

CAREERS 360

JKBOSE CLASS 12th MATHEMATICS

MODEL QUESTION PAPER

Long Answer Questions

Q1. Define symmetric and skew symmetric matrices. Prove that every square can be expressed as a sum of symmetric and skew symmetric matrices.

Or

Define determinant of a matrix and using the properties of determinants
 Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3 \quad \text{06 marks}$$

Q2. Define differentiability and continuity of a function. What is the relationship between them, justify your answer.

Or

If $(x-a)^2 + (y-b)^2 = c^2$ for some $c > 0$, prove that

$$\frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\frac{d^2 y}{dx^2}}, \text{ is a constant independent of } a \text{ and } b! \quad \text{6 marks}$$

Q3. Evaluate $\int e^{ax} \sin bx \, dx$

$$\text{Hence, deduce } I = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left(bx + \tan^{-1} \frac{b}{a} \right) + c$$

Or

Define definite integral of a function and find the area under $y = (x + c^{2x})$ between the limits 0 and 4: 6 marks

Q4. Determine graphically the minimum value of the objective function $Z = -50x + 20y$ subject to the constraints.

$$2x-y \geq -5, \quad 3x+y \geq 3, \quad 2x-3y \leq 12, \quad x \geq 0, y \geq 0$$

Or

One kind of cake requires 200 kg of flour and 25g of fat and another kind of requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5Kg of flour and 1Kg of ft assuming that there is no shortage of the other ingredients used in making the cakes. 6 marks

Q5. Define skew lines. Find the shortest distance between two skew lines with equations

$$\vec{r} = \vec{a}_1 + \pi \vec{b}_1 \text{ and}$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2 \text{ in vector form}$$

Or

For any two vectors \vec{a} and \vec{b}

$$\left| \vec{a} + \vec{b} \right| \leq \left| \vec{a} \right| + \left| \vec{b} \right| \text{ prove it and if } \left| \vec{a} + \vec{b} \right| = \left| \vec{a} \right| + \left| \vec{b} \right|, \text{ then what happens?}$$

6 marks

Short Answer Type Questions

Q6 Define bijective function and show that the function $F : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 + 1$ is a bijection. **4 marks**

Q7. Define reflexive, symmetric and transitive relation with an example to each. **4 marks**

Q8. Using elementary transformation. Find the inverse of $\begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix}$. **4 marks**

Q9. State Lagrange's Mean value theorem and interpret geometrically. **4 marks**

Q10 The side of a square sheet of metal is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long? **4 marks**

Q11. Evaluate $\int \frac{1}{1 + \tan x} dx$ **4 marks**

Q12 Find the equation of tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_0, y_0) . **4 marks**

Q13. Find the equation of the plane passing through the points (1, 2, 3) and perpendicular to the plane

$$\begin{aligned} 2x + 3y + 4z - 5 &= 0 \\ 4x + 6y + 8z - 15 &= 0 \end{aligned} \quad \text{4 marks}$$

Q14. Prove by vector method an angle in a semi circle is a right angle. **4 marks**

Q15. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. **4 marks**

Very Short Answer Type

Q16. If $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x . **2 marks**

Q17. Find x and y if,

$$2 \begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} y & 0 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 1 & 8 \end{vmatrix} \quad \text{2 marks}$$

Q18. Is the function defined by

$$f(x) = \begin{cases} x+5, & \text{if } x \leq 1 \\ x-5 & \text{if } x > 1 \end{cases}$$

2 marks

Q19. Find $\frac{dy}{dx}$ if $2x + 3y = \sin y$

2 marks

Q20. Evaluate $\int \sec x \, dx$

2 marks

Q21. State 2nd fundamental theorem of integral calculus.

2 marks

Q22. Solve $xy \frac{dy}{dx} = e^x$

2 marks

Q23. Prove that the scalar product between the given vector is commutative

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} \quad \forall \quad a_1, b_1 \in \mathbb{R}$$

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} \quad a_2, b_2 \in \mathbb{R}$$

2 marks

Q24. A die is rolled, if the outcome is an even number. What is the probability that it is prime number?

2 marks

Q25. Evaluate $P(A \cup B)$

$$\text{If } 2P(A) = P(B) = \frac{5}{13}$$

$$\text{and } P\left(\frac{A}{B}\right) = \frac{2}{5}$$

2 marks

Objective Type questions

Q26. When two coins are tossed, what is the probability of at most two heads?

- a) 1
- b) -1
- c) 0
- d) None of this

1 mark

Q27. If A and B are two $n \times n$ non-singular matrices, then

- a) AB is non-singular
- b) AB is singular
- c) $(AB)^{-1} = A^{-1} B^{-1}$
- d) $(AB)^{-1}$ does not exist

1 mark

Q28. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \infty$ then $\frac{dy}{dx} =$

a) 0 b) 1 c) x d) y 1 mark

Q29. $\frac{d}{dx}(\tan^{-1} x + \cos^{-1} x)$ is equal to
a) $\frac{1}{1+x^2}$ b) 0 c) $\frac{-1}{1+x^2}$ d) None of these 1 mark

Q30. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to
a) $\tan x + \cot x + c$ b) $\tan x - \cot x + c$
c) $\tan x + \cot 2x + c$ d) $\tan x - \cot 2x + c$ 1 mark

Q31. $\int \frac{1}{e^x + 1} dx$ is equal to
a) $\log(e^x + 1) + c$ b) $\log(1 + e^x) + c$
c) $-\log(1 + e^{-x}) + c$ d) None of these 1 mark

Q32. The differential equation of all circles of radius a is of order
a) 2 b) 3 c) 4 d) none of these 1 mark

Q33. $|a \times b|$ is equal to
a) $ab \sin \theta$ b) $ab \cos \theta$
c) $ab \sin \theta \hat{n}$ d) none of these 1 mark

Q34. Solution of the differential equation $Xdy - Ydx = 0$ represents
a. A rectangular hyperbola
b. A straight line passing through origin
c. Parabola whose vertex is at the origin
d. Circle whose centre is at the origin 1 mark

Q35. If $A + B$ are independent events $P(A \cup B)$ is equal to
a) $P(A) + P(B)$ b) $P(A) + P(B) - P(A \cap B)$
c) $P(A) \cdot P(B)$ d) None of these 1 mark