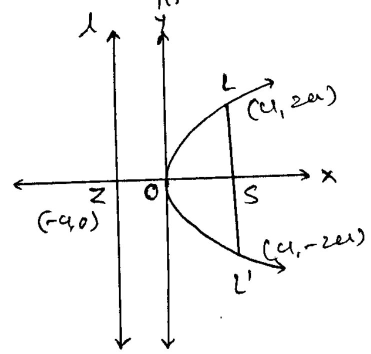
if $\lambda = -\frac{7}{3}$ then by (A) $(2x-5y+1) - \frac{7}{3}(x-2y-2) = 0$

$\therefore -x-y+17=0$

$\therefore \boxed{x+y-17=0}$

which are the reqd. equations.

Que: 3 (A) (i) Latus - Rectum: A focal-chord which is $\perp^{ar}$ to its axis is called the latus-rectum of a parabola



Here LL' is a latus-rectum of a parabola $y^2=4ax$ $(a>0)$. So L and L' are the end-points of a latus-Rectum

Now LL' is $\perp^{ar}$ to x-axis.

$\therefore LL' = 2LS$

(∵ The parabola is symmetric w.r.t its axis)

$\therefore LL' = 2SZ$

$= 2(2a)$

$= 4a$

∴ LS = length of semi latus-Rectum = 2a

∴ The equation of the line containing the latus-rectum is $x=a$ and the coordinates of the end-points L and L' are $(a,2a)$ and $(a,-2a)$

(ii) Suppose the tangent with slope m to the parabola $y^2=4ax$ is $y=mx+\frac{a}{m}$, $m \neq 0$.

If this line is tangent to $x^2=4by$ then substituting $y=mx+\frac{a}{m}$ in $x^2=4by$

$\therefore x^2 = 4b\left(mx+\frac{a}{m}\right)$

$\therefore x^2 = 4bmx + \frac{4ab}{m}$

$\therefore mx^2 - 4bm^2x - 4ab = 0$

Now for the common tangent $\Delta = 0$

$\therefore \Delta = (-4bm^2)^2 - 4(m)(-4ab) = 0$

$\therefore 16b^2m^4 + 16mab = 0$

$\therefore 16mb\,(bm^3+a) = 0$

but $m \neq 0$; $b \neq 0$ $\Rightarrow bm^3 + a = 0$

$\therefore m^3 = -\frac{a}{b}$ $\Rightarrow m = \left(-\frac{a}{b}\right)^{1/3}$

∴ the eqn. of the tangent is

$y = \left(-\frac{a}{b}\right)^{1/3}x - \frac{a}{\left(\frac{a}{b}\right)^{1/3}}$

$\therefore y = \frac{-a^{1/3}x}{b^{1/3}} - \frac{a}{a^{1/3}}\cdot b^{1/3}$

$$\therefore y = \frac{-a^{1/3}x}{b^{1/3}} - a^{1-1/3}\, b^{1/3}$$

$$\therefore b^{1/3}y = -a^{1/3}x - a^{2/3}\cdot b^{1/3+1/3}$$

$$\therefore a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$$

$$\therefore a^{1/3}x + b^{1/3}y + (ab)^{2/3} = 0.$$

OR

(i) Suppose the equations of the tangents are $y = \frac{x}{t_1} + at_1$ & $y = \frac{x}{t_2} + at_2$ where $\frac{1}{t_1}$ and $\frac{1}{t_2}$ are the slopes of the tangents resp.

Now $\frac{1}{t_1} + \frac{1}{t_2} = k$. ($k$ = constant) —(1)

The point of intersection of both equations is $(at_1t_2,\ a(t_1+t_2))$

by (1), $\frac{t_1+t_2}{t_1t_2} = k$

$\Rightarrow t_1 + t_2 = k(t_1t_2)$

$\Rightarrow a(t_1+t_2) = k(at_1t_2)$

$\Rightarrow y = kx$

$\Rightarrow \boxed{kx - y = 0}$

(ii) Now $\frac{1}{t_1}\cdot\frac{1}{t_2} = k$ (k = constant)

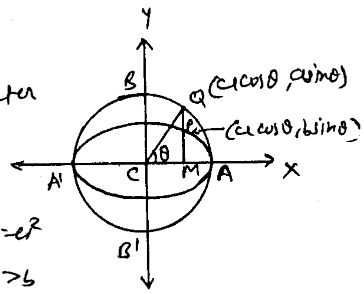
$\Rightarrow 1 = kt_1t_2$

$\Rightarrow a = k(at_1t_2)$

$\Rightarrow a = kx$

$\Rightarrow \boxed{kx - a = 0}$

(B) (i) **Auxiliary Circle of an Ellipse:-**



Auxiliary circle: The circle whose diameter is the major axis of the ellipse is called the auxiliary circle.
The equation of this circle is $x^2 + y^2 = a^2$ for $a > b$

Sup. the coordinates of a point p on the ellipse are $(a\cos\theta, b\sin\theta)$. Now M is the foot of $\perp^{ar}$ from P to x-axis and $\overleftrightarrow{PM}$ intersects the auxiliary circle at Q. Hence $\overleftrightarrow{PM}$ is a vertical line

$\therefore$ The x-coordinate of Q is $a\cos\theta$.

but the equation of auxiliary circle is $x^2+y^2=a^2$

$\therefore \; x^2 = a^2 - y^2$

$\therefore \; y^2 = a^2 - x^2$

$= a^2 - a^2\cos^2\theta$

$= a^2\sin^2\theta$

$\therefore \; y = a\sin\theta$

$\therefore$ The coordinates of Q are $(a\cos\theta, a\sin\theta)$

If $0<\theta<\pi$, then Q is in the upper half plane and $m\angle ACQ = \theta$

If $-\pi<\theta<0$, then Q is in the lower half plane.

where θ = eccentric angle of point P.

The eccentric angles of A and A' are 0 and π resp.

P and Q are called coherent points of the ellipse and auxiliary circle resp.

(2) The equation of the tangent at the point $P(\theta)$ is

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ——— (1)

The equation of the tangents at A and A' are $x=a$ —(2) and $x=-a$ resp. —(3)

by solving (1) and (2), the point of intersection

$R\left[a, \frac{b(1-\cos\theta)}{\sin\theta}\right]$ is obtained

l^{ly} the point of intersection of (1) and (3) is

$R'\left[-a, \frac{b(1+\cos\theta)}{\sin\theta}\right]$

$\therefore$ (The slope of $\overline{SR}$)(The slope of $\overline{SR'}$)

$= \frac{b(1-\cos\theta)}{(a-ae)\sin\theta} \cdot \frac{b(1+\cos\theta)}{(-a-ae)\sin\theta}$

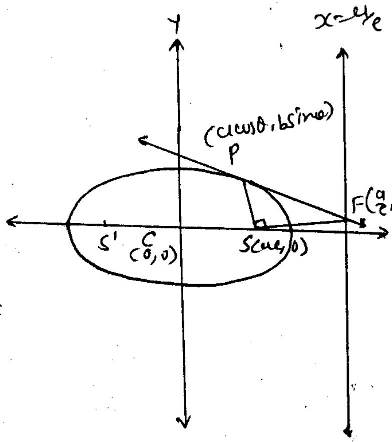
$$= \frac{-b^2}{a^2(1-e^2)} \cdot \frac{1-\cos^2\theta}{\sin^2\theta} = -1$$

$\therefore \overline{SR} \perp \overline{SR'}$

$\therefore m\angle RSR' = \frac{\pi}{2}$

Similarly it can be proved that $m\angle RS'R' = \frac{\pi}{2}$.

OR



The equation of the tangent at $P(a\cos\theta, b\sin\theta)$ to ellipse is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \quad \text{--- (1)}$$

(1) intersects the directrix at F

$\therefore$ substituting $x = \frac{a}{e}$ in (1)

$\therefore \frac{\cos\theta}{e} + \frac{y}{b}\sin\theta = 1$

$$\therefore y = \frac{b(1-\cos\theta/e)}{\sin\theta} = \frac{b(e-\cos\theta)}{e\sin\theta}$$

$\therefore$ The coordinates of F are $\left(\frac{a}{e}, \frac{b(e-\cos\theta)}{e\sin\theta}\right)$

Now slope of $\overleftrightarrow{SF} = \frac{\left(\frac{b(e-\cos\theta)}{e\sin\theta}\right)}{\left(\frac{a}{e} - ae\right)}$

$$= \frac{b(e-\cos\theta)}{e\sin\theta} \times \frac{e}{a(1-e^2)}$$

$$= \frac{b(e-\cos\theta)}{a(1-e^2)\sin\theta}$$

and slope of $\overleftrightarrow{SP} = \frac{0 - b\sin\theta}{ae - a\cos\theta}$

$$= \frac{-b\sin\theta}{a(e-\cos\theta)}$$

$\therefore$ (slope of $\overleftrightarrow{SF}$)(slope of $\overleftrightarrow{SP}$) $= \frac{b(e-\cos\theta)}{\sin\theta \, a(1-e^2)} \times \frac{(-b\sin\theta)}{a(e-\cos\theta)}$

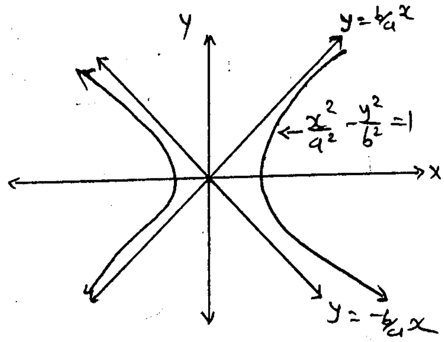
$$= \frac{-b^2}{a^2(1-e^2)}$$

$$= \frac{-b^2}{b^2}$$

$$= -1.$$

$\therefore \overline{PF}$ forms a right angle at the corresponding focus.

(c) (i) Asymptotes: Let $y = f(x)$ be a curve and $y = mx + c$ be a line such that $\lim_{|x| \to \infty} |f(x) - mx - c| = 0$. Then the line $y = mx + c$ is called an asymptote of the curve $y = f(x)$.



The standard equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

$$\therefore b^2\left(\frac{x^2 - a^2}{a^2}\right) = y^2$$

$$\therefore y^2 = \frac{b^2}{a^2}(x^2 - a^2)$$

$$\therefore y = \pm \frac{b}{a}\sqrt{x^2 - a^2} = f(x)$$

Sup. the line $y = mx + c$ is an asymptote of the hyperbola then $\lim_{|x| \to \infty} |f(x) - mx - c| = 0$

$$\therefore \lim_{|x| \to \infty} |f(x) - mx - c|$$

$$= \lim_{|x| \to \infty} \left|\pm \frac{b}{a}\sqrt{x^2 - a^2} - mx - c\right|$$

$$= \lim_{|x| \to \infty} \left|\pm \frac{b}{a}\sqrt{x^2\left(1 - \frac{a^2}{x^2}\right)} - mx - c\right|$$

$$= \lim_{|x| \to \infty} \left|\pm \frac{b}{a} \cdot x\sqrt{1 - \frac{a^2}{x^2}} - mx - c\right|$$

$$= \lim_{|x| \to \infty} \left|x\left(\pm \frac{b}{a}\sqrt{1 - \frac{a^2}{x^2}} - m\right) - c\right|$$

If this limit will be 0 then the line $y = mx + c$ be an asymptote hence if $c = 0$ and $m = \pm \frac{b}{a}$

$(\because \text{ as } x \to \infty \Rightarrow \frac{1}{x} \to 0)$

$$\therefore \lim_{|x| \to \infty} \left|\pm \frac{b}{a}\sqrt{x^2 - a^2} - mx - c\right|$$

$$= \lim_{|x| \to \infty} \left|\pm \frac{b}{a}\sqrt{x^2 - a^2} \pm \frac{b}{a}(-x)\right|$$

$$= \lim_{|x| \to \infty} \left|\pm \frac{b}{a}\left(\sqrt{x^2 - a^2} - x\right)\right|$$

$$= \lim_{|x| \to \infty} \left|\pm \frac{b}{a}\left(\frac{\{\sqrt{x^2 - a^2} - x\}\{\sqrt{x^2 - a^2} + x\}}{\sqrt{x^2 - a^2} + x}\right)\right|$$

$$= \lim_{|x|\to\infty} \left| \pm \frac{b}{a} \left(\frac{x^2-a^2-x^2}{\sqrt{x^2-a^2}+x} \right) \right|$$

$$= \lim_{|x|\to\infty} \left| \pm \frac{b}{a} \left(\frac{-a^2}{\sqrt{x^2(1-a^2/x^2)}+x} \right) \right|$$

$$= \lim_{|x|\to\infty} \left| \frac{\pm ab}{x\left(\sqrt{1-a^2/x^2}+1\right)} \right|$$

as $x \to \infty \Rightarrow 1/x \to 0$

$= 0$.

$\therefore \lim_{|x|\to\infty} |f(x) - mx - c| = 0$

$\therefore y = f(x) = \pm \frac{b}{a} x$

$\therefore$ The equation of asymptotes are $y = \pm \frac{b}{a} x$

(2) The equation of the hyperbola is $3x^2 - 4y^2 = 12$.

$\therefore \frac{x^2}{4} - \frac{y^2}{3} = 1 \quad \Rightarrow a^2 = 4, \ b^2 = 3.$

The equation of the tangent to the hyperbola whose slope is m ie $y = mx \pm \sqrt{a^2m^2 - b^2}$. $\Rightarrow y = mx \pm \sqrt{4m^2-3}$ —(1)

Now the equation of the parabola is $y^2 = 4x$

$\therefore a = 1$

The equation of the tangent to the parabola whose slope is m ie $y = mx + \frac{a}{m}$

$\Rightarrow y = mx + \frac{1}{m}$. — (2)

Now (1) and (2) represents a common tangent to the hyperbola and parabola

$\therefore mx \pm \sqrt{4m^2-3} = mx + \frac{1}{m}$

$\therefore 4m^2 - 3 = \frac{1}{m^2}$

$\therefore 4m^4 - 3m^2 - 1 = 0$

$\therefore 4m^4 - 4m^2 + m^2 - 1 = 0$

$\therefore (4m^2+1)(m^2-1) = 0$

$\therefore m^2 = -1/4$ or $m^2 = 1$

90

but $m^2 = -1/4$ is impossible

$\therefore m^2 = 1 \Rightarrow m = \pm 1$.

if $m = 1$ then the equation of the common tangent is $y = x + 1 \Rightarrow \boxed{x - y + 1 = 0}$

if $m = -1$ then the equation of the common tangent is $y = -x - 1 \Rightarrow \boxed{x + y + 1 = 0}$

D (1) Now the equation of the tangent to the hyperbola is $y = mx \pm \sqrt{a^2m^2 - b^2}$ — (A)

which cuts the equal intercepts on the axes then $-c/a = -c/b \Rightarrow a = b$. —(1)

If the equation of a tangent is the form of $ax + by + c = 0$ $\therefore$ slope $m = -\frac{a}{b}$

but by (1) $m = -1$

$\therefore$ substitute $m = -1$ in (A) then the equation of the tangent is $y = -x \pm \sqrt{a^2 - b^2}$

$\therefore x + y \mp \sqrt{a^2 - b^2} = 0$ are the equations of the tangents.

(2) $S_1: x^2 + y^2 + 2gx + a^2 = 0 \Rightarrow C_1$ is the centre of a $\odot S_1$

ie $C_1 = (-g, 0)$

$r_1 = \sqrt{g^2 + f^2 - c} = \sqrt{g^2 - a^2}$

$S_2: x^2 + y^2 + 2fy + a^2 = 0 \Rightarrow C_2$ is the centre of a $\odot S_2$

ie $C_2 = (0, -f)$

$\therefore r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{f^2 - a^2}$

Now $\odot S_1$ and S_2 touches each other then

$C_1C_2 = r_1 \pm r_2$

$\therefore \sqrt{(-g-0)^2 + (0+f)^2} = \sqrt{g^2 - a^2} \pm \sqrt{f^2 - a^2}$

$\therefore \sqrt{g^2 + f^2} = \sqrt{g^2 - a^2} \pm \sqrt{f^2 - a^2}$

$\therefore g^2 + f^2 = g^2 - a^2 + f^2 - a^2 \pm 2\sqrt{(g^2 - a^2)(f^2 - a^2)}$

$\therefore 2a^2 = \pm 2\sqrt{g^2f^2 - g^2a^2 - f^2a^2 + a^4}$

$\therefore a^2 = \pm\sqrt{g^2f^2 - g^2a^2 - f^2a^2 + a^4}$

$\therefore a^4 = g^2f^2 - g^2a^2 - f^2a^2 + a^4$

$\therefore g^2f^2 - g^2a^2 - f^2a^2 = 0$

$\therefore g^2a^2 + f^2a^2 = g^2f^2$

$\therefore \frac{a^2}{f^2} + \frac{a^2}{g^2} = 1$ $\quad (\because g \neq 0; f \neq 0 \Rightarrow g^2f^2 \neq 0)$

$\therefore a^2\left(\frac{1}{f^2} + \frac{1}{g^2}\right) = 1$

$\therefore \frac{1}{g^2} + \frac{1}{f^2} = \frac{1}{a^2}$

$\therefore \boxed{g^{-2} + f^{-2} = a^{-2}}$

Que: 4 (A) Here $a = b$, $\theta = \frac{\pi}{4}$. $h \neq 0$.

$\therefore$ Rotate the axes by $\theta = \pi/4$.

Hence $x = \frac{x' - y'}{\sqrt{2}}$, $y = \frac{x' + y'}{\sqrt{2}}$.

Here, the equation is $7x^2 - 2xy + 7y^2 + 12\sqrt{2}x - 36\sqrt{2}y + 72 = 0$

$\therefore 7\left(\frac{x'-y'}{\sqrt{2}}\right)^2 - 2\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) + 7\left(\frac{x'+y'}{\sqrt{2}}\right)^2 + 12\sqrt{2}\left(\frac{x'-y'}{\sqrt{2}}\right) - 36\sqrt{2}\left(\frac{x'+y'}{\sqrt{2}}\right) + 72 = 0$

$\therefore 7\left(\frac{x'^2 - 2x'y' + y'^2}{2}\right) - 2\left(\frac{x'^2 - y'^2}{2}\right) + 7\left(\frac{x'^2 + 2x'y' + y'^2}{2}\right) + \frac{24(x'-y')}{2} - \frac{72(x'+y')}{2} + \frac{144}{2} = 0$

$\therefore 7x'^2 - 14x'y' + 7y'^2 - 2x'^2 + 2y'^2 + 7x'^2 + 14x'y' + 7y'^2 + 24x' - 24y' - 72x' - 72y' + 144 = 0$

$\therefore 12x'^2 + 16y'^2 - 48x' - 96y' + 144 = 0$

$\therefore 6x'^2 + 8y'^2 - 24x' - 48y' + 72 = 0.$

$\therefore 6(x'^2 - 4x' + 4) + 8(y'^2 - 6y' + 9) = 24$

$\therefore 6(x'-2)^2 + 8(y'-3)^2 = 24$

$\therefore \frac{(x'-2)^2}{4} + \frac{(y'-3)^2}{3} = 1$

We shift the origin to $(2,3)$ then taking $x'-2 = X$, $y'-3 = Y$

$\therefore \frac{X^2}{4} + \frac{Y^2}{3} = 1$

which represents an ellipse (10)

where $x' = \frac{x+y}{\sqrt{2}}$, $y' = \frac{-x+y}{\sqrt{2}}$

$\therefore$ $x' = x+2$, $y' = y+3$.

Here $a^2 = 4$, $b^2 = 3$

$\therefore$ $a = 2$, $b = \sqrt{3}$. where $a > b$

$\therefore$ The length of the major axis $= 2a = 4$

" of the minor axis $= 2b = 2\sqrt{3}$

from $b^2 = a^2(1-e^2)$

$\therefore 3 = 4(1-e^2) \Rightarrow 1-e^2 = 3/4$

$\therefore e^2 = 1/4 \Rightarrow e = \frac{1}{2}$ ($\because e<1$)

$\therefore$ The centre in (X, Y)-system is $(0,0)$

The centre in (x', y')-system is $(2,3)$ ($\because$ we shift the origin to $(2,3)$)

x-coordinate $= \frac{x'-y'}{\sqrt{2}} = \frac{2-3}{\sqrt{2}} = -1/\sqrt{2}$

y-coordinate $= \frac{x'+y'}{\sqrt{2}} = \frac{2+3}{\sqrt{2}} = 5/\sqrt{2}$

$\therefore$ The centre in original system is $\left(-\frac{1}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$

Focii $(\pm ae, 0) = (\pm 2(\frac{1}{2}), 0) = (\pm 1, 0)$

In (x', y')-system focii are $(1,3)$ and $(3,3)$

In original-system focii are $(-\sqrt{2}, 2\sqrt{2})$ and $(0, 3\sqrt{2})$

Equations of the directrices $X = \pm 4$.

ie $x' - 2 = \pm 4$

ie $\frac{x+y}{\sqrt{2}} - 2 = \pm 4$

$\therefore$ The equations of directrices are

$x+y = -2\sqrt{2}$ and $x+y = 6\sqrt{2}$

OR

(P.T.O.)

OR

(1) $x^2+y^2-10xy+18x+6y-14=0$ — (1)

by (1) $a=b=1,\ h=-5$

$a=b \Rightarrow \theta=\pi/4 \Rightarrow x=\frac{x'-y'}{\sqrt{2}},\ y=\frac{x'+y'}{\sqrt{2}}$.

$\therefore$ by (1)

$\left(\frac{x'-y'}{\sqrt{2}}\right)^2+\left(\frac{x'+y'}{\sqrt{2}}\right)^2-10\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right)+18\left(\frac{x'-y'}{\sqrt{2}}\right)+6\left(\frac{x'+y'}{\sqrt{2}}\right)-14=0$

$\therefore \frac{x'^2-2x'y'+y'^2}{2}+\frac{x'^2+2x'y'+y'^2}{2}-\frac{10(x'^2-y'^2)}{2}+\frac{18\sqrt{2}(x'-y')}{2}+\frac{6\sqrt{2}(x'+y')}{2}-\frac{28}{2}=0$

$\therefore x'^2-2x'y'+y'^2+x'^2+2x'y'+y'^2-10x'^2+10y'^2+18\sqrt{2}x'-18\sqrt{2}y'+6\sqrt{2}x'+6\sqrt{2}y'-28=0$

$\therefore -8x'^2+12y'^2+24\sqrt{2}x'-12\sqrt{2}y'-28=0$

$\therefore 2x'^2-3y'^2-6\sqrt{2}x'+3\sqrt{2}y'+7=0$

$\therefore 2\left(x'^2-3\sqrt{2}x'+\frac{9}{2}\right)-3\left(y'^2-\sqrt{2}y'+\frac{1}{2}\right)=\frac{1}{2}$

$\therefore 2\left(x'-\frac{3}{\sqrt{2}}\right)^2-3\left(y'-\frac{1}{\sqrt{2}}\right)^2=\left(\frac{1}{\sqrt{2}}\right)^2$

$\therefore \frac{2(x'-3/\sqrt{2})^2}{1/2}-\frac{3(y'-1/\sqrt{2})^2}{1/2}=1$

$\therefore 4(x'-3/\sqrt{2})^2-6(y'-1/\sqrt{2})^2=1$

Shifting the origin at $\left(3/\sqrt{2},1/\sqrt{2}\right)$ and taking

$x'-\frac{3}{\sqrt{2}}=X;\quad y'-\frac{1}{\sqrt{2}}=Y$ then

$4X^2-6Y^2=1$

which represents hyperbola

(2) $4x^2-y^2+4x+2y-3=0$ — (1)

by (1) $a=4,\ b=-1.\quad a\neq b,\ h=0$

$\therefore \tan 2\theta=\frac{2h}{a-b}=0 \Rightarrow \theta=0$

$\therefore \cos 2\theta=1 \Rightarrow \cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}=\sqrt{2/2}=1$ and $\sin\theta=0$

$\therefore x=x';\quad y=y'$

(12)

$\therefore\ 4x'^2+4x'-y'^2+2y'-3=0$

$\therefore\ 4(x'^2+x'+\frac{1}{4})-(y'^2-2y'+1)=3$

$\therefore\ 4(x'+\frac{1}{2})^2-(y'-1)^2=3$

$\therefore\ \frac{4(x'+\frac{1}{2})^2}{3}-\frac{(y'-1)^2}{3}=1$

Shifting the origin at $(-\frac{1}{2},1)$ then taking $x'+\frac{1}{2}=X$, $y'-1=Y$ so that the above equation will be

$\frac{4X^2}{3}-\frac{Y^2}{3}=1$

which represents hyperbola

(B) (1) Angle between two non-null vectors:

Let $\bar{x},\bar{y}$ be non-null vectors of R^2 or R^3.

I) If the direction of two vectors are same then the angle between them is 0.

If the direction of two vectors are opposite then the angle between them is π.

II) The directions of two vectors are different:

If $\bar{x}=k\bar{y}$ then $|\bar{x}\cdot\bar{y}|=|k\bar{y}\cdot\bar{y}|=|k||\bar{y}|^2$

$=|k\bar{y}||\bar{y}|$

$=|\bar{x}||\bar{y}|$

if $|\bar{x}\cdot\bar{y}|=|\bar{x}||\bar{y}|$ then $|\bar{x}\cdot\bar{y}|^2=|\bar{x}|^2|\bar{y}|^2$ —(1)

we have $|\bar{x}\times\bar{y}|^2+|\bar{x}\cdot\bar{y}|^2=|\bar{x}|^2|\bar{y}|^2$

by (1) $|\bar{x}\times\bar{y}|^2=|\bar{x}|^2|\bar{y}|^2-|\bar{x}\cdot\bar{y}|^2$

$=|\bar{x}|^2|\bar{y}|^2-|\bar{x}|^2|\bar{y}|^2$

$=0$

$\therefore\ |\bar{x}\times\bar{y}|=0$

$\therefore\ (x_2y_3-x_3y_2,\ x_3y_1-x_1y_3,\ x_1y_2-x_2y_1)=(0,0,0)$

$\therefore\ x_2y_3=x_3y_2$ and $x_3y_1=x_1y_3$; $x_1y_2=x_2y_1$

$\therefore\ \frac{x_2}{y_3}=\frac{y_2}{y_3} \Rightarrow \frac{x_2}{y_2}=\frac{x_3}{y_3},\ \frac{x_1}{y_1}=\frac{x_3}{y_3},\ \frac{x_1}{y_1}=\frac{x_2}{y_2}$

$\therefore\ \frac{x_1}{y_1}=\frac{x_2}{y_2}=\frac{x_3}{y_3}=k$

$\therefore\ x_1=ky_1,\ x_2=ky_2,\ x_3=ky_3$

$\therefore (x_1, x_2, x_3) = k(y_1, y_2, y_3)$

$\therefore \quad \bar{x} = k\bar{y}$ then angles are 0 or π.

Thus from schwartz inequality we have

$|\bar{x}\cdot\bar{y}| < |\bar{x}||\bar{y}|$

$\therefore \dfrac{|\bar{x}\cdot\bar{y}|}{|\bar{x}||\bar{y}|} < 1 \qquad (\because \bar{x}\neq\theta, \bar{y}\neq\theta)$

$\therefore -1 < \dfrac{\bar{x}\cdot\bar{y}}{|\bar{x}||\bar{y}|} < 1$

If angle between two vectors is α then $\alpha\in(0,\pi)$

such that $\cos\alpha = \dfrac{\bar{x}\cdot\bar{y}}{|\bar{x}||\bar{y}|}$

$\therefore \alpha = \cos^{-1}\dfrac{\bar{x}\cdot\bar{y}}{|\bar{x}||\bar{y}|}.$

(2) $\bar{x} = (2,-6,3) \quad \bar{y} = (1,2,-2) \quad , \theta = (\hat{\bar{x}\bar{y}})$

we have $|\bar{x}\times\bar{y}| = |\bar{x}||\bar{y}|\sin\theta$

$\therefore \sin\theta = \dfrac{|\bar{x}\times\bar{y}|}{|\bar{x}||\bar{y}|} \quad —(1)$

Now, $|\bar{x}| = \sqrt{4+36+9} = 7$ and

$|\bar{y}| = \sqrt{1+4+4} = 3$

$$\bar{x}\times\bar{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & 3 \\ 1 & 2 & -2 \end{vmatrix}$$

$= \hat{i}(6) - \hat{j}(-7) + \hat{k}(10)$

$= (6,7,10)$

$\Rightarrow |\bar{x}\times\bar{y}| = \sqrt{36+49+100} = \sqrt{185}$

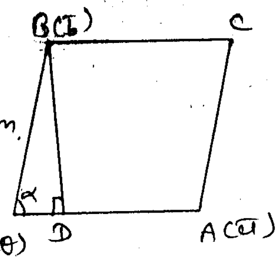
$\therefore$ by (1) $\sin\theta = \dfrac{\sqrt{185}}{21}$

Now unit vector $\perp^{ar}$ to both $\bar{x}$ and $\bar{y}$ is $\pm\dfrac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|}$

$\therefore \pm\dfrac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|} = \pm\dfrac{(6,7,10)}{\sqrt{185}}$

(13)

(C) (i) <u>Geometrical</u> <u>Interpretation</u> <u>Of $|\bar{a}\times\bar{b}|$</u>



Let $\bar{a}, \bar{b}$ be non-null vectors which are not collinear Relative to O(θ).
Let A($\bar{a}$) and B($\bar{b}$), then OACB is a $\square^m$.
Now we draw a $\perp^{ar}$ from B on $\overrightarrow{OA}$
$\therefore \overline{BD} \perp \overrightarrow{OA}$.

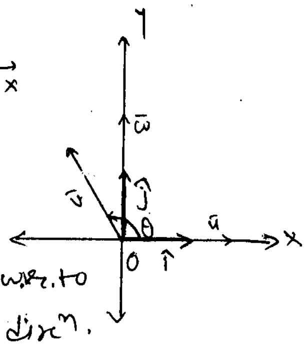
If α is the magnitude of angle between vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ ie $\bar{a}$ and $\bar{b}$ then $\sin\alpha = \frac{BD}{|\overrightarrow{OB}|} = \frac{BD}{|\bar{b}|}$

$\therefore BD = |\bar{b}|\sin\alpha$ —— (1)

Now the area of a $\square^m$ OACB = base $\times$ altitude
$= |\overrightarrow{OA}| \times BD$
$= |\bar{a}|\,|\bar{b}|\sin\alpha$ ($\because$ by (1))
$= |\bar{a}\times\bar{b}|$

$\therefore$ The area of a $\square^m$ OACB = $|\overrightarrow{OA}\times\overrightarrow{OB}|$

(2) Sup. the dir^n. of a flow of river is $\overrightarrow{OX}$



The velocity of the river flow is $\bar{u}$.
then $\bar{u} = 5\hat{i} + 0\cdot\hat{j} = (5, 0)$

Suppose, the swimmer makes θ angle w.r.to the river flows to cross the river in dir^n. $\perp^{ar}$ to the flow.

also, swimmer swims at the speed of 8km/h

$\therefore$ the velocity of the swimmer is $\bar{v}$.

$\therefore \bar{v} = 8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j} = (8\cos\theta, 8\sin\theta)$

$\therefore$ The resultant velocity is $\bar{w} = \bar{u} + \bar{v}$.

$\therefore \bar{w} = (8\cos\theta + 5, 8\sin\theta)$

Now, $\bar{w} \perp \overrightarrow{OX} \Rightarrow \bar{w}\cdot\overrightarrow{OX} = 0$

$\therefore (8\cos\theta + 5, 8\sin\theta)\cdot(1, 0) = 0$

$\therefore 8\cos\theta + 5 = 0$

$\therefore \cos\theta = -5/8$

$\therefore \theta = \cos^{-1}(-5/8) = \pi - \cos^{-1} 5/8$.

$\therefore$ swimmer has to swim in the direction $\perp^{ar}$ to the flow at the angle $\theta = \pi - \cos^{-1} 5/8$.

(D) (1) $A(1,2,4)$, $B(-1,1,1)$, $C(6,3,8)$, $D(2,1,2)$ are given.

$\overrightarrow{AB} = (-2,-1,-3)$ $\quad\quad$ $\overrightarrow{CD} = (-4,-2,-6) = 2(-2,-1,-3)$

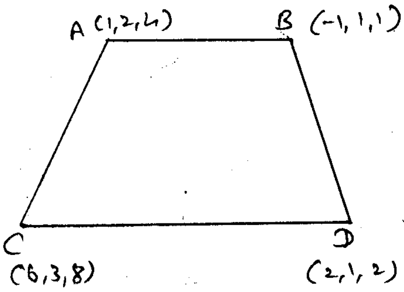
$\overrightarrow{BC} = (7,2,7)$ $\quad\quad$ $\overrightarrow{DA} = (-1,1,2)$

Here $\overrightarrow{CD} = 2\overrightarrow{AB}$ and $\overrightarrow{BC}$ & $\overrightarrow{DA}$ are not parallel

$\therefore$ A, B, C, D are the vertices of a trapezium

$\therefore$ trapezium ABDC exist.

$\therefore$ Now $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are the diagonals.



$\therefore$ $\overrightarrow{AD} = (1,-1,-2)$,

$$\overrightarrow{AD} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ 7 & 2 & 7 \end{vmatrix}$$

$= \hat{i}(-3) - \hat{j}(21) + \hat{k}(9)$

$= (-3,-21,9)$

$|\overrightarrow{AD} \times \overrightarrow{BC}| = \sqrt{9+441+81}$

$= \sqrt{531}$

$\therefore$ The area of a trapezium $= \frac{1}{2} |\overrightarrow{AD} \times \overrightarrow{BC}|$

$= \frac{1}{2}\sqrt{531}$

$= \frac{1}{2}\sqrt{9 \times 59}$

$= \frac{3}{2}\sqrt{59}$ units.

(2) $\bar{x} \neq 0$, $\bar{y} \neq 0 \in R^3$.

$\bar{x}, \bar{y}$ are non-collinear then $\bar{x} \times \bar{y} \neq 0$. —(1)

Now, $\bar{x} \cdot [\bar{y} \times (\bar{x} \times \bar{y})] = \bar{x} \cdot [(\bar{y}\cdot\bar{y})\bar{x} - (\bar{y}\cdot\bar{x})\bar{y}]$

$= (\bar{x}\cdot\bar{x})(\bar{y}\cdot\bar{y}) - (\bar{x}\cdot\bar{y})(\bar{y}\cdot\bar{x})$

$= |\bar{x}|^2|\bar{y}|^2 - |\bar{x}\cdot\bar{y}|^2$ $\quad (\because \bar{x}\cdot\bar{y} = \bar{y}\cdot\bar{x})$.

$= |\bar{x} \times \bar{y}|^2$

$\neq 0$ $\quad (\because$ by (1))

$\therefore$ $\bar{x}, \bar{y}$ and $\bar{x} \times \bar{y}$ are non-coplanar

(NE)

Necessary Part

Que: 5 (A) (1) Suppose A, B, C are collinear

$\therefore C(\bar{c}) \in \overleftrightarrow{AB}$

Hence $C(\bar{c})$ satisfies the equation

$\bar{r} - \bar{a} = k(\bar{b} - \bar{a})$; $k \in R$

$\therefore \bar{c} - \bar{a} = k(\bar{b} - \bar{a})$

$\therefore \bar{c} - \bar{a} - k\bar{b} + k\bar{a} = \bar{0}$

$\therefore (k-1)\bar{a} + (-k)\bar{b} + 1 \cdot \bar{c} = \bar{0}$

For $k=1$ we get $\bar{b} = \bar{c}$ and

For $k=0$ we get $\bar{a} = \bar{c}$

but A, B, C are distinct points hence $k \neq 0, k \neq 1$

Taking Now $l = k-1$, $m = -k$, $n = 1$ then $l\bar{a} + m\bar{b} + n\bar{c} = \bar{0}$

and $l + m + n = k - 1 - k + 1 = 0$. where $l, m, n \neq 0$.

Sufficient Part:

let us suppose that $l, m, n \in R - \{0\}$ such that

$l\bar{a} + m\bar{b} + n\bar{c} = \bar{0}$ and $l + m + n = 0$.

taking $l = -m - n$ in $l\bar{a} + m\bar{b} + n\bar{c} = \bar{0}$ then we get

$(-m-n)\bar{a} + m\bar{b} + n\bar{c} = \bar{0}$

$\therefore -m\bar{a} + m\bar{b} + n\bar{c} - n\bar{a} = \bar{0}$

$\therefore m(\bar{b} - \bar{a}) + n(\bar{c} - \bar{a}) = \bar{0}$

$\therefore \bar{c} - \bar{a} = -\frac{m}{n}(\bar{b} - \bar{a})$ $(n \neq 0)$

$= k(\bar{b} - \bar{a})$ $\left(k = \frac{-m}{n}\right)$ $k \in R$

Since $m \neq 0$; k is non-zero.

$m \neq -n$; $k \neq 1$ then $m + n \neq 0$

if $m + n = 0$ then from $l + m + n = 0$ we get $l = 0$

which is contradiction

$\therefore l, m, n \in R - \{0\}$, $k \neq 0$; $k \neq 1$

Thus $\bar{r} = \bar{c}$ satisfies $\bar{r} = \bar{a} + k(\bar{b} - \bar{a})$; $k \in R$

$\therefore C \in \overleftrightarrow{AB}$ and $C \neq A, B$

$\therefore$ A, B, C are collinear.

(2) let $a_1x+b_1y+c_1z-d_1=0$ and $a_2x+b_2y+c_2z-d_2=0$ be two planes.

where $\bar{n}_1=(a_1,b_1,c_1)$ and $\bar{n}_2=(a_2,b_2,c_2)$

If $\bar{n}_1$ and $\bar{n}_2$ are in the same direction then $\bar{n}_1\times\bar{n}_2=\bar{0}$ then planes are parallel

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = (0,0,0) = \bar{n}_1\times\bar{n}_2$$

$$\therefore (b_1c_2-b_2c_1,\ a_2c_1-a_1c_2,\ a_1b_2-a_2b_1)=(0,0,0)$$

$$\therefore b_1c_2=b_2c_1,\quad a_2c_1=a_1c_2,\quad a_1b_2=a_2b_1$$

$$\therefore \frac{b_1}{c_1}=\frac{b_2}{c_2};\quad \frac{a_1}{c_1}=\frac{a_2}{c_2};\quad \frac{a_1}{b_1}=\frac{a_2}{b_2}$$

$$\therefore \frac{a_1}{a_2}=\frac{c_1}{c_2};\quad \frac{a_1}{a_2}=\frac{b_1}{b_2};\quad \frac{b_1}{b_2}=\frac{c_1}{c_2}$$

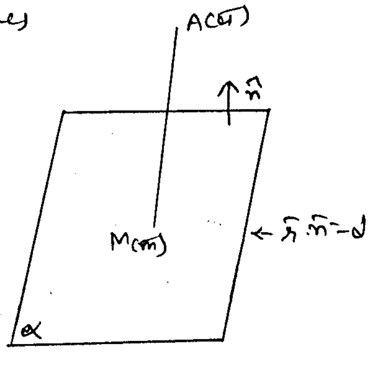
$\therefore \boxed{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}}$ which is the condition for the $\parallel^{el}$ planes

If $\bar{n}_1\cdot\bar{n}_2=0$ then the planes are $\perp^{ar}$ to each other

$$\therefore (a_1,b_1,c_1)\cdot(a_2,b_2,c_2)=0$$

$\therefore \boxed{a_1a_2+b_1b_2+c_1c_2=0}$ is the reqd. condition for the perpendicular planes

OR



(2) let $\bar{r}\cdot\bar{n}=d$ be the equation of a given plane α and $A(\bar{a})$ be a point in R^3. where $A\notin\alpha$.

Now we draw a $\perp^{ar}$ from A on α which intersect at M

$\therefore \overline{AM}\perp\alpha$

Now $M(\bar{m})\in\alpha$ and $\bar{n}$ is the normal to the plane, then the equation of the line thro' A $\perp^{ar}$ to the plane α is

$$\bar{r}=\bar{a}+k\bar{n},\quad k\in R$$

Now $M(\bar{m}) \in \alpha$ as well as on the line $\overleftrightarrow{AM}$. (15)

so we get $\bar{m}\cdot\bar{n} = d$ —(1)

and $\bar{m} = \bar{a} + k_1\bar{n}$ where k_1 is a definite real no. } (2)

$\therefore$ substitute the value of (2) in (1)

$(\bar{a}+k_1\bar{n})\cdot\bar{n} = d.$

$\therefore \bar{a}\cdot\bar{n} + k_1\bar{n}\cdot\bar{n} = d$

$\therefore k_1 = \dfrac{d-(\bar{a}\cdot\bar{n})}{|\bar{n}|^2}$ which is unique

So the foot of $\perp^{ar}$ from A on the plane is a unique point.

Now $AM = \perp^{ar}$ distance from A to plane α

$\therefore AM = |\bar{m}-\bar{a}|$

$= |k_1\bar{n}|$

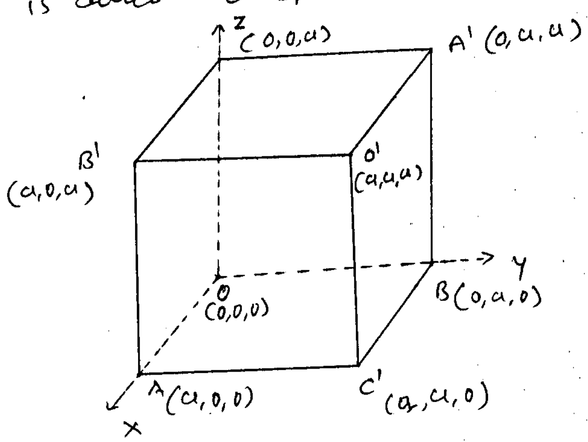
$= \left|\dfrac{d-(\bar{a}\cdot\bar{n})}{|\bar{n}|^2}\right| \cdot |\bar{n}|$

$= \dfrac{|d-(\bar{a}\cdot\bar{n})|}{|\bar{n}|}$

which is the length of $\perp^{ar}$ or $\perp^{ar}$ distance from A to the plane α

(B) (1) Sphere:- The set of all points of R^3 which are at the constant distance from a given fixed point is called a sphere

(2)



Take origin O as a one vertex of the cube, $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ as the +ve direction of the x-axis, y-axis and z-axis resp.

sides of cube are $OA = OB = OC = a$ (sup.)

$\therefore$ we get four diagonals $\vec{AA'}, \vec{BB'}, \vec{CC'}, \vec{OO'}$

$\therefore$ The diagonal vector $\vec{OO'} = (a,a,a) - (0,0,0) = (a,a,a)$

similarly, the diagonal vector $\vec{AA'} = (0,a,a) - (a,0,0)$
$= (-a,a,a)$

the diagonal vector $\vec{BB'} = (a,-a,a)$

the diagonal vector $\vec{CC'} = (a,a,-a)$

Also suppose l, m, n are the direction cosine of a given line let $\bar{x} = (l,m,n)$ $\Rightarrow l^2+m^2+n^2=1$

Here a line makes an angle $\alpha, \beta, \gamma, \delta$ with the diagonals $\vec{OO'}, \vec{AA'}, \vec{BB'}, \vec{CC'}$ of a cube resp.

$$\therefore \cos\alpha = \frac{\vec{OO'} \cdot (l,m,n)}{|\vec{OO'}|\,|\bar{x}|} = \frac{(a,a,a)\cdot(l,m,n)}{\sqrt{3}a\sqrt{l^2+m^2+n^2}}$$

$$= \frac{al+ma+na}{\sqrt{3}\,a}$$

$$= \frac{a(l+m+n)}{\sqrt{3}\,a}$$

$$= \frac{l+m+n}{\sqrt{3}}$$

Similarly $\cos\beta = \dfrac{-l+m+n}{\sqrt{3}}$, $\cos\gamma = \dfrac{l-m+n}{\sqrt{3}}$ and

$$\cos\delta = \frac{l+m-n}{\sqrt{3}}$$

$$\text{L.H.S} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$$

$$= 1-\cos^2\alpha + 1-\cos^2\beta + 1-\cos^2\gamma + 1-\cos^2\delta$$

$$= 4 - \left[\left(\frac{l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{-l+m+n}{\sqrt{3}}\right)^2 + \left(\frac{l-m+n}{\sqrt{3}}\right)^2 + \left(\frac{l+m-n}{\sqrt{3}}\right)^2\right]$$

$$= 4 - \left[\frac{4l^2+4m^2+4n^2+4lm+4ln+4mn-4lm-4ln-4mn}{3}\right]$$

$$= 4 - \left[\frac{4(l^2+m^2+n^2)}{3}\right]$$

$$= 4 - 4/3$$

$$= 8/3 = \text{R.H.S}$$

(15)

OR

From $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{5}$ we get

(by comparing $\frac{x-x_1}{l_1} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$)

$x_1 = 1,\ y_1 = -1,\ z_1 = 2 \quad \therefore \bar{a} = (1,-1,2).,\ \bar{l} = (2,-3,5)$

From $\frac{x}{3} = \frac{y-1}{2} = \frac{z-1}{1}$, we get

$x_2 = 0,\ y_2 = 1,\ z_2 = 1 \quad \therefore \bar{b} = (0,1,1),\ \bar{m} = (3,2,1)$

Here $\bar{l} \times \bar{m} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 3 & 2 & 1 \end{vmatrix}$

$= \hat{i}(-13) - \hat{j}(-13) + \hat{k}(13)$

$= (-13, 13, 13)$

$= 13(-1,1,1)$

$|\bar{l} \times \bar{m}| = 13\sqrt{3} \neq 0$

$\therefore$ The lines are neither parallel nor coincident

Now $(\bar{a} - \bar{b}) = (1,-1,2) - (0,1,1) = (1,-2,1)$

$(\bar{a} - \bar{b}) \cdot (\bar{l} \times \bar{m}) = (1,-2,1) \cdot (-13,13,13)$

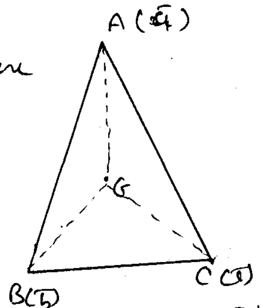
$= -13 - 26 + 13 = -26 \neq 0$

$\therefore$ lines not intersect to each other

$\therefore$ The lines are skew.

The shortest distance between them is $\left| \frac{(\bar{a}-\bar{b}) \cdot (\bar{l} \times \bar{m})}{|\bar{l} \times \bar{m}|} \right|$

$= \left| \frac{-26}{13\sqrt{3}} \right| = \frac{2}{\sqrt{3}}$ units.



(C.1) Suppose the position vectors of A, B, C are $\bar{a}, \bar{b}$ and $\bar{c}$ resp.

Here $\triangle ABC$ is an equilateral $\triangle$.

$\therefore AB = BC = CA$

$\therefore c = a = b = p$ (sup.) —(1)

$\therefore$ The position vector of centroid of $\triangle ABC$ is $\left(\frac{\bar{a}+\bar{b}+\bar{c}}{3}\right)$

and the position vector of Incentre of $\triangle ABC$ is

$\frac{a\bar{x} + b\bar{y} + c\bar{z}}{a+b+c} = \frac{p\bar{x} + p\bar{y} + p\bar{z}}{3p}$ ($\because$ by (1))

$$= \frac{p(\bar{x}+\bar{y}+\bar{z})}{3p}$$

$$= \frac{\bar{x}+\bar{y}+\bar{z}}{3}$$

= The position vector of centroid.

$\therefore$ The position vector of centroid and incentre are same

$\therefore$ For an equilateral Δ centroid and incentre are same

Now $A(6,4,6)$, $B(12,4,0)$, $C(4,2,-2)$ are the vertices of ΔABC then

$$AB = \sqrt{36+0+36} = 6\sqrt{2}$$

$$BC = \sqrt{64+4+4} = 6\sqrt{2}$$

$$CA = \sqrt{4+4+64} = 6\sqrt{2}$$

$\therefore AB = BC = CA$

$\therefore \Delta ABC$ is an equilateral Δ

$\therefore$ The centroid of ΔABC = The incentre of ΔABC

$\therefore$ The incentre of $\Delta ABC = \left(\frac{6+12+4}{3}, \frac{4+4+2}{3}, \frac{6+0-2}{3}\right)$

$$= \left(\frac{22}{3}, \frac{10}{3}, \frac{4}{3}\right).$$

(2) Here, $x^2+y^2+z^2=r^2$ is the equation of a sphere then the centre is $(0,0,0)$, and radius $= r$

Now the equation of a plane is $ax+by+cz=p$ $(p\neq 0)$

$\therefore \bar{n} = (a,b,c)$

If a sphere touches the plane then the $\perp^{ar}$ distance from the centre of a sphere to the plane = radius of a sphere

$$\therefore \frac{|\bar{a}\cdot\bar{n}-d|}{|\bar{n}|} = r$$

$$\therefore \frac{|(0,0,0)\cdot(a,b,c)-p|}{\sqrt{a^2+b^2+c^2}} = r.$$

$$\therefore \frac{|p|}{\sqrt{a^2+b^2+c^2}} = r$$

$\therefore p^2 = r^2(a^2+b^2+c^2)$ is the reqd. condition

(17)

(D) Here the equation of a plane is $2x-3y+4z=44$

$\therefore \bar{n}=(2,-3,4),\ d=44,\ \bar{a}=(2,-1,2).$

The equation of $\perp^{ar}$ line is $\bar{r}=\bar{a}+k\bar{n}$; $k\in R$

$\therefore\ \bar{r}=(2,-1,2)+k_1(2,-3,4)$; $k_1\in R$ —(1)

where $k_1=\dfrac{d-\bar{a}\cdot\bar{n}}{|\bar{n}|^2}$

here $|\bar{n}|^2=4+9+16=29$

$\therefore k_1=\dfrac{44-(2,-1,2)\cdot(2,-3,4)}{29}$

$=\dfrac{44-(4+3+8)}{29}$

$=1$

$\therefore$ by (1) the coordinates of foot of $\perp^{ar}$ one

$\bar{r}=(2,-1,2)+(2,-3,4)$

$=(4,-4,6)$ $\therefore \bar{r}=\bar{m}$ $\therefore \bar{m}=(4,-4,6)$

Now $M(\bar{m})$ & $\overleftrightarrow{AM}$

The length of $\perp^{ar}$ line is $|\overrightarrow{AM}|$

$=|\bar{m}-\bar{a}|$

$=|(4,-4,6)-(2,-1,2)|$

$=|(2,-3,4)|$

$|\overrightarrow{AM}|=\sqrt{4+9+16}=\underline{\sqrt{29}}$ units.

OR

Here $\bar{n}_1=(1,2,-3)$ $(\because x+2y-3z=6)$

$\bar{n}_2=(2,-1,1)$ $(\because 2x-y+z=17)$

$\therefore \bar{n}=\bar{n}_1\times\bar{n}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-3\\2&-1&1\end{vmatrix}$

$=\hat{i}(-1)-\hat{j}(7)+\hat{k}(-5)$

$=(-1,-7,-5)$

$\therefore (1,7,5)$ is the direction of common line of intersection of two planes. we get a point common to both planes, by taking $z=0$

$\therefore\ x+2y=6 \quad -(1)$ and $2x-y=7. \quad -(2)$

by solving (1) and (2) we get $x=4,\ y=1,\ z=0$

$\therefore\ (4,1,0)$ is the common point of two planes.

$\therefore$ The equation of common section of planes are

$$\boxed{\frac{x-4}{1}=\frac{y-1}{7}=\frac{z}{5}.}$$

* * *